

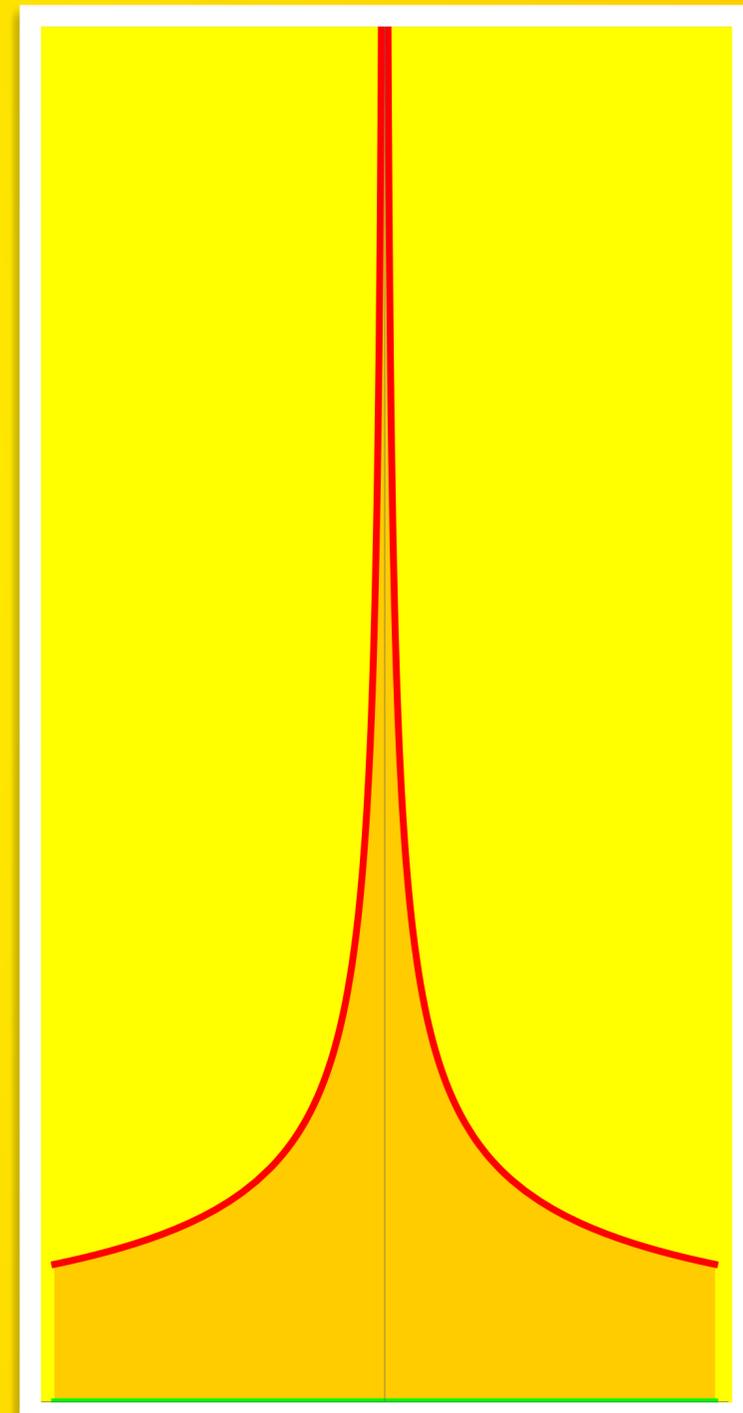
22

Improper Integrals



$$\int_a^{\infty} f(x) dx$$

$$\int_{-1}^1 \frac{1}{\sqrt{|x|}} dx$$



PLAN

1. Poll

2. Improper Types

3. A shocking example

4. The trumpet

5. Jam

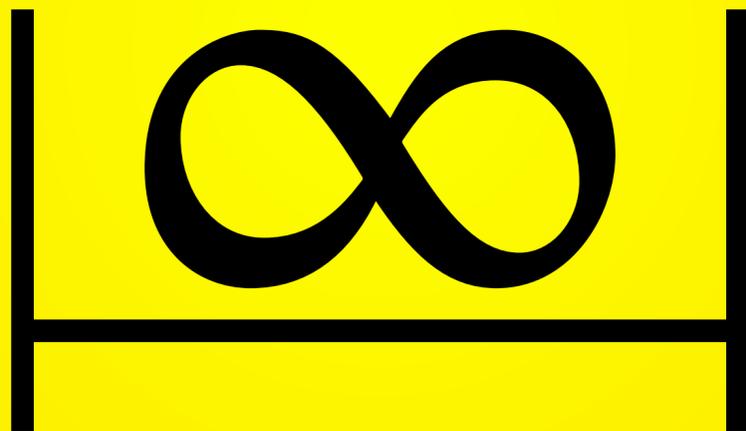
Poll

Guess!

*Do you think
