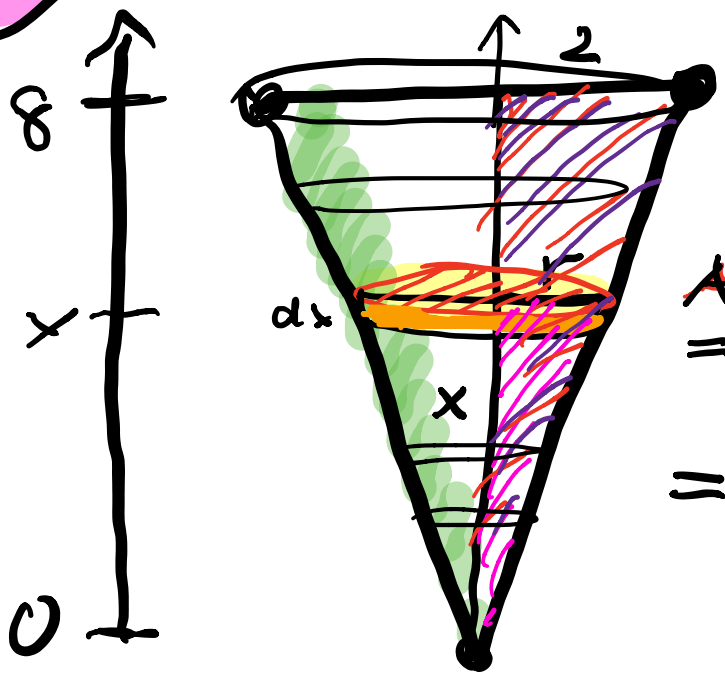
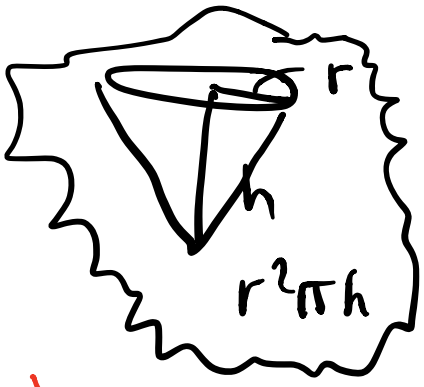
A

Unit 21





$A(x) = r(x)^2\pi$

$= \frac{x^2}{16}\pi$

$r = ?$

similar triangles!

$$\frac{2}{8} = \frac{r}{x}$$

$$r = \frac{x}{4}$$

Volume:

