① $\int_1^\infty \frac{1}{x^3}\,dx$ Improper

x^{-3}

$= \lim_{b\to\infty} \int_1^b \frac{1}{x^3}\,dx$ Proper

f

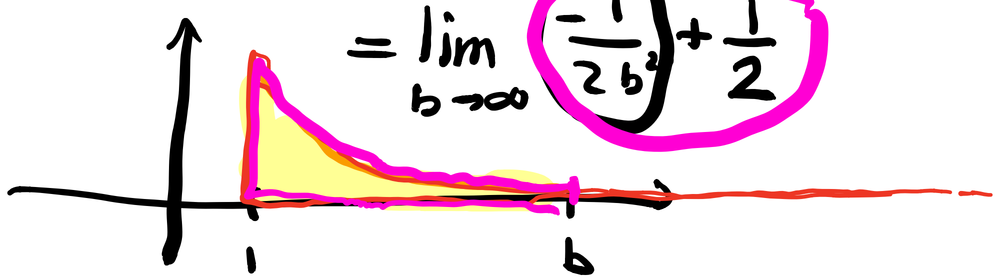
$F' = f$ F

$= \lim_{b\to\infty} \left. \frac{-1}{2x^2} \right|_1^b$

this is the area from 1 to b

0

$= \lim_{b\to\infty} \left(\frac{-1}{2b^2}\right) + \frac{1}{2}$



What happens if $b \to \infty$

The limit is $\boxed{\frac{1}{2}}$

We know! $\lim_{b\to\infty} \frac{1}{b^2} = 0$

take this for granted

$$= \frac{1}{\infty^2} = 0$$

$$\lim_{b \to \infty} e^{-b} = 0$$

2

$$\boxed{\int_1^{\infty} \frac{1}{x^{1/3}}\, dx}$$

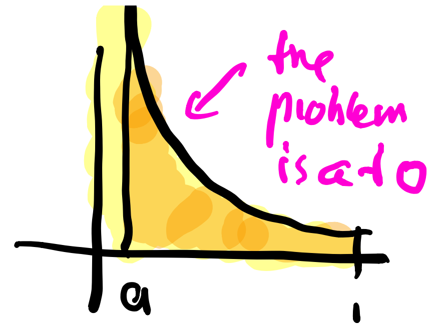
also improper.

$$= \lim_{b \to \infty} \int_1^{b} x^{-1/3}\, dx$$

$$= \lim_{b \to \infty} \frac{3}{2}\left(b^{2/3} - 1\right)$$

$$= \infty$$

③ $\boxed{\int_0^1 x^{-3}\,dx}$



$$= \lim_{a\to 0} \int_a^1 x^{-3}\,dx$$

$$= \lim_{a\to 0} -\frac{x^{-2}}{2}\Big|_a^1$$

$$= \lim_{a\to 0} \left(-\frac{1}{2} + \frac{a^{-2}}{2}\right)$$

What happens if $a \to 0$?

Goes to ∞! and beyond

This area is infinite. We call it divergent!