the integral
to the right is
finite or
infinite?*

$$\int_{-1}^1 x^{-2} dx$$

Infinity



Improper integral A

Region is infinite

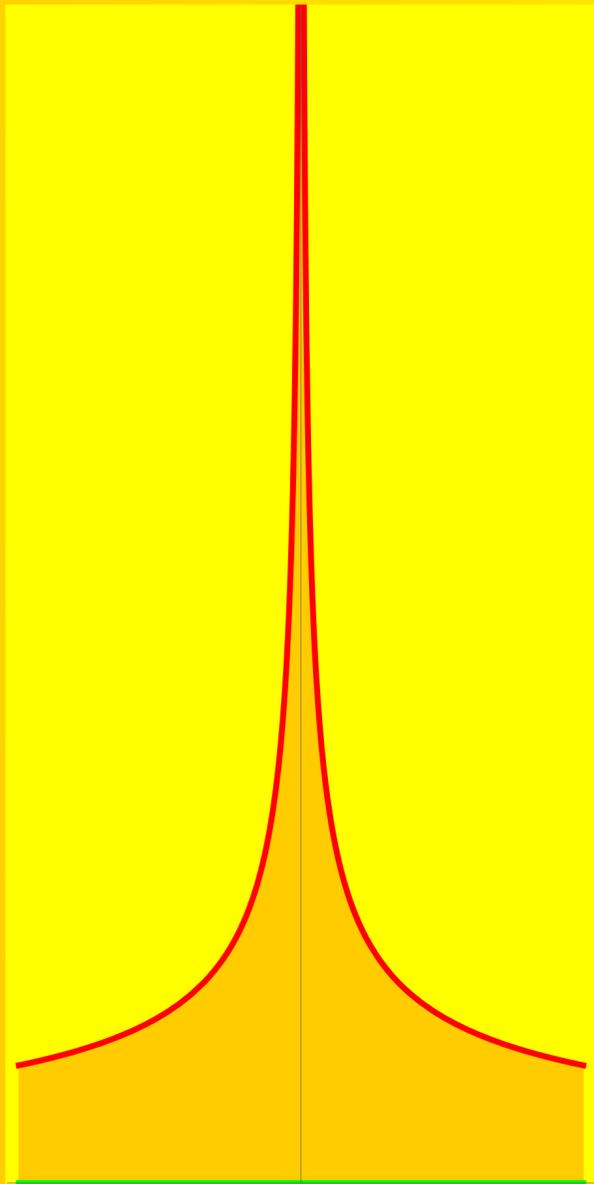


$$\int_a^{\infty} \sin(x) dx$$

Example

Improper integral B

Function is unbounded



$$\int_{-1}^1 \frac{1}{x^3} dx$$

Why?

