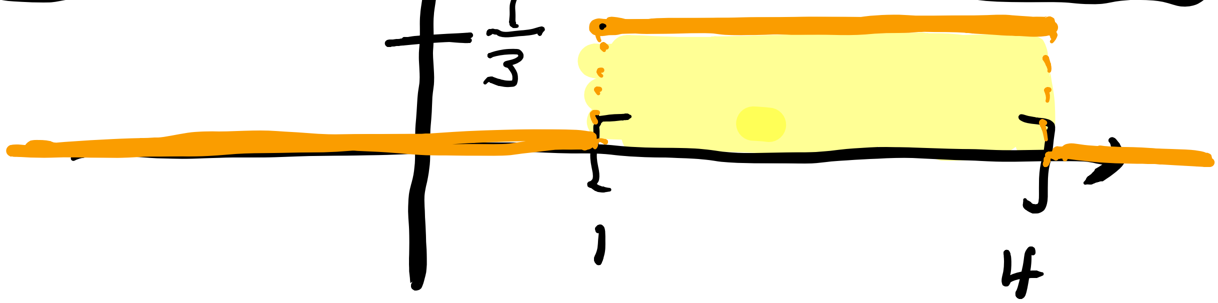
Unit 23

a)

uniform distribution



$$f(x) = \begin{cases} \frac{1}{3}, \\ 0 & \text{if } x < 1 \text{ or } x > 4 \end{cases}$$

This is a PDF:

a) piecewise cont ✓

b) ≥ 0

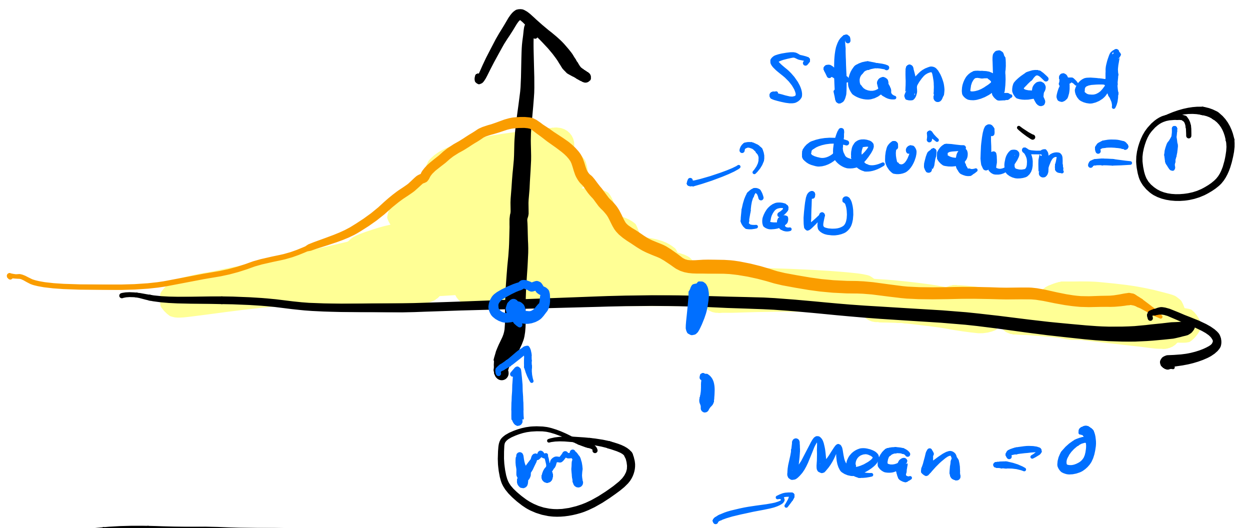
c) $\int_{-\infty}^{\infty} f(x)\,dx$

split it up

$$= \int_{-\infty}^{1} 0\,dx + \int_{1}^{4} \frac{1}{3}\,dx + \int_{4}^{\infty} 0\,dx$$