$$-\frac{1}{2}+\frac{1}{2a^2}=\frac{1}{2a^2}-\frac{1}{2}$$

if $a \longrightarrow 0$

$$a=\frac{1}{1000}, \quad \frac{1}{2a^2}=\frac{1}{2\cdot 1000000}$$

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4 $$\int_0^1 x^{-1/3}\,dx$$

$$=\lim_{a\to 0}\int_a^1 x^{-1/3}\,dx$$

$$=\frac{2}{3}x^{2/3}\Big|_a^1=$$

$$=\frac{2}{3}\left(1-a^{2/3}\right)$$

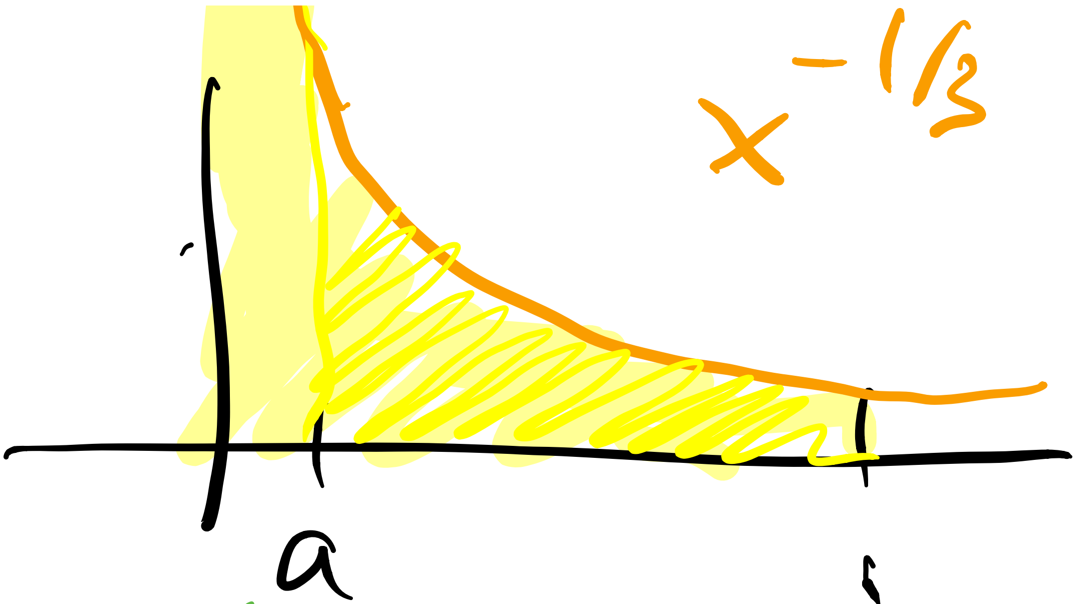
What happens, if

$a \to 0$.

The limit is $\frac{2}{3}$

We call this integral <u>convergent</u>

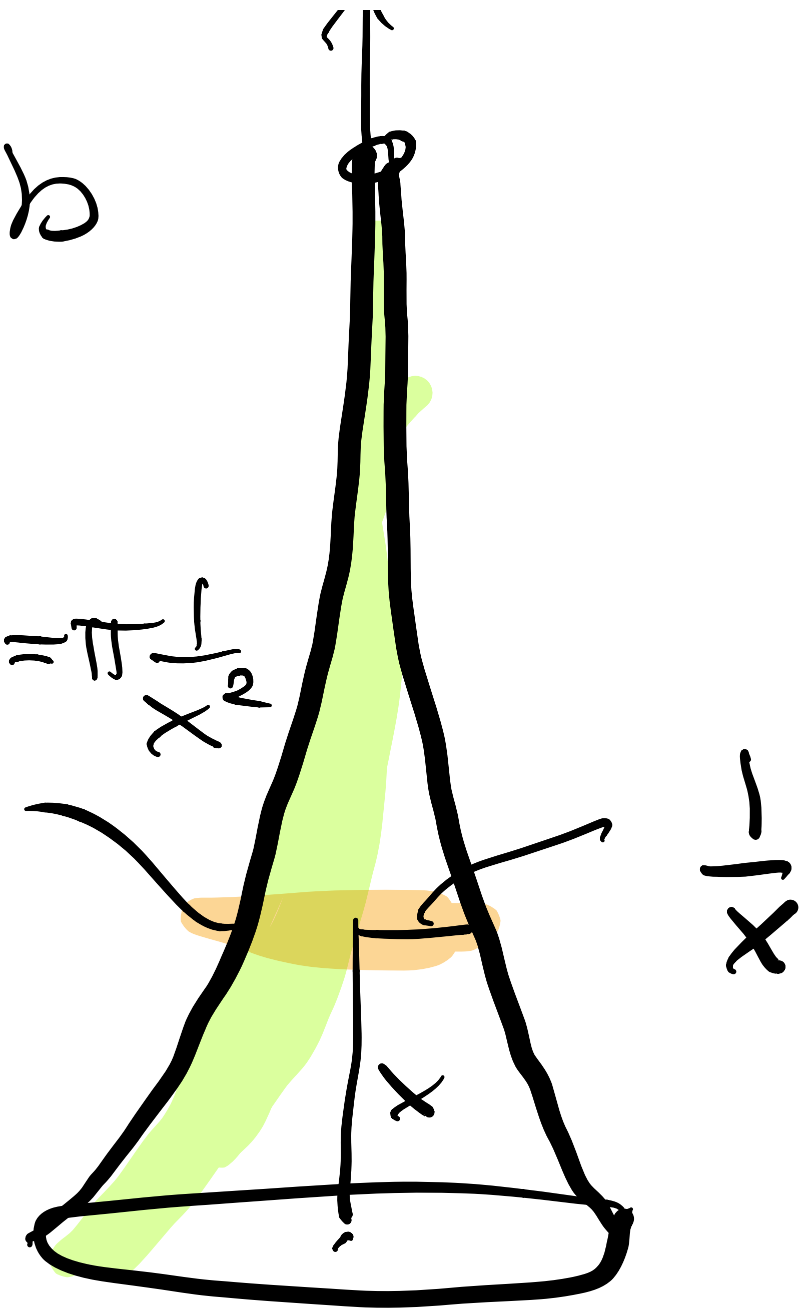
It produces a finite area.