In Stats, many distributions are defined on unbounded domains or are unbounded

$$\frac{e^{-x^2/2}}{\sqrt{2\pi}}$$

$$\frac{1}{\sqrt{1-x^2}} \frac{1}{\pi}$$

A journal



2011 edition



2020 edition

THE 1A ISSUE *March 12-25, 2021

the Improper Integral

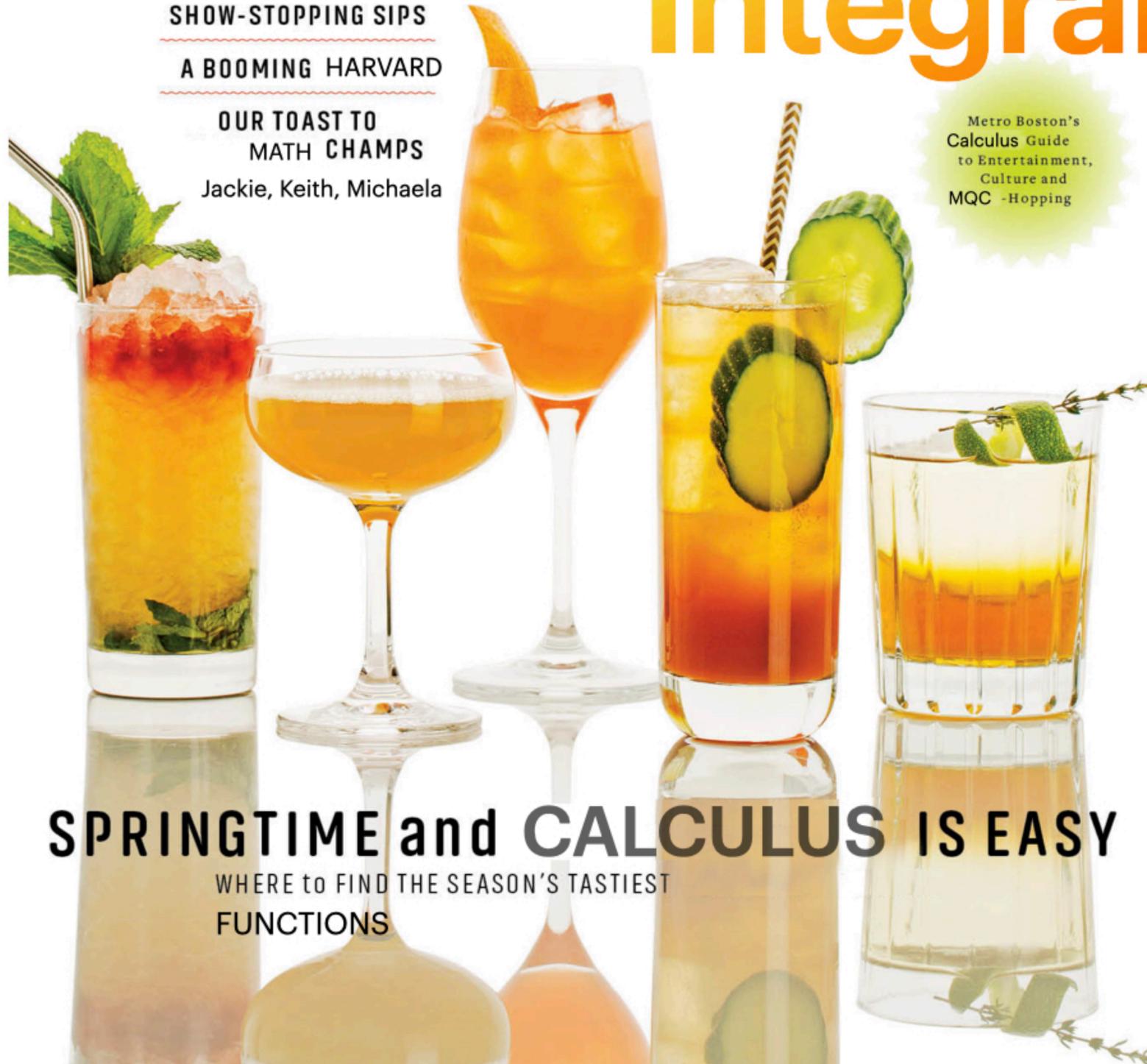
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A BOOMING HARVARD

OUR TOAST TO
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Jackie, Keith, Michaela

Metro Boston's
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Culture and
MQC -Hopping



