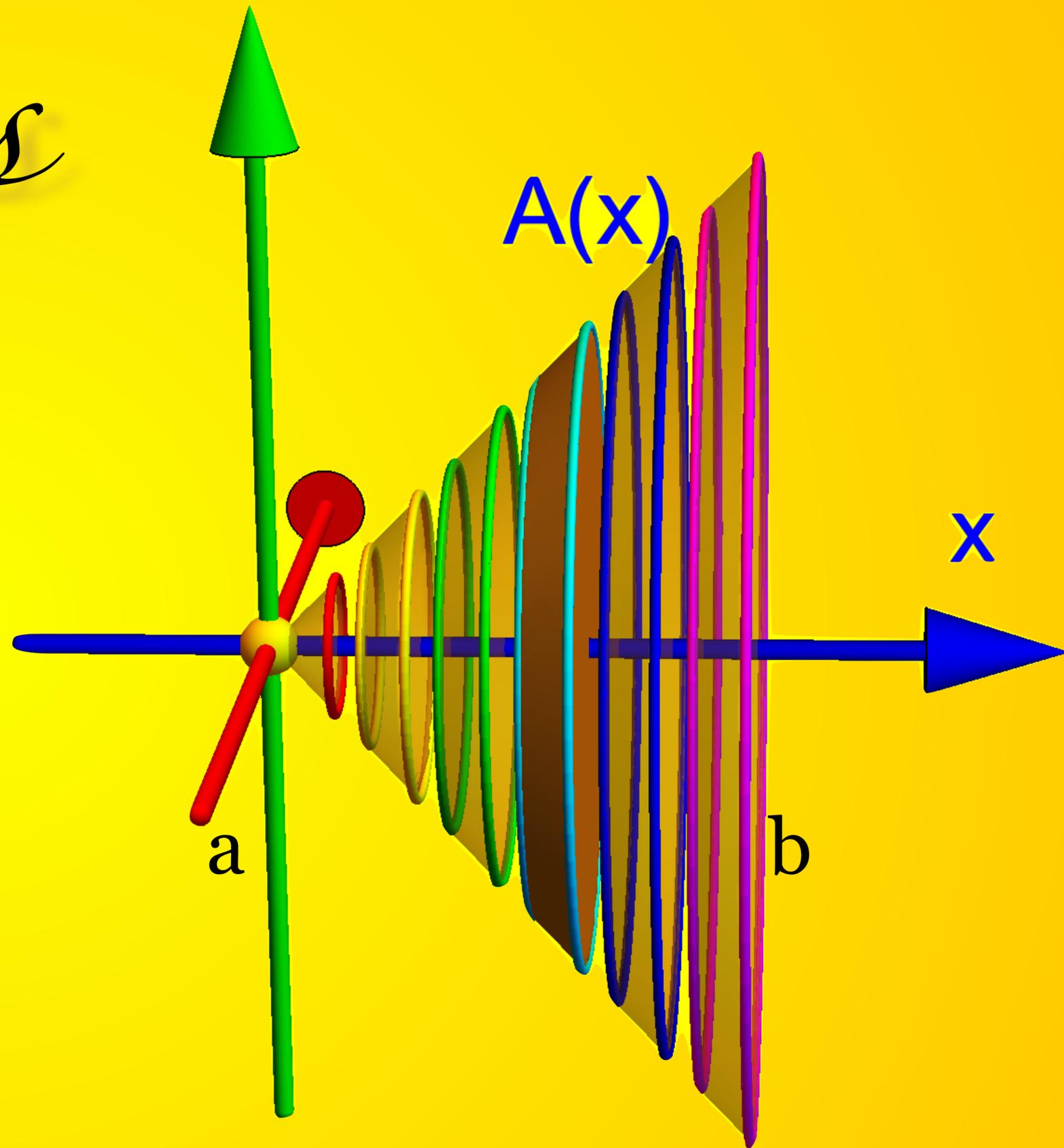


21

Volumes

Volume if $A(x)$
is area of slice.

$$V = \int_a^b A(x) dx$$



PLAN

1. Poll

2. Archimedes Formula

3. Cone, Paraboloid, Sphere, Lemon

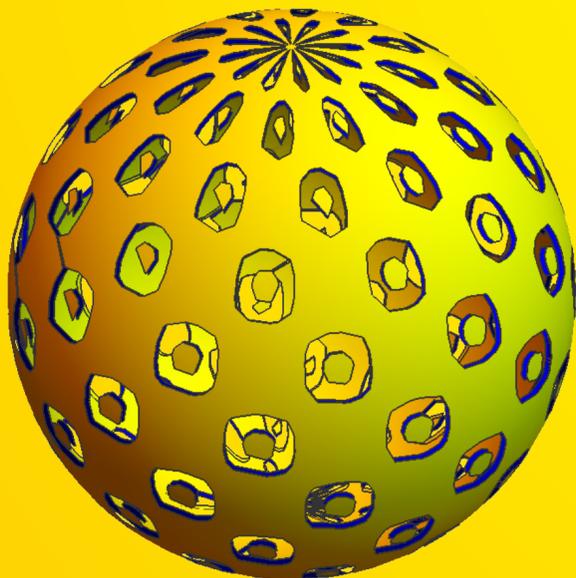
4. Sapace X SN10

6. Jam

5. Pappus Torus

POLL

Which
formula is the
volume of
the sphere?



