

Unit 25

a) $\int x e^x dx$

u
 dv

$= x e^x - \int x \cdot e^x dx$

$\frac{d}{dx} x = 1$

$= x e^x - e^x + C$

Method

$v' = \frac{dv}{dx}, dv = v'dx$

$\int (uv)' dx = \int u \cdot v' dx + \int v u' dx$

$uv = \int u dv + \int v du$

Solve for:

$u' = \frac{du}{dx}$

$du = u' dx$

$\int u dv = uv - \int v du$

b) $\int x \log x \, dx$

Diagram: A green oval encloses the entire integrand. A pink oval encloses $\log x$ with a pink u above it. A red arrow points down from $\log x$ to the text below. A red arrow points up from x to the text below. A green dv is written to the right of the integral.

differentiale a log (make a
fixe!)

Diagram: A green oval encloses $\frac{x^2}{2} \log x$ with a green v above it. A green oval encloses $\int \frac{x^2}{2} \frac{1}{x} dx$ with a green u above it. A red $\frac{1}{x}$ is written to the right of the integral. A green du is written below the integral.

$$\frac{x^2}{2} \log x - \int \frac{x}{2} dx$$

Final result in a yellow box:

$$\frac{x^2}{2} \log x - \frac{x^2}{4} + C$$

Diagram: A red arrow points down from the $\frac{x^2}{4}$ term to the text below.

c) $\int x^2 \cos x \, dx$

We need 2 integrals by part

$$x^2 \sin x - \int 2x \sin x \, dx$$

$$x^2 \sin(x) + 2x \cos x - \int 2 \cos x \, dx$$

$$x^2 \sin x + 2x \cos x - 2 \sin x + C$$

→ This can be done better with Tic Tac Toe.

What functions
do we
differentiate first?



every thing harder
to integrate, you
differentiate

(A)

→ TIKTAC toe

$$\int x^2 e^x dx =$$

↓ ↑

$$= \boxed{x^2 e^x - 2x e^x + 2e^x}$$

(B)

$$\int (x-3) \cos(x+5) dx$$

↓ ↑

$$(x-3) \sin(x+5) - \int 1 \cdot \sin(x+5) dx$$

$$= \boxed{(x-3) \sin(x+5) + \cos(x+5)}$$

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Prototype!

$$\int \log x \cdot 1 \, dx$$

There is only one function!
Just add!

$$\int \log x \cdot 1 \, dx$$

$$\log x \cdot x - \int \frac{x}{x} \, dx$$

$$= \log x \cdot x - \int 1 \, dx$$

$$= \log x \cdot x - x + C$$

(A) Merry go round **I**

$$\int e^x \cos x dx$$

$$e^x \sin x - \int e^x \sin x dx$$

$$e^x \sin x + e^x \cos x - \int e^{-x} \cos x dx$$

we have gone in circles!

Solve for I

$$2I = (e^x \sin x + e^x \cos x) / 2$$

$$\int \arctan(x) \cdot 1 dx$$

$$\arctan x \cdot x - \int \frac{x}{1+x^2} dx$$

LIATE

$$\int x^5 e^x dx$$

Tic Tac Toe

Different	Integrate	
x^5	e^x	
$5x^4$	e^x	+
$20x^3$	e^x	-
$60x^2$	e^x	+
$120x$	e^x	-
120	e^x	+
0	e^x	-

Write down the solution

$$\begin{aligned} & x^5 e^x - 5x^4 e^x + 20x^3 e^x \\ & - 60x^2 e^x + 120x e^x \\ & - 120 e^x + C \end{aligned}$$