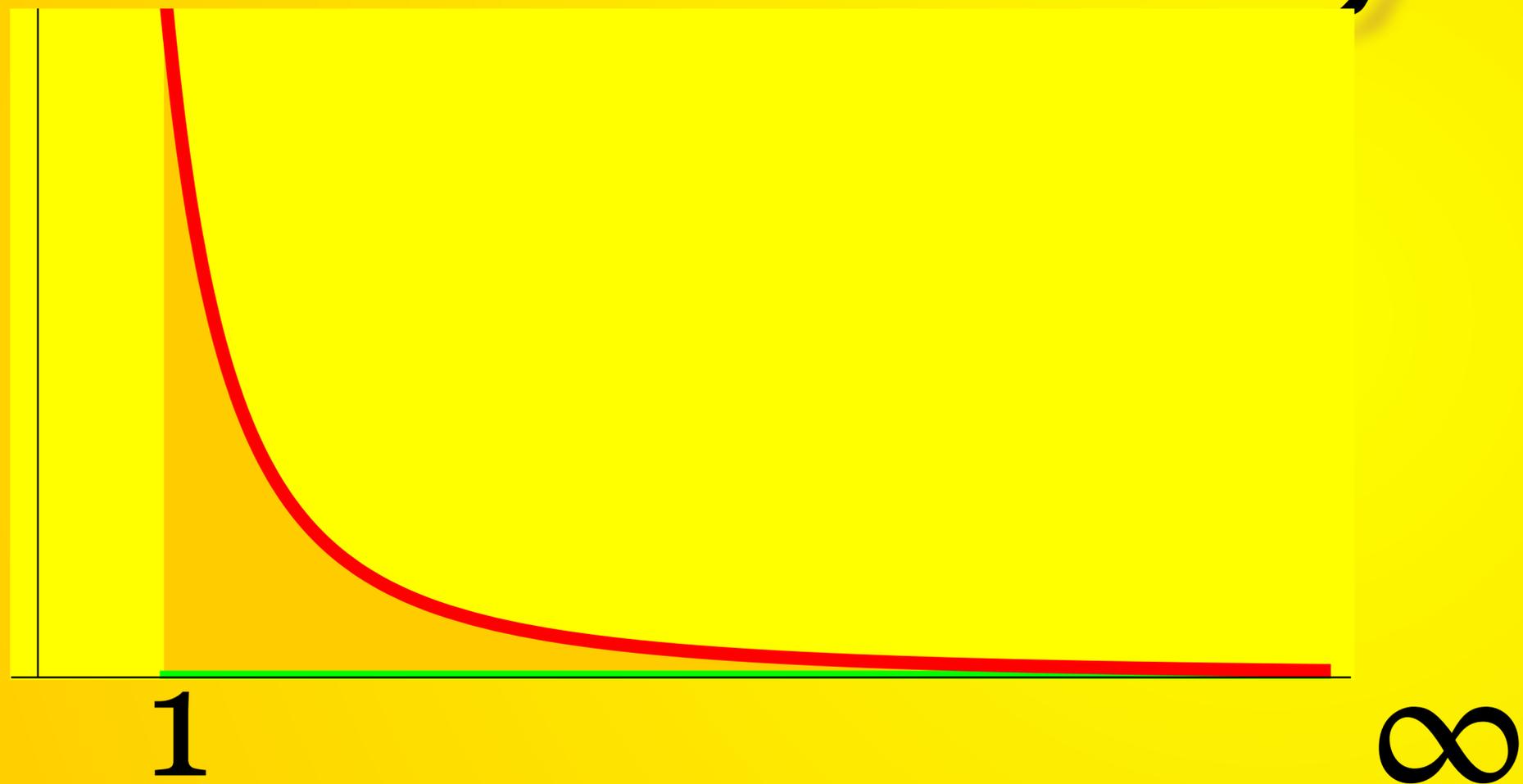
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