$$\int_0^8 \frac{x^2}{16}\pi\,dx$$

$$= \frac{x^3}{48}\pi \Big|_0^8 = \frac{8^3}{48}\pi$$

B

Paraboloid

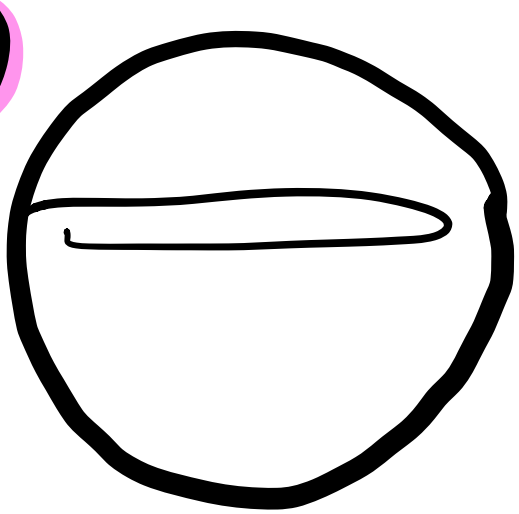
$$\int_0^3 \frac{x}{9}\pi \, dx$$

$$= \frac{x^2}{18}\pi \Big|_0^3$$

Volume

$$= \frac{9}{18}\pi = \frac{\pi}{2}$$

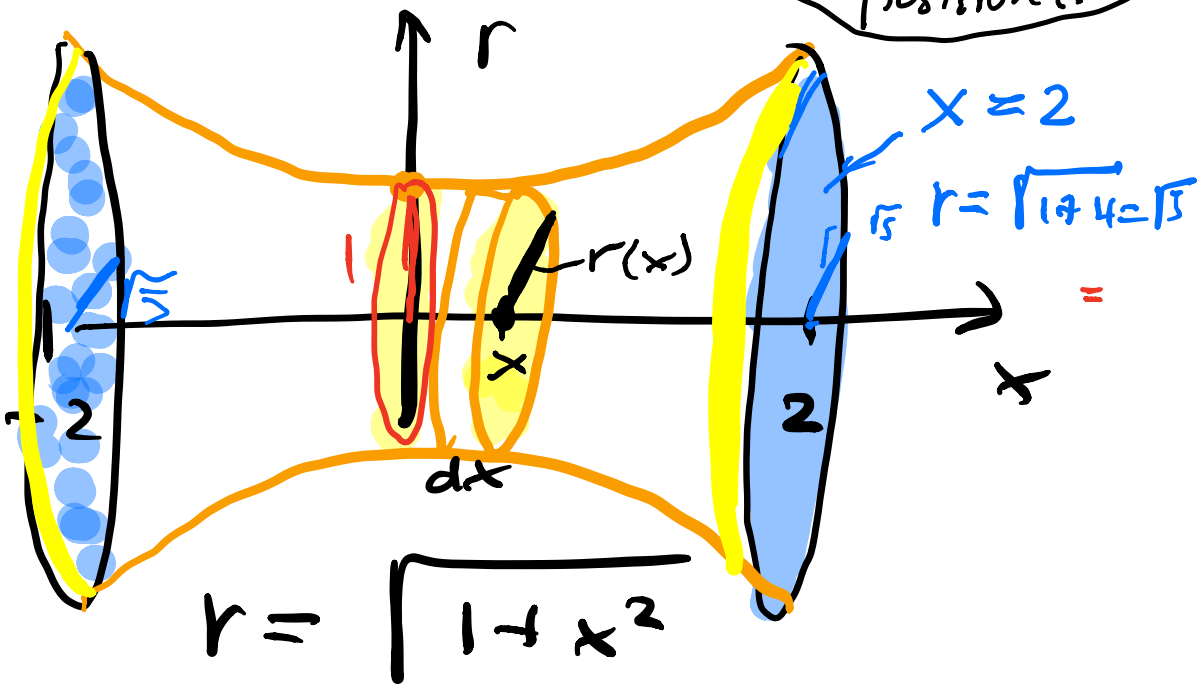
C



Sphere

(b) $r^2 - x^2 = 1$

$r(x)$ radius of disc at position x



$r = \sqrt{1 + x^2}$

Volume: $\displaystyle\int_{-2}^{2} \pi (1 + x^2)\, dx$

$A(x) = \pi r(x)^2 = \pi (1 + x^2)$

E

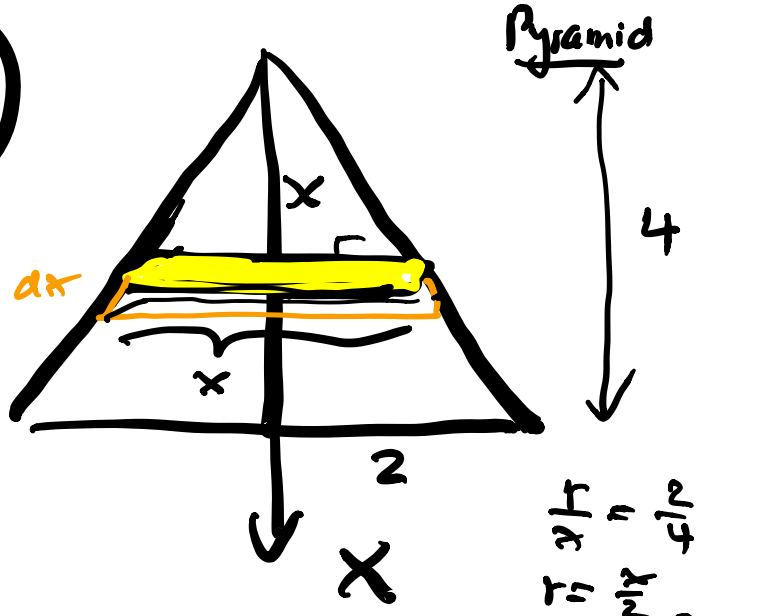
Lemon

$$\int_0^{\pi} \pi \sin^2 x \, dx$$