$a \to 0$

$\boxed{\text{Gabriel's trumpet}}$

$A(x) = \pi \frac{1}{x^2}$



What is the

$\boxed{\text{Volume}}$

$$\int_1^b \frac{\pi}{x^2}\,dx$$

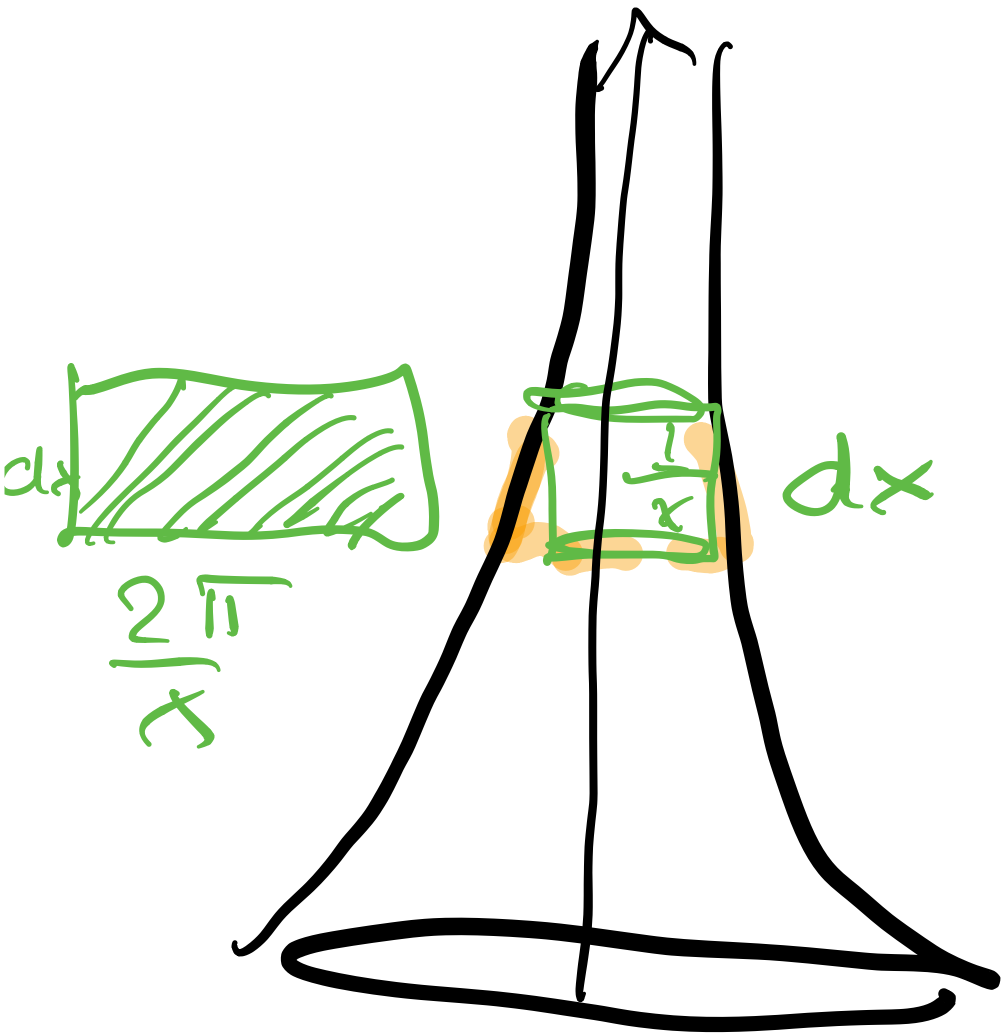
$$= -\frac{\pi}{x}\Big|_1^b$$

$$= -\frac{\pi}{b} + \pi$$

$$= \pi\left(1 - \frac{1}{b}\right)$$

$$\underset{b \to \infty}{=} \boxed{\pi} \qquad 0 \quad b \to \infty$$