A

$$4\pi r^3$$

B

$$4\pi r^3 / 3$$

C

$$\pi r^3$$

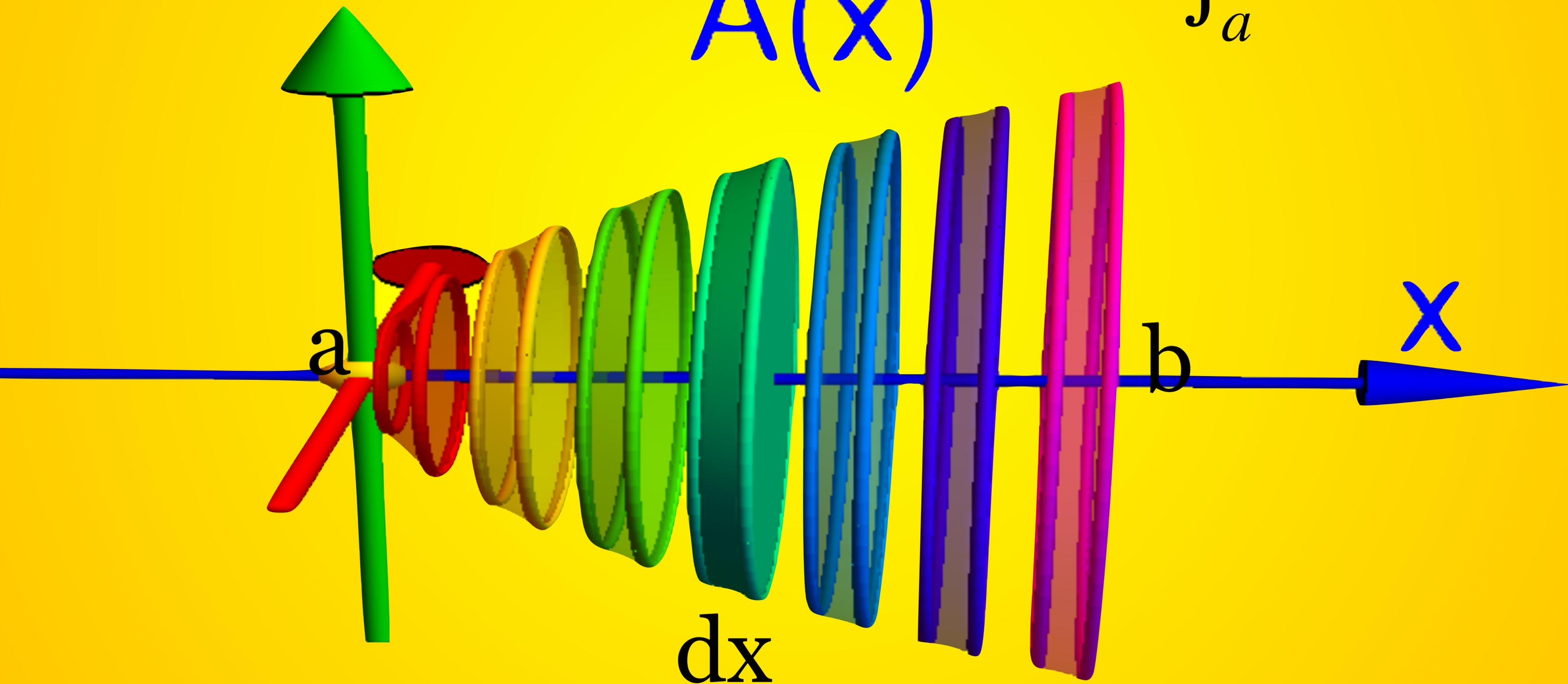
D

$$r^3$$

Formula

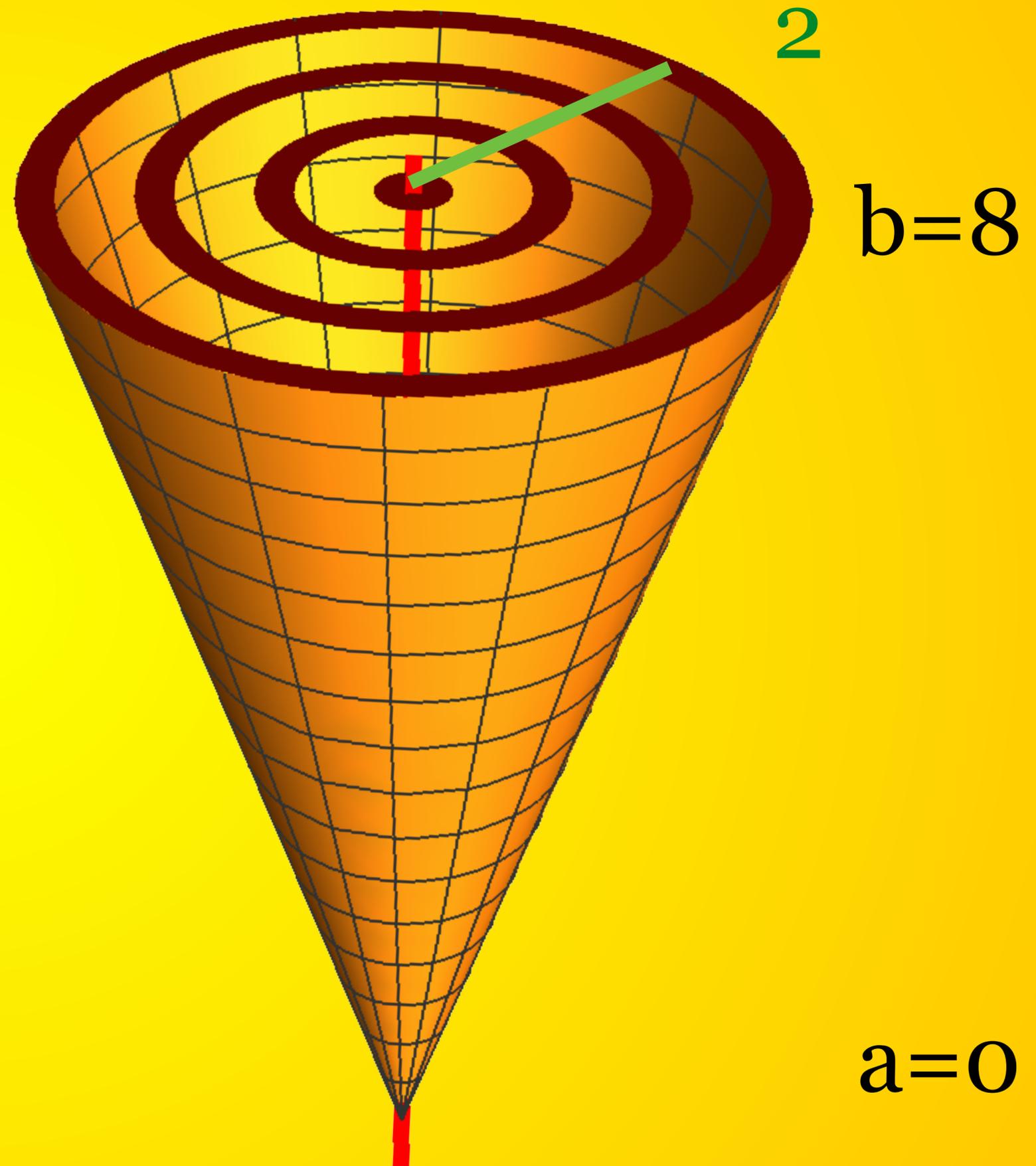
$$V = \int_a^b A(x) dx$$