b) **Normal distribution**

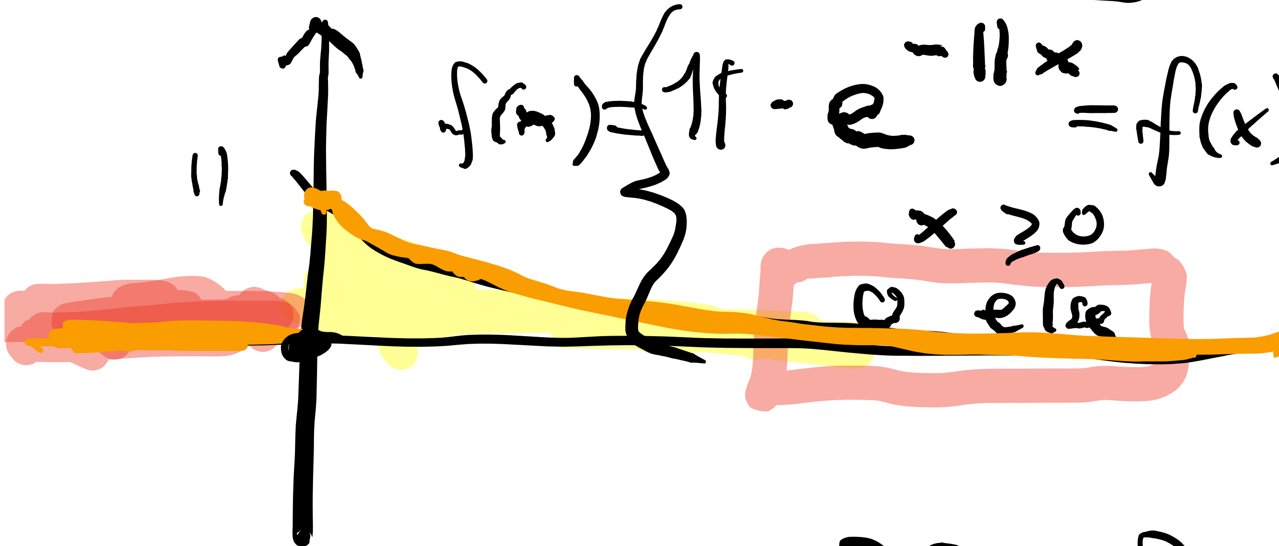
$$\boxed{f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}}$$



Theorem:

Central limit theorem justifies the importance of N(0,1)

c) $\boxed{\text{Exponential}}$



$$f(x) = \begin{cases} 11 \cdot e^{-11x} = f(x) & x \geq 0 \\ \boxed{0 \quad \text{else}} & \end{cases}$$

Is this a PDF?

a) p.c. ✓

b) ≥ 0 ✓

c)

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

Check this!

Split up:

$$\int_{-\infty}^{0} \boxed{0}\, dx + 11\int_{0}^{\infty} e^{-11x}\, dx$$

$$= \cancel{11} \cdot \frac{-e^{-11x}}{\cancel{11}} \Big|_{0}^{\infty}$$

improper integral

$$= e^{-11\cdot 0} - e^{-11\cdot\infty}$$

If you are a "pro",
you compute without limit

$$\lim_{b\to\infty} 11 \int_0^b e^{-11x}\,dx$$

$$\lim_{b\to\infty} 11 \left.\frac{e^{-11x}}{11}\right|_0^b$$

$$= \ldots = 1$$

proper way

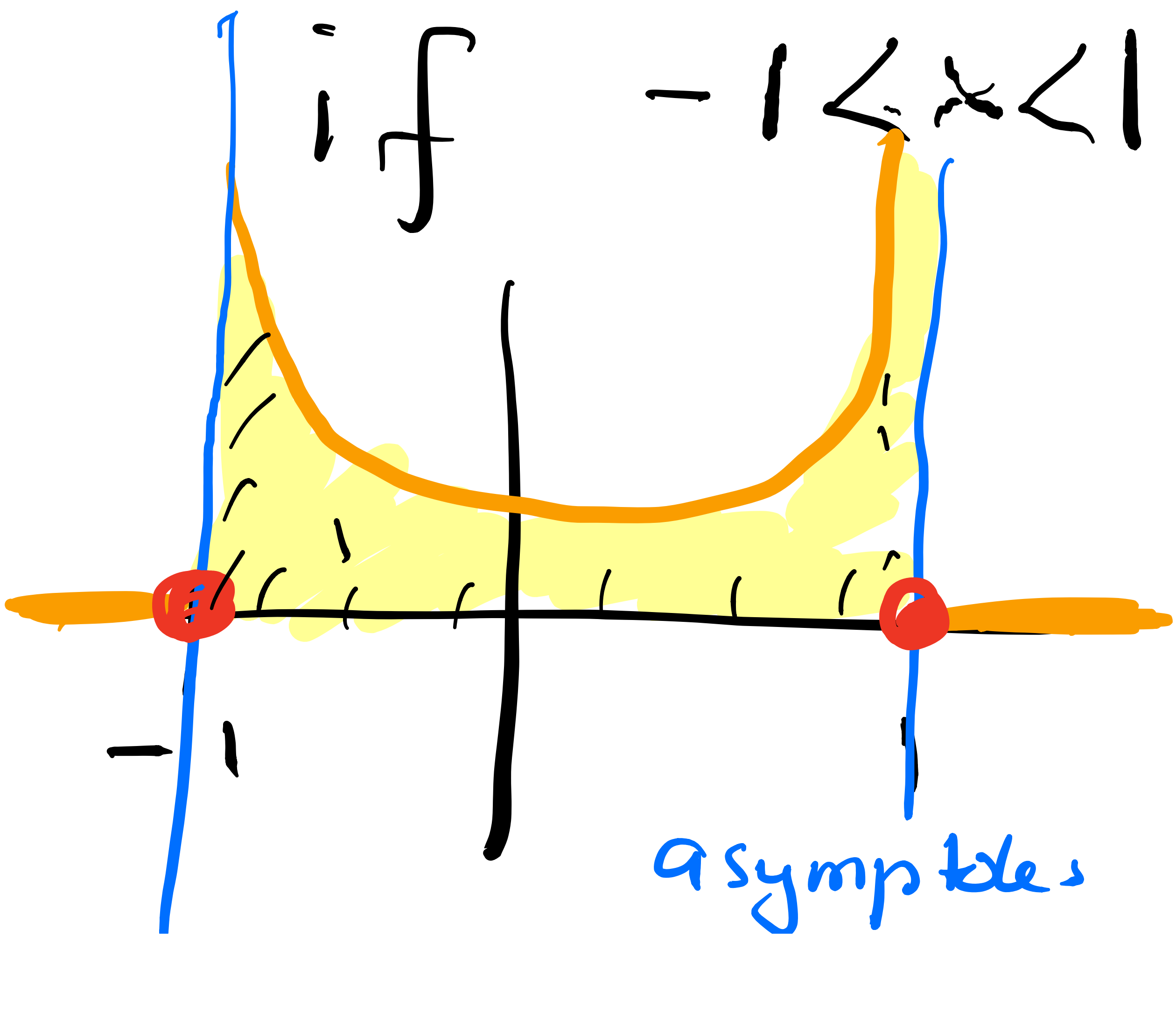
Waiting times.

wait for a bus.

d) arcsin

$$f(x) = \frac{1}{\pi} \frac{1}{\sqrt{x^2-1}}$$

if $-1 < x < 1$



in HW 22 4)
you have verified
property c)