Surface area



Surface Area

$$\geq \int_1^\infty \frac{2\pi}{x}\,dx$$

$$= 2\pi \int_1^\infty \frac{1}{x}\,dx$$

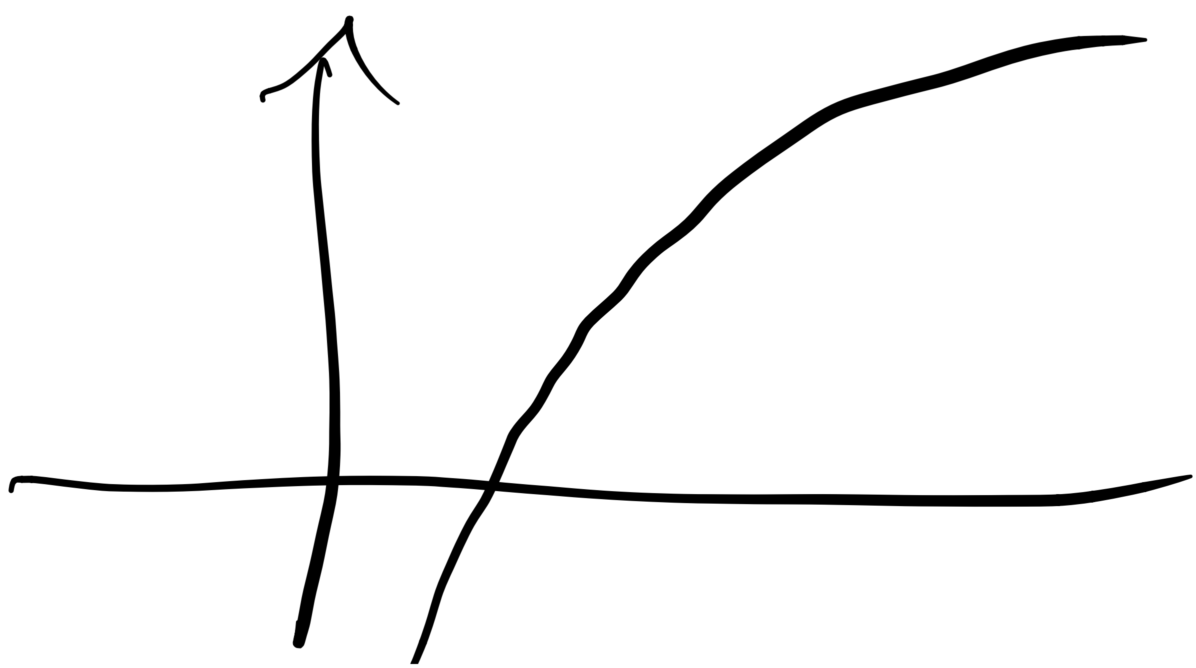
$$= \lim_{b\to\infty} 2\pi \int_1^b \frac{1}{x}\,dx$$

$$= \lim_{b\to\infty} 2\pi\left(\log(b) - \log(1)\right)$$

$$= \lim_{b \to \infty} 2\pi \log b$$

What happens if $b \to \infty$?

this diverges.

