Unit 24

1. $\int e^{x+1}\,dx$
2. $\int \cos(x^2)2x\,dx$
3. $\int \frac{x}{1+x^2}\,dx$

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1. $\int e^{x+1}\,dx =$

Substitute

$u = x+1$
$du = dx$

$= \int e^u\,du = e^u + C$

Integrate

$= \boxed{e^{x+1} + C}$

Backsubstitute

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2. $\int \cos(x^2)\,2x\,dx =$

$u = x^2$
$du = 2x\,dx$

$= \int \cos(u)\,du = \sin u + C$

$= \boxed{\sin(x^2) + C}$

③ $\int \frac{x}{1+x^2}\, dx$

mean of the Cauchy distribution

$$= \int \frac{du}{u \cdot 2} = \frac{\log u}{2} + C$$

$u = 1+x^2$

$du = 2x\,dx$

$dx = \frac{du}{x \cdot 2}$

$$= \boxed{\frac{\log(1+x^2)}{2} + C}$$

$u = 1 + x^2$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

A bit more difficult

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1. $\int \sqrt{x+2}\, dx$
2. $\int \sin(x^5) x^4\, dx$
3. $\int \sin(\sin x)\cos x\, dx$

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1. $\int \sqrt{x+2}\, dx$

$$\boxed{\begin{aligned} u &= x+2 \\ du &= dx \end{aligned}}$$

$$= \int \sqrt{u}\, du = u^{3/2} \cdot \frac{2}{3} + C$$

$$= (x+2)^{3/2} \cdot \frac{2}{3} + C$$

2. $$\int \sin(x^5)\, x^4\, dx$$

$u = x^5$

$du = 5x^4\, dx$

$dx = \dfrac{du}{5x^4}$

$$= \int \sin u \cdot \frac{du}{5\cancel{x^4}} \cancel{x^4}$$

Cancel

$$= -\frac{\cos u}{5} + C$$

$$= \boxed{-\frac{\cos(x^5)}{5} + C}$$

③ $\int \sin(\sin(x))\cos x \, dx$

$\int g(u(x))\, u'(x)\,dx$

$= G(u(x))$

chain rule

insight for the method

$$u = \sin x$$
$$du = \cos x \, dx$$
$$dx = \frac{du}{\cos x}$$

$$= \int \sin u \, du$$

$$= -\cos u + C$$

$$= \boxed{-\cos(\sin x) + C}$$

Harder!

$$+ \; A \int \frac{1+x}{1+x^2}\,dx$$

$$- \; B \int e^{-4x^2} x \, dx$$

$$+ \; C \int \frac{1+x}{1-x^2}\,dx$$

A **Split it up!**

$$\int \frac{1}{1+x^2}\,dx + \int \frac{x}{1+x^2}\,dx$$

$$= \arctan(x) + \int \frac{du}{u \cdot 2}$$

$u = x^2+1$

$du = 2x\,dx$

for second int

$$= \arctan(x) + \frac{\log u}{2} + c$$

$$= \arctan(x) + \log(1+x^2)\,c$$

$$du = 2x\,dx$$

$$\boxed{dx = \frac{du}{2x}}$$

$$\int \frac{x\,dx}{1+x^2} = \int \frac{\cancel{x}}{u}\,\frac{du}{2\cancel{x}}$$

$$= \int \frac{du}{u} + C$$

B

without details

$$\frac{e^{-4x^2}}{8} + C$$

$u = -4x^2$

$du = -8x\,dx$

do this [illegible]

C

$$\int \frac{1+x}{1-x^2}\,dx$$

$$= \int \frac{\cancel{1+x}}{(1-x)\cancel{(1+x)}}\,dx$$

$$= \int \frac{1}{1-x}\, dx$$

$$= -\int \frac{1}{u}\, du$$

$u = 1-x$

$du = -dx$

$$= -\log(u) + c$$

$$= -\log(1-x) + c$$

If you do not see this

$$\int \frac{1}{1-x^2}\, dx + \int \frac{x}{1-x^2}\, dx$$

$$\int \frac{1/2}{1-x} + \frac{1/2}{1+x}\,dx$$

try: $u = 1 - x^2$

$du = -2x\,dx$

hw10(a)

A $\displaystyle\int \frac{x}{1+x^4}\,dx$ du

B $\displaystyle\int \frac{x^3}{\sqrt{1+x^2}}\,dx$

A

$u = x^4$ $\quad u = 1 + x^4$

$\rightsquigarrow$ difficult deadend.

$u = x^2$

$du = 2x\,dx$

$dx = \frac{du}{2x}$

$$\int \frac{du}{2(1+u^2)}$$

$$= \frac{1}{2}\arctan(u) + C$$

$$= \frac{1}{2}\arctan(x^2) + C.$$

3

$$u = x^2$$
$$du = 2x\,dx$$
$$dx = \frac{du}{2x}$$

$$\int \frac{x^{\cancel{3}\ 2}}{\sqrt{1+u}}\,\frac{du}{2\cancel{x}}$$

$$= \int \frac{x^2}{\sqrt{1+u}}\,du$$

$$= \int \frac{u}{\sqrt{1+u}}\,du$$

Still stuck!

Do again a u substitution:

$$v = 1+u$$
$$dv = du$$

$$= \int \frac{v-1}{\sqrt{v}}\, dv$$

What do we do now!

Splitting it up!

$$\int \sqrt{v} - \frac{1}{\sqrt{v}}\,dv$$

$$= v^{3/2}\cdot\frac{2}{3} - 2\sqrt{v} + C$$

$$= (1+u)^{3/2}\cdot\frac{2}{3} - 2\sqrt{1+u} + C$$

$$\boxed{= (1+x^2)^{3/2}\,\frac{2}{3} - 2\sqrt{1+x^2} + C}$$