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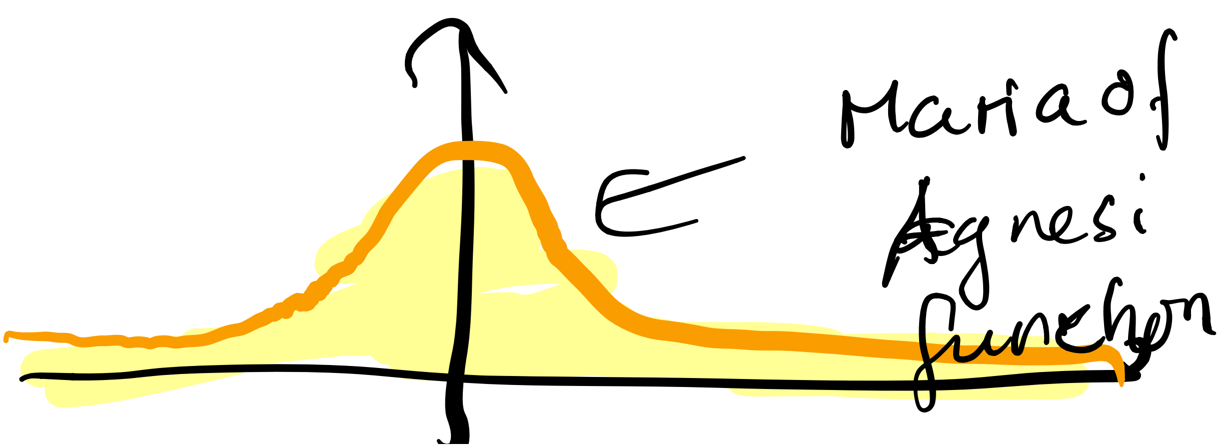
e) $\boxed{\text{Cauchy}}$

$$f(x) = \frac{1}{\pi(1+x^2)}$$



$$\int_{-\infty}^{\infty} \frac{1}{\pi(1+x^2)}\,dx$$

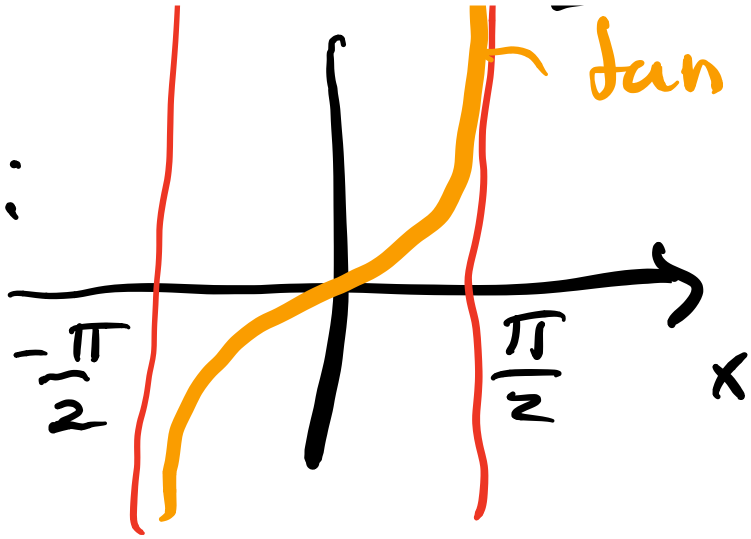
$$\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx$$

$$\frac{1}{\pi}\left(\arctan(\infty)-\arctan(-\infty)\right)$$

(without limits)

$$\frac{1}{\pi}\left(\frac{\pi}{2}+\frac{\pi}{2}\right) = \boxed{1}$$

reminder:



This is a high risk distribution.

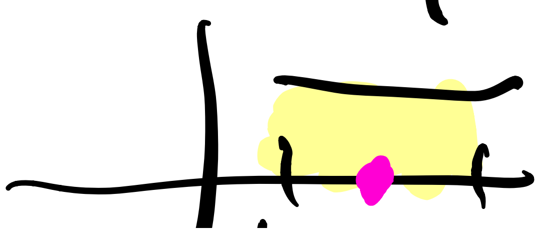
The standard deviation is ∞

Expectation

$$m = \int_{-\infty}^{\infty} x f(x)\,dx$$

Task: compute this for

$$f(x) = \begin{cases} \frac{1}{3} & 1 \leq x \leq 4 \\ 0 & \text{else} \end{cases}$$



PDF

What is m?

$$m = \frac{5}{2}$$

$$\cancel{\int_{-\infty}^{1} x \cdot 0 \, dx} + \int_{1}^{4} x \cdot \frac{1}{3} \, dx + \cancel{\int_{4}^{\infty} x \cdot 0 \, dx}$$

$$= \frac{x^2}{6} \Big|_{1}^{4} = \frac{16-1}{6}$$

$$= \frac{15}{6} = \boxed{\frac{5}{2}}$$