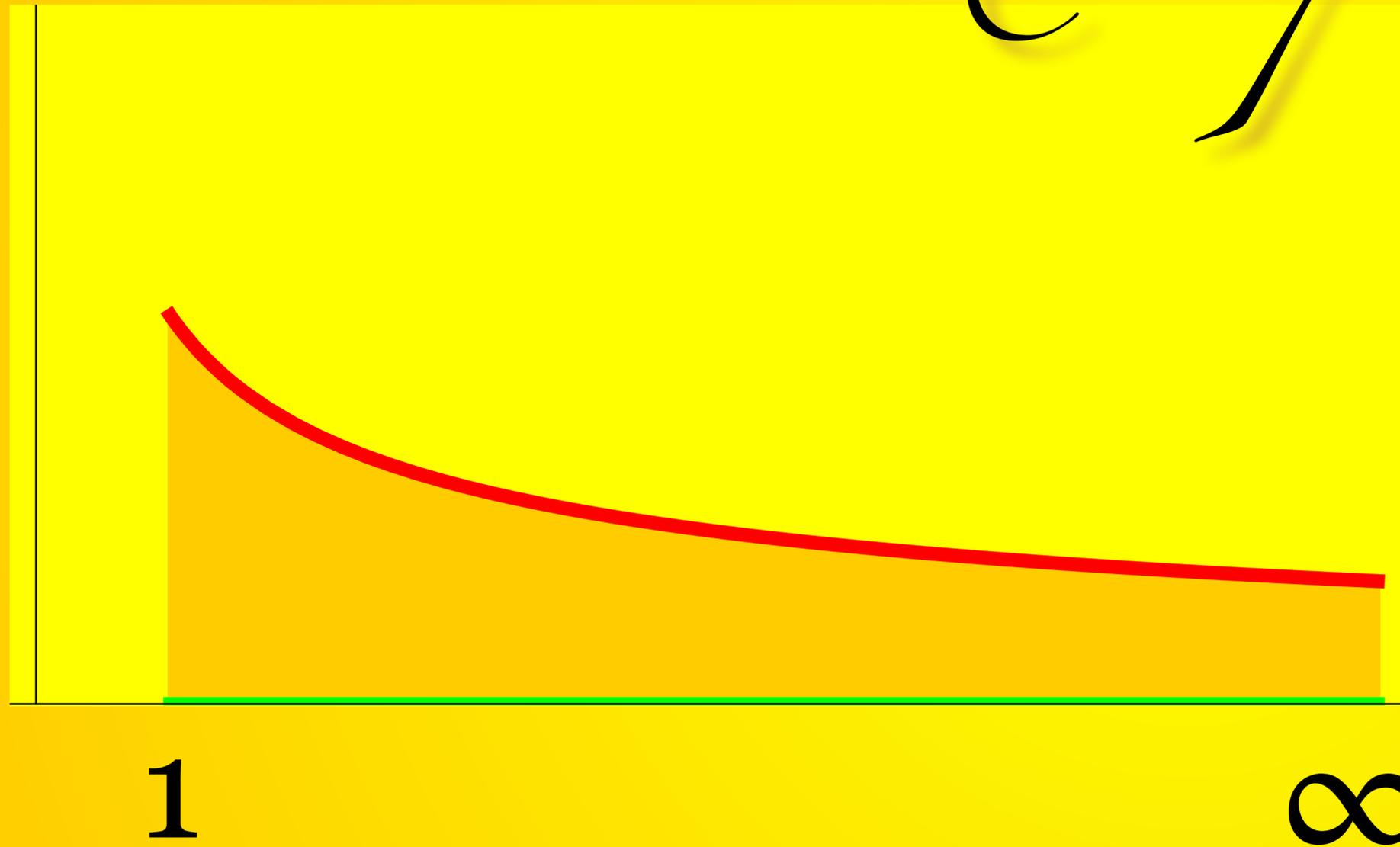
Example 1