$A(x)$



Cone

Assume a cone has height 8 and base radius 2.
What is the volume?

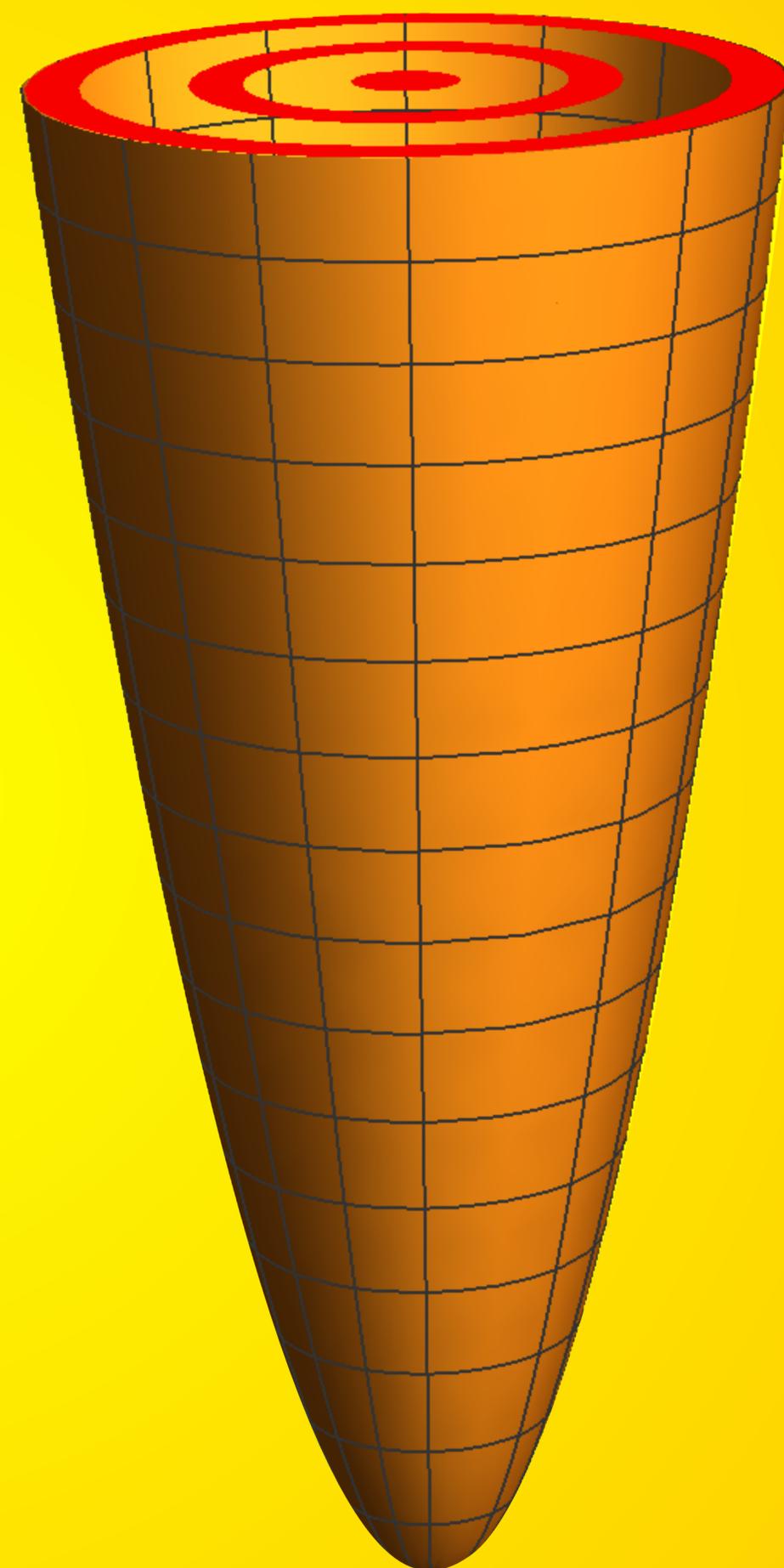


Paraboloid

Assume

$$r(x) = \sqrt{x/9}$$

Find the
volume.



