$$\boxed{\text{Unit 25}}$$

u

d·v

a) $\int x\, e^x\, dx$

$\frac{d}{dx} x = 1$

$$= x e^x - \int 1 \cdot e^x\, dx$$

$$= \boxed{x e^x - e^x + C}$$

Method

$V' = \frac{dv}{dx}$, $dv = v\,dx$

$$\int (uv)'_{dx} = \int u \cdot V'\, dx + \int V u'\, dx$$

$$\boxed{uv} = \boxed{\int u\, dv} + \boxed{v\, du}$$

Solve for

$u' = \frac{du}{dx}$

$du = u'\, dx$

$$\boxed{\int u\, dv = uv - \int v\, du}$$

b) $\displaystyle\int x \underbrace{\log x}_{u} \, dx$

dv

differentiate a log (make a fire!)

$\dfrac{1}{x}$

$$\dfrac{x^2}{2} \log x - \int \dfrac{x^2}{2} \dfrac{1}{x} \, dx$$

du

$$\dfrac{x^2}{2} \log x - \int \dfrac{x}{2} \, dx$$

$$= \boxed{\dfrac{x^2}{2} \log x - \dfrac{x^2}{4} + C}$$

c) $\int x^2 \cos x \, dx$

We need 2 integs by part

$x^2 \sin x - \int 2x \sin x \, dx$

$x^2 \sin(x) + 2x \cos x - \int 2 \cos x \, dx$

$x^2 \sin x + 2x \cos x - 2 \sin x + C$

$\longrightarrow$ This can be done better with Tic Tac Toe.

what functions
do we
differentiate first?

LIATE ← arithmetic

LIPTE → thing

polynomial

log   inverse   exp

"differentiate the log"

everything harder to integrate, you differentiale

A

$$\int x^2 e^x \, dx =$$

↓ ↑

$$\cdots\cdots \quad = \quad \boxed{x^2 e^x - 2x e^x + 2 e^x}$$

B

$$\int (x-3) \cos(x+5) \, dx$$

↓ ↑

$$(x-3) \sin(x+5) - \int 1 \cdot \sin(x+5) \, dx = \boxed{(x-3)\sin(x+5) + \cos(x+5)}$$

$$\int \log x \cdot 1 \, dx$$

u

dv

There is only one function !

Just add 1

$$\int \log x \cdot 1 \, dx$$

$$\log x \cdot x - \int \frac{x}{x} \, dx$$

$$= \log x \cdot x - \int 1 \, dx$$

$$= \boxed{\log x \cdot x - x + C}$$

(A) $\int e^{x} \cos x \, dx$ (Merry go round) I

$\downarrow$ $\uparrow$

$e^{x} \sin x - \int e^{x} \sin x \, dx$

$\downarrow$ $\uparrow$

$e^{x} \sin x + e^{x} \cos x - \int e^{x} \cos x \, dx$ I

we have gone in circles!

Solve for I

$2I = (e^{x} \sin x + e^{x} \cos x)/2$

$\int \arctan(x) \cdot 1 \, dx$

$\downarrow$ $\uparrow$

$\arctan x \cdot x - \int \left(\frac{x}{1+x^2} \right) dx$ LIATE

$$\int x^5 e^x \, dx$$

Tic Tac Toe

Differenti	Integrale	
x^5	e^x	
$5x^4$	e^x	+
$20x^3$	e^x	−
$60x^2$	e^x	+
$120x$	e^x	−
120	e^x	+
0	e^x	−

Write down the solu

$$x^5 e^x - 5x^4 e^x + 20x^3 e^x$$
$$- 60 x^2 e^x + 120 x e^x$$
$$- 120 e^x + c$$