$$\int_1^{\infty} \frac{1}{x^3} dx$$

Region is unbounded

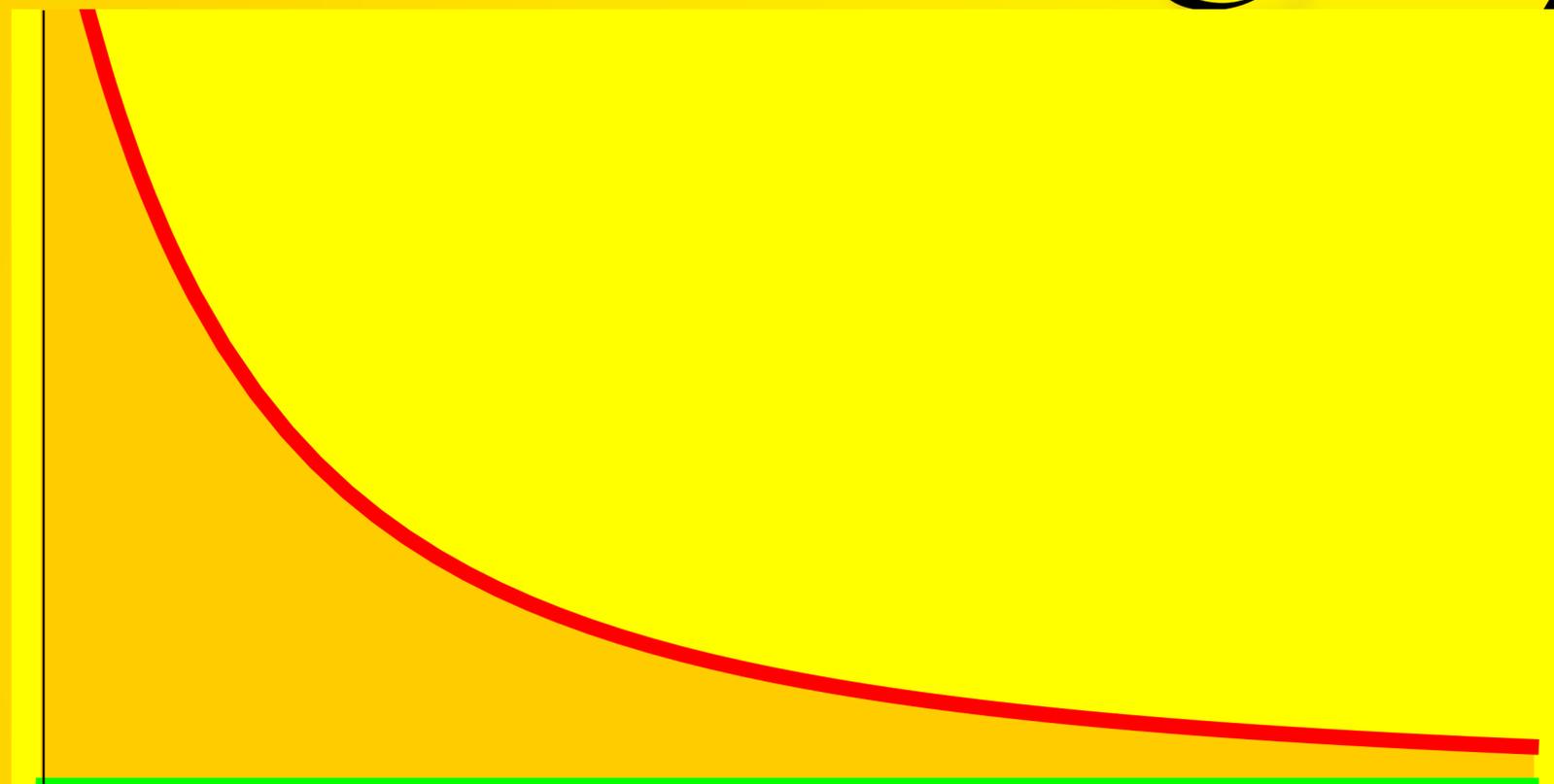
Example 2



$$\int_1^{\infty} \frac{1}{x^{1/3}} dx$$

Region is unbounded

Example 3



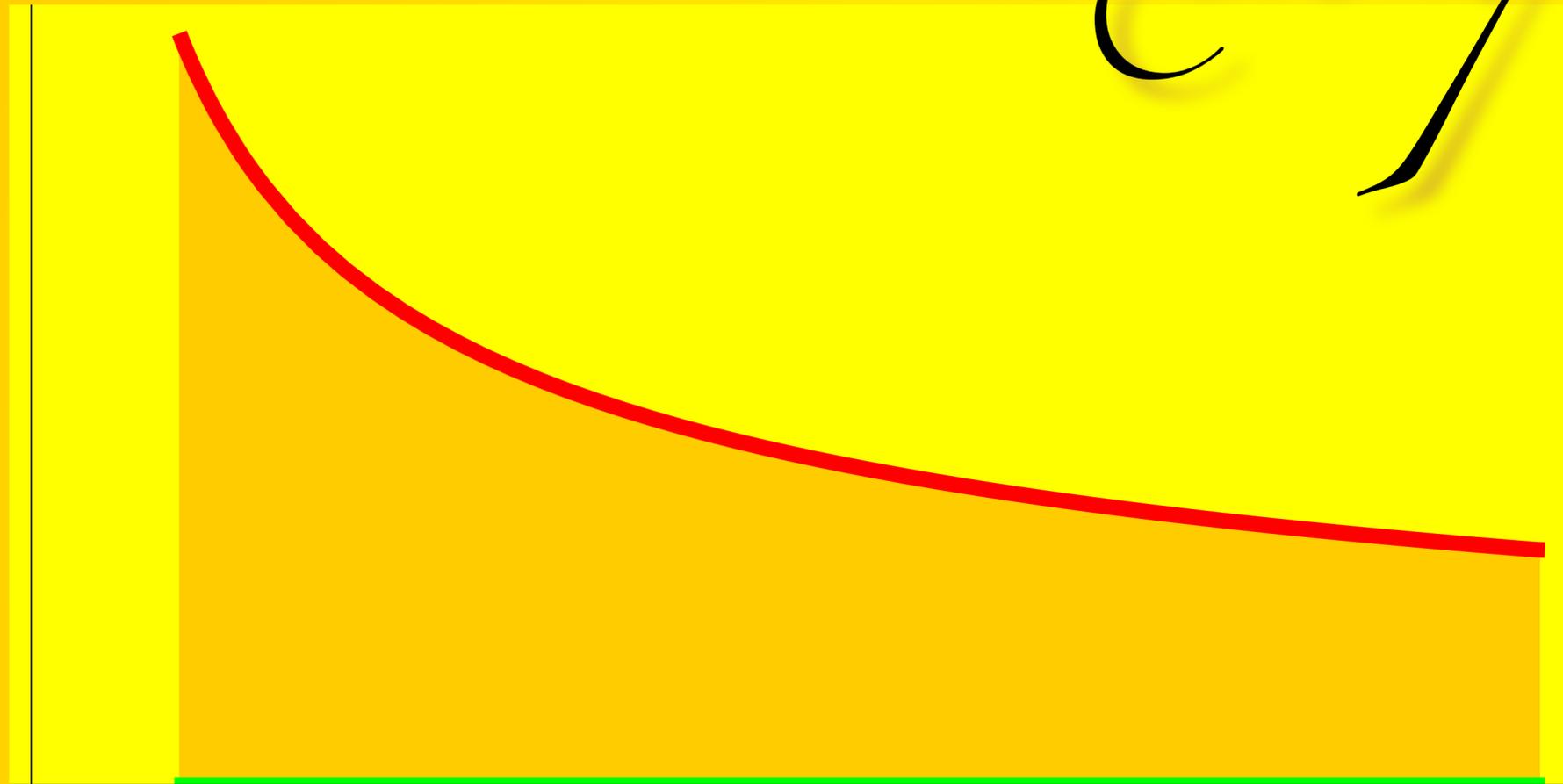
$$\int_0^1 \frac{1}{x^3} dx$$

0

1

Function is unbounded

Example 4



$$\int_0^1 \frac{1}{x^{1/3}} dx$$

0

1

Function is unbounded

b

The Trumpet

$$r = \frac{1}{x}$$

1



Surface

Area \geq

$$\int_1^b \frac{2\pi}{x} dx$$

Volume:

$$\int_1^b \frac{\pi}{x^2} dx$$

See jam

Jam

$$\int_1^{\infty} \frac{1}{x^2} dx$$

$$\int_0^1 \frac{1}{x^2} dx$$

finite

$$\int_1^{\infty} \frac{1}{\sqrt{x}} dx$$

$$\int_0^1 \frac{1}{\sqrt{x}} dx$$

or

infinite?

$$\int_1^{\infty} \frac{1}{x} dx$$

$$\int_0^1 \frac{1}{x} dx$$

b



The Trumpet

Punch line!

1

The End