$$b=3$$

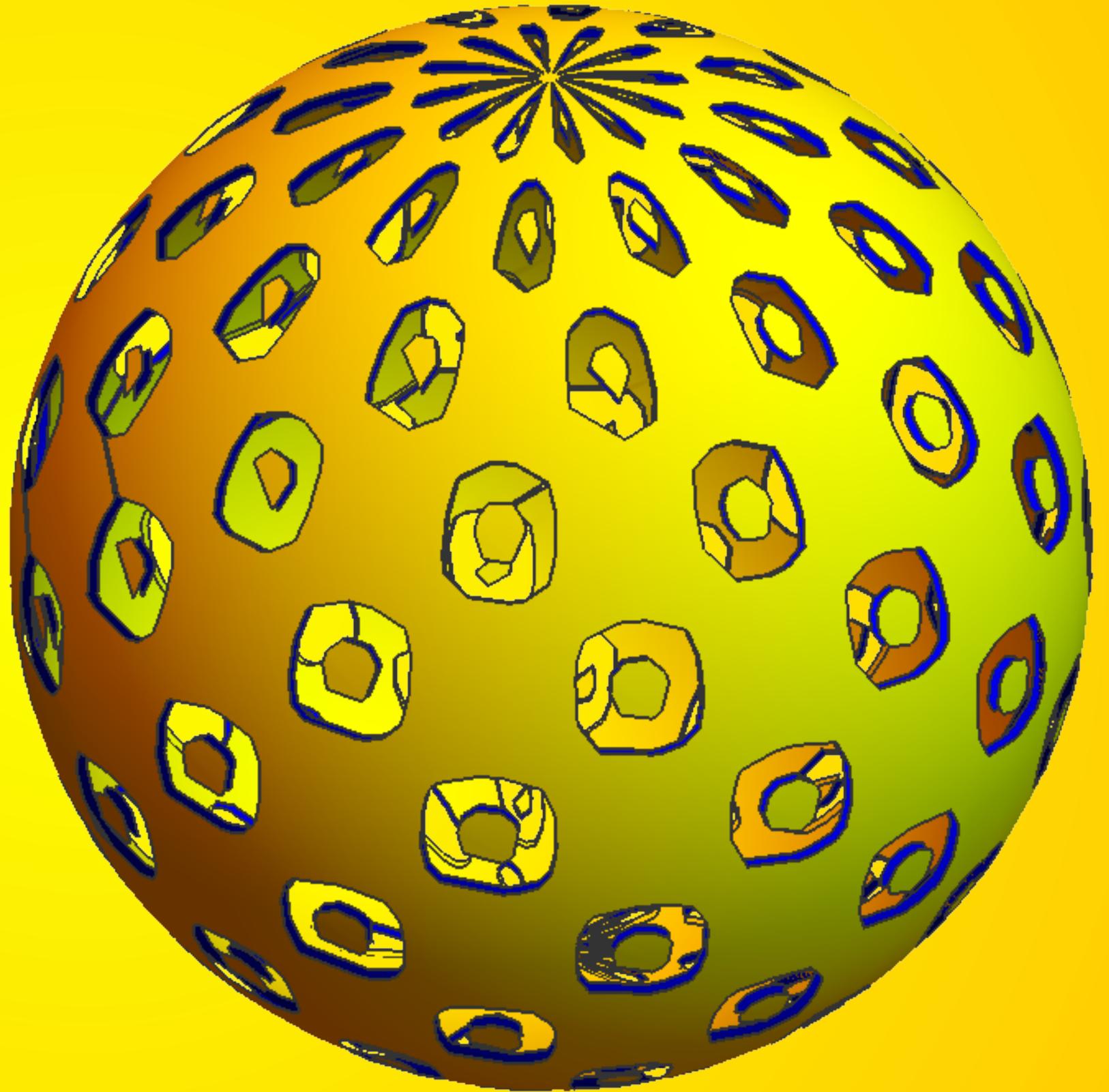
$$a=0$$

Sphere

We have

$$r(x) = \sqrt{1 - x^2}$$

What is the volume?



Hyperboloid

At point x
the radius
of the
disc is
 r .

Assume

$$r^2 - x^2 = 1$$

and $|x| < 2$. Draw
the situation
and Find the
volume.