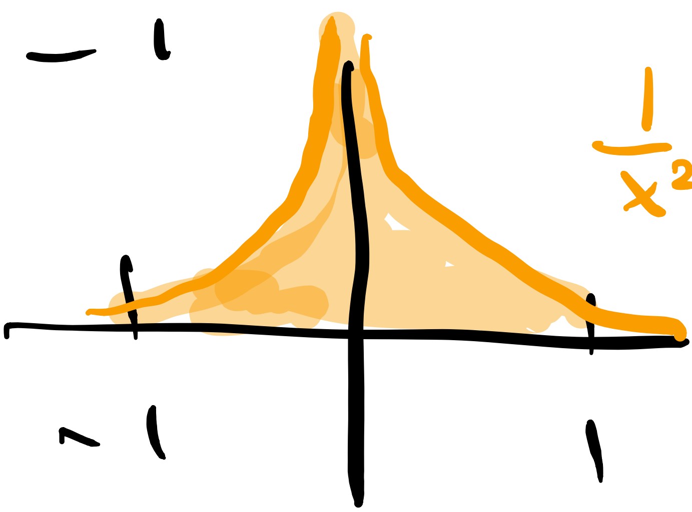


This trumpet has ∞ area, but finite volume.

You can not paint it, but you can fill it with paint!

To shock somebody:

$$\int_{-1}^{1} \frac{1}{x^2}\, dx$$



$\frac{1}{x^2} \geq 0$

positive

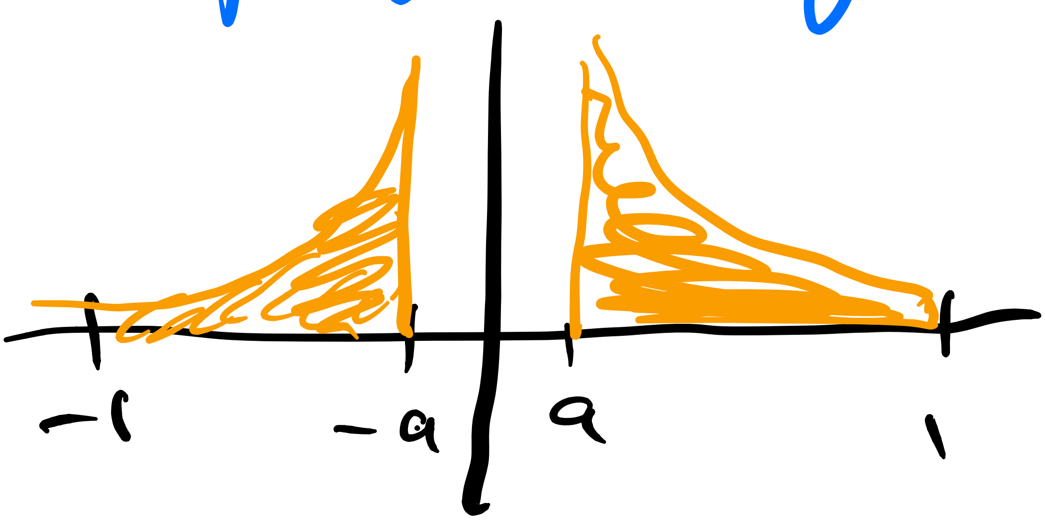
$$= -\frac{1}{x}\Big|_{-1}^{1} = \frac{-1}{1} + \frac{-1}{1}$$

$= \boxed{-2}$

negative area!

What is going on?

This is an improper integral,



if $a \to 0$

this diverges