$$= \int_0^{\pi} \pi \left(\frac{1 - \cos 2x}{2} \right) dx$$

$$= \boxed{\frac{\pi^2}{2}}$$

F



$\frac{r}{x} = \frac{2}{4}$

$r = \frac{x}{2}$ $\quad 2r = x$

$A(x) =$ Area of cross section

x^2

$$\int_0^4 x^2\,dx = \frac{x^3}{3}\Big|_0^4$$

$$= \frac{64}{3}$$

which agrees with

$$\frac{\text{Base area} \cdot \text{height}}{3}$$

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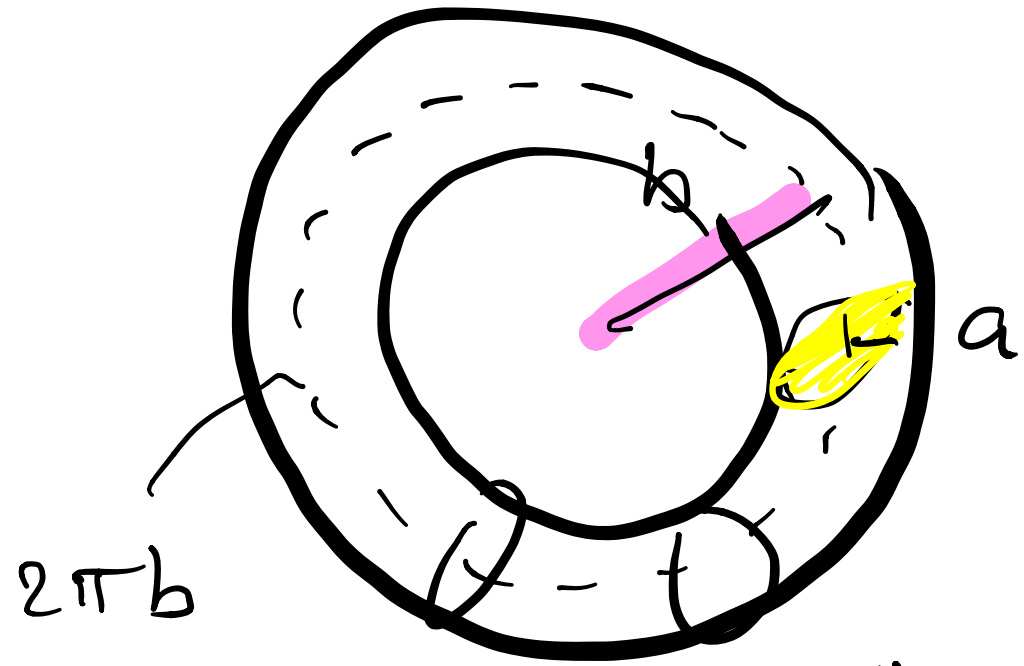
6) Doughnuts



tubes.

$$A(x) = \pi a^2$$

$$V = \int_0^{2\pi b} \pi a^2 \, dx = \boxed{2\pi^2 a^2 b}$$

Volume of doughnut.

The same formula of Archimedes

$$\int_a^b A(x)\,dx$$

applies for tubes