Lemon

Draw the region

$$r(x) = \sin(x)$$

with $0 \leq x \leq \pi$

and compute the
volume

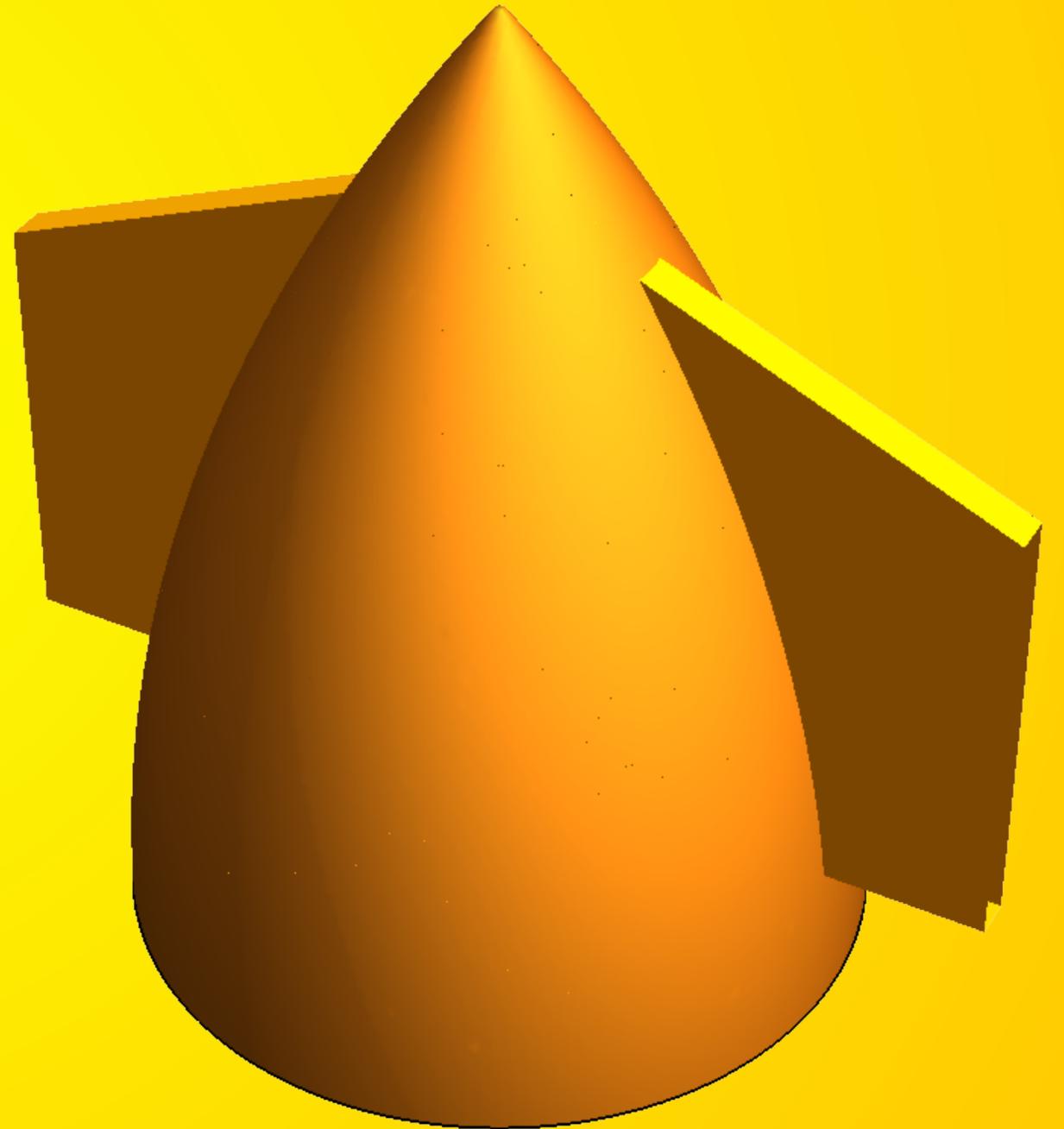


Space X



Homework

$$r(x) = \sqrt{9 - x^2}$$





- 00:01.0

Space X, 20 times faster

Moscow Papyrus

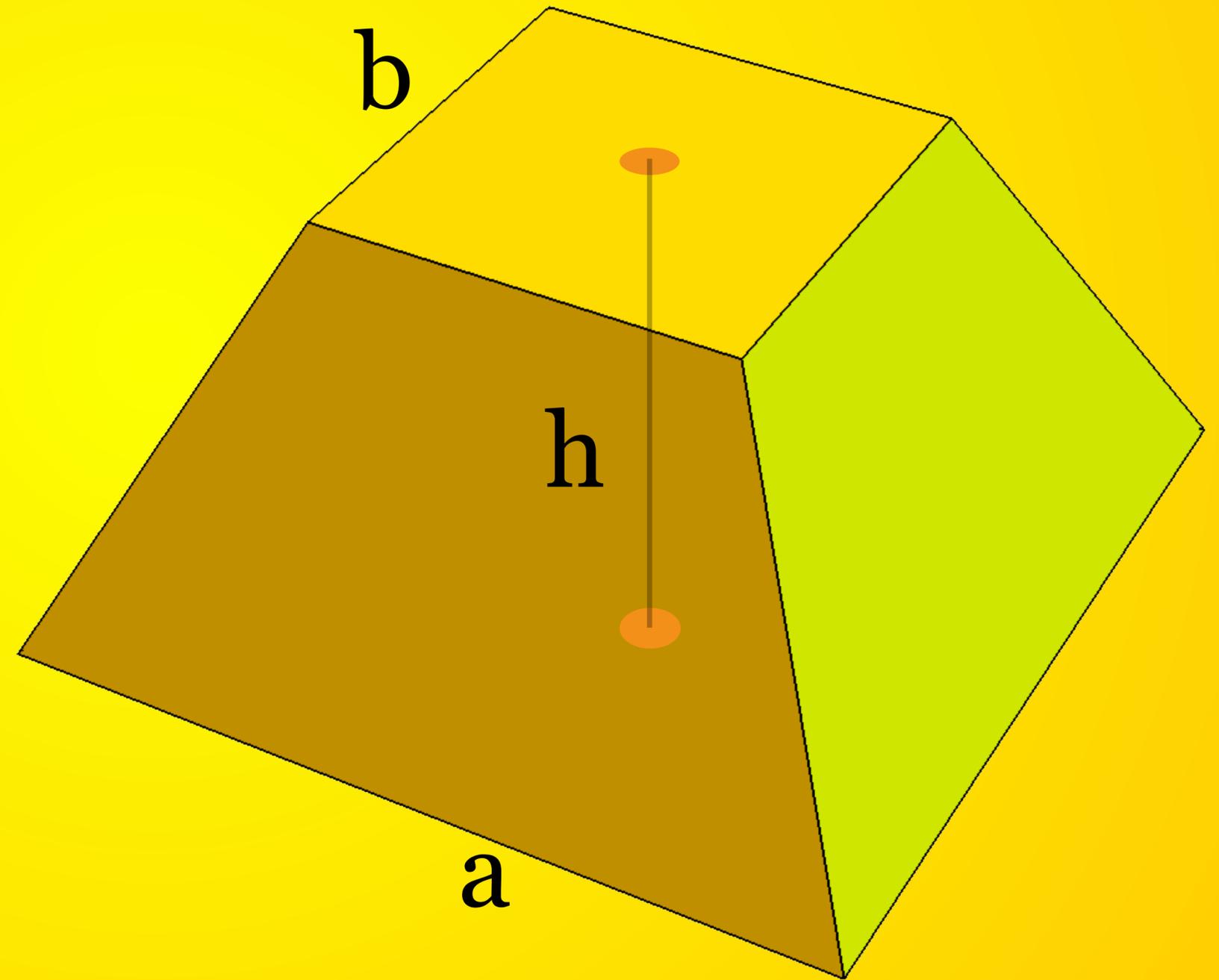
1850 BC

Handwritten Egyptian hieroglyphs on the left side of the papyrus fragment, including a large trapezoidal diagram with the number '2' inside and various symbols above and below it.

Handwritten Egyptian hieroglyphs on the right side of the papyrus fragment, arranged in several lines of text.

Volume Frustum

$$V = \frac{h}{3}(a^2 + ab + b^2)$$

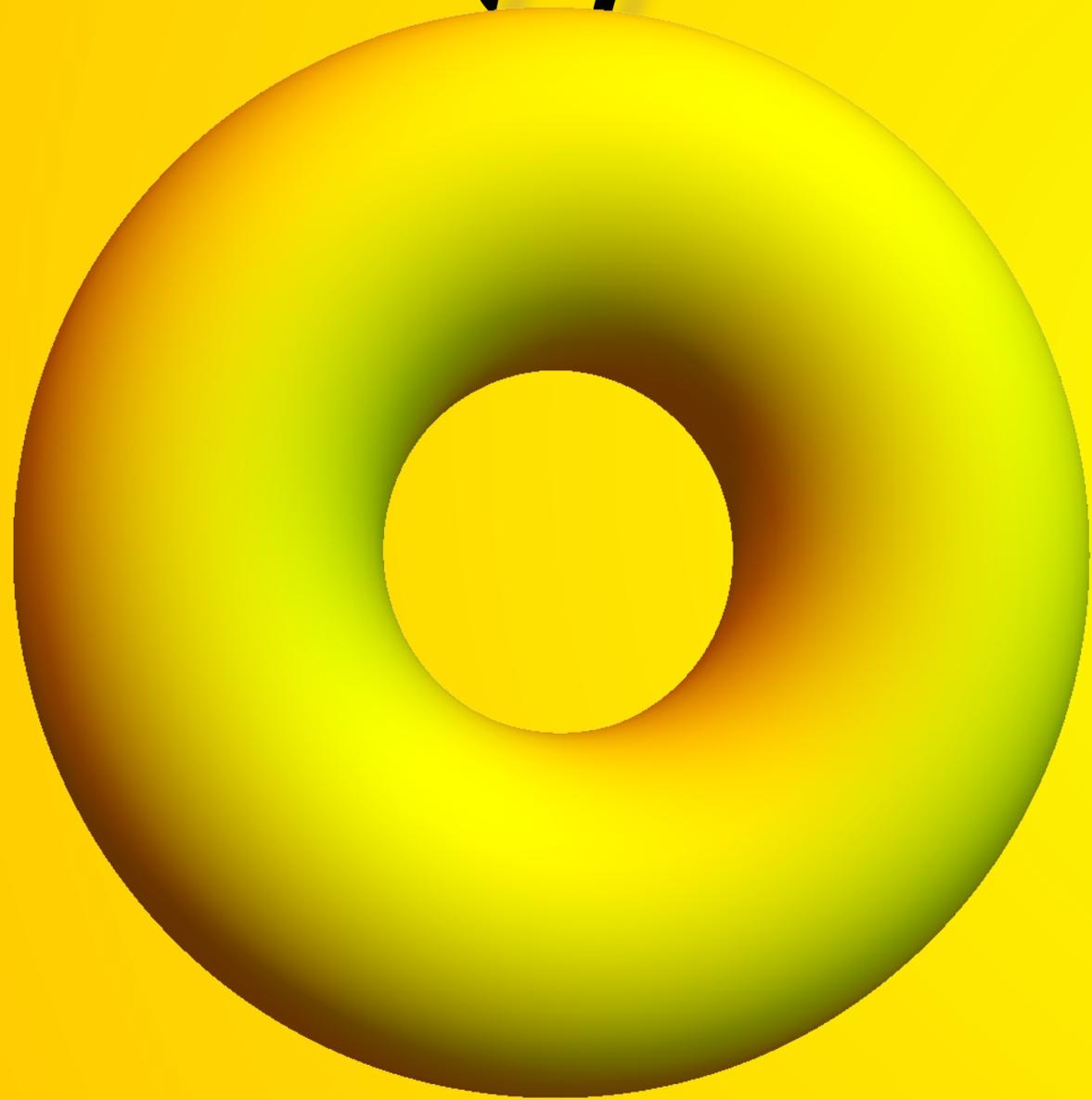


Pyramid

height 4,
base length 4



Pappus Centroid result

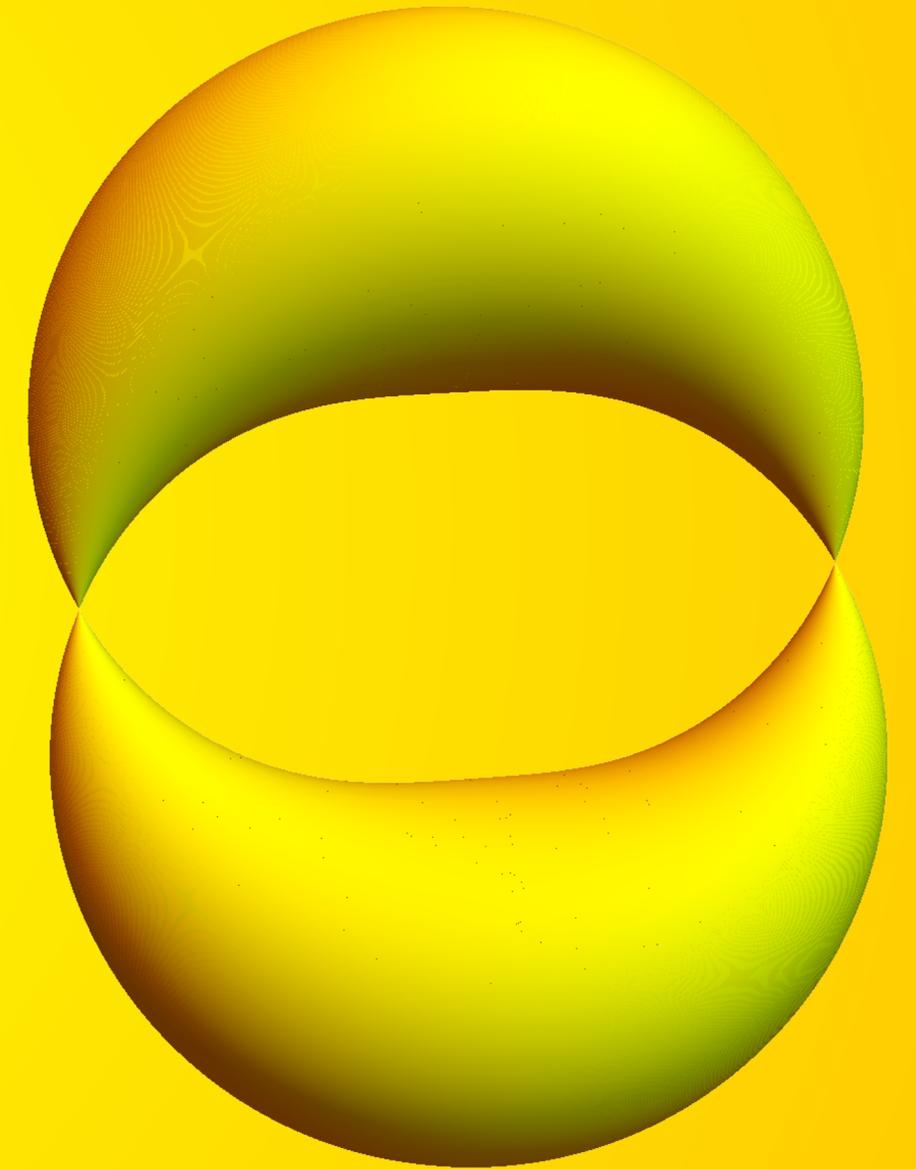


$$A(x) = \pi/4$$

The formula

$$V = \int_0^{2\pi} A(x) dx$$

also holds
if the center
moves along a
curve



$$A(x) = \pi \sin^2(x)/4$$

String Beans

The area at
position x is

$$A(x) = \pi \sin^2(12x)$$

The string is
parametrized by

$$0 \leq x \leq 2\pi$$

Find its volume

$$V = \int_0^{2\pi} A(x) dx$$



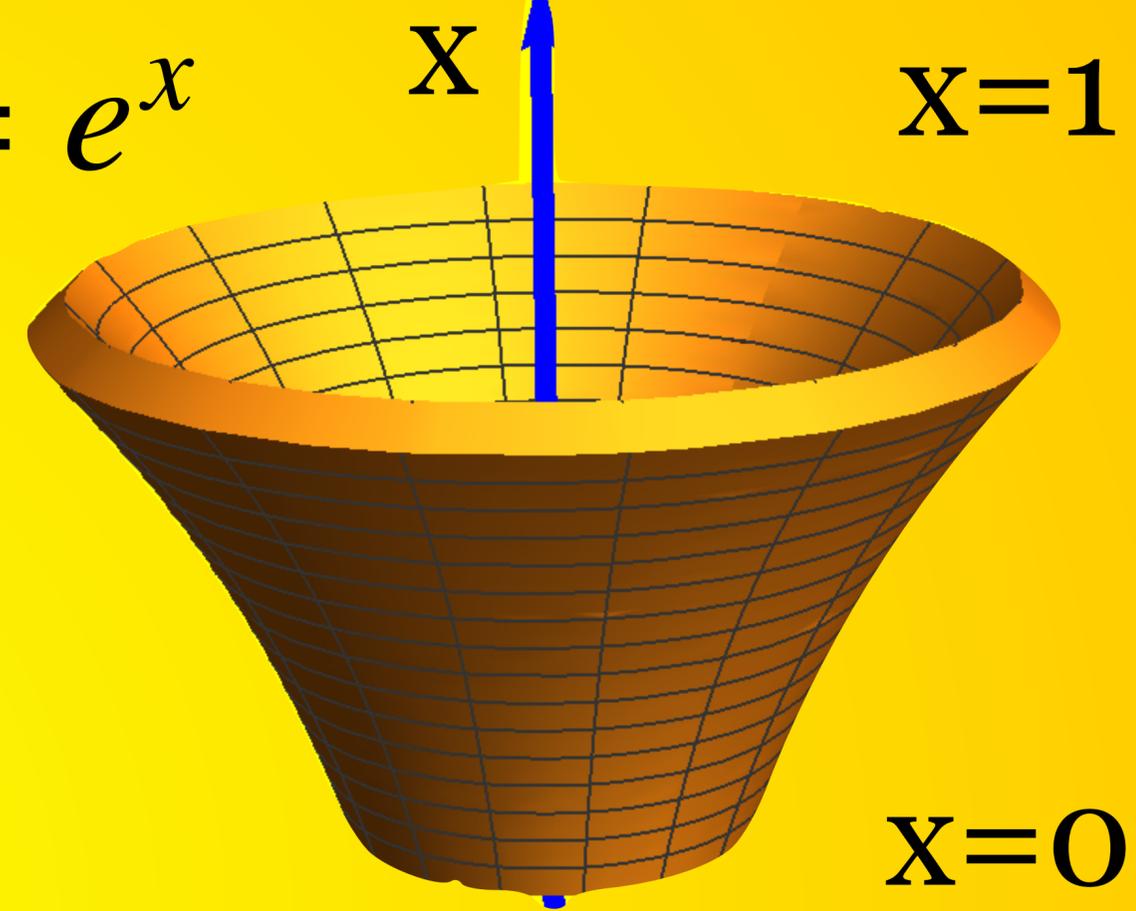
Tam

Find the volumes
of the solids for
which the radius of the
cross section is given.

In the second
case the cross section
is a half circle.

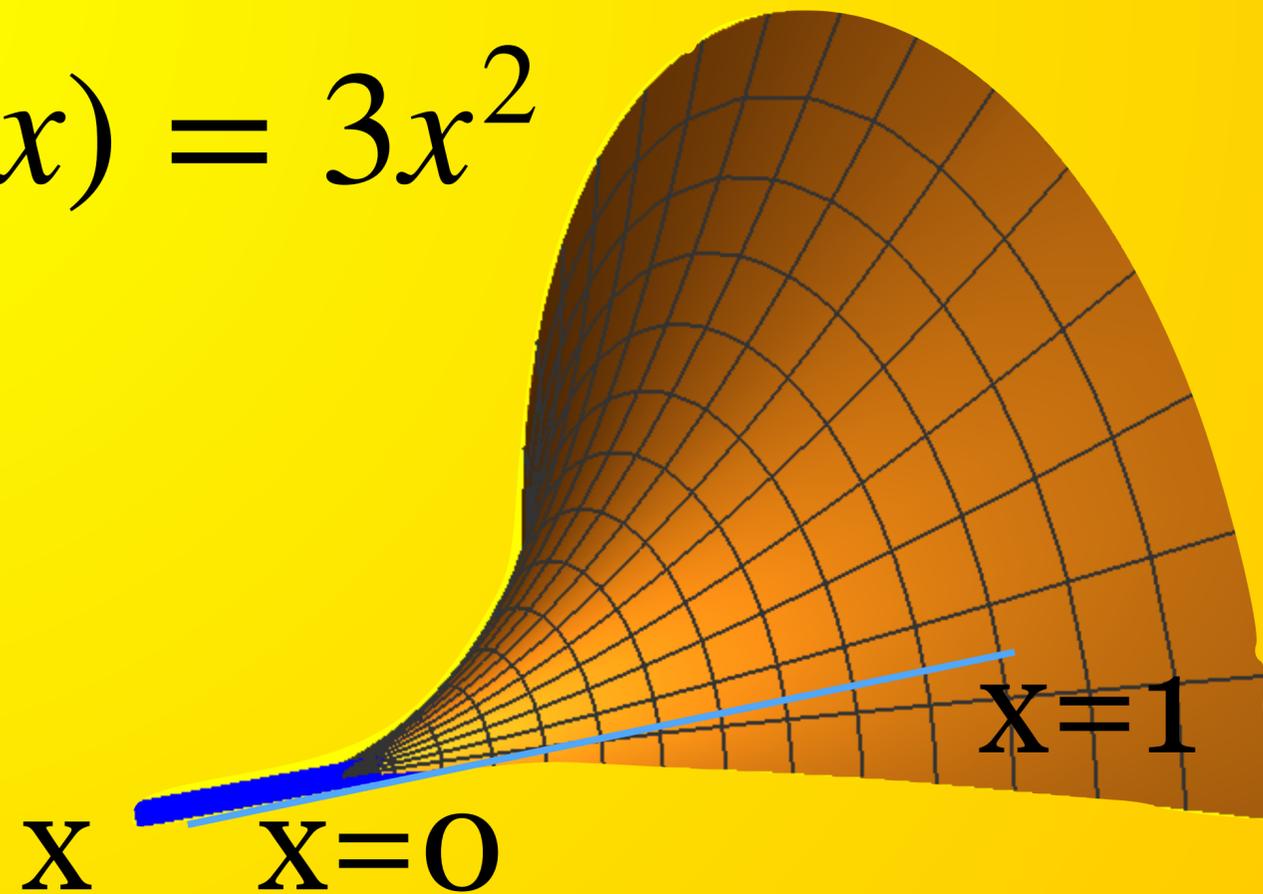
A

$$r(x) = e^x$$



B

$$r(x) = 3x^2$$



The End