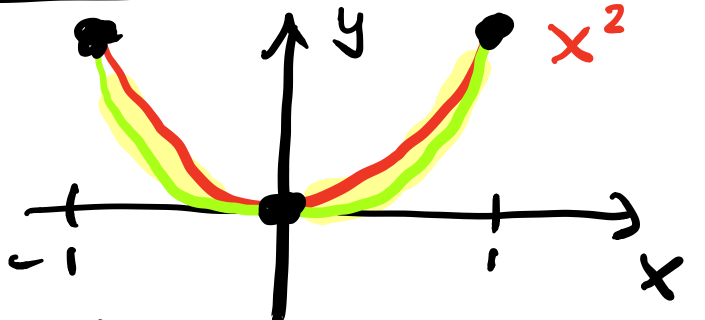
Unit 20

(A) $y = x^6, \; y = x^2$

(B) $y = x, \; y = x^3, \; x=0, \; x=2$

(C) $x = y, \; x = 2y, \; y = 1$

(A)



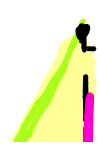
Intersect at $x^6 = x^2 \Rightarrow x = \pm 1$ or $x = 0$

Bounds!
Picture!
What is above!
Symmetry

$$\int_{-1}^{1} x^2 - x^6 \, dx$$

$$= \frac{x^3}{3} - \frac{x^7}{7}\Big|_{-1}^{1} = \left(\frac{1}{3} - \frac{1}{7}\right)\cdot 2$$

$$2 \cdot \int^{1} x^2 - x^6 \, dx \qquad = \boxed{\frac{8}{7}}$$



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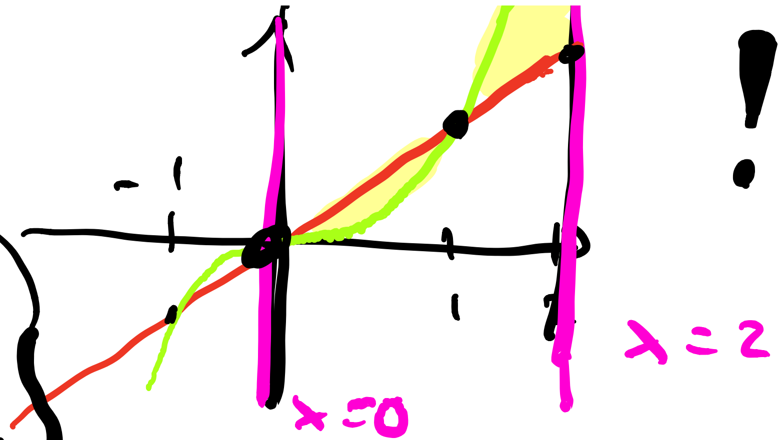
Make a picture of the situation



2 separate integrals:

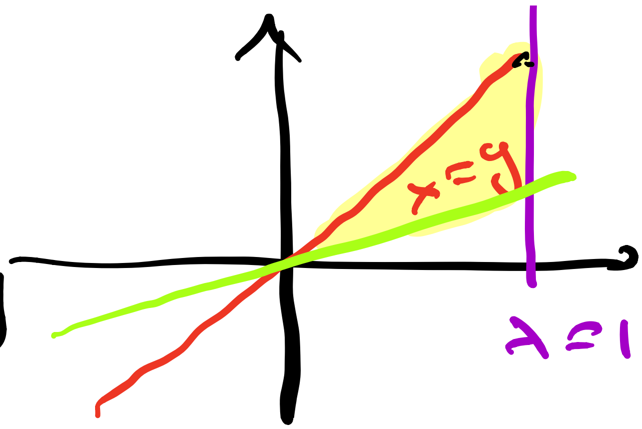
$$\int_0^1 x - x^3\,dx + \int_1^2 x^3 - x\,dx$$

$$= \frac{x^2}{2} - \frac{x^4}{4}\Big|_0^1 + \frac{x^4}{4} - \frac{x^2}{2}\Big|_1^2$$

$$= \frac{1}{2} - \frac{1}{4} + \left(\frac{16}{4} - \frac{4}{2}\right) - \left(\frac{1}{4} - \frac{1}{2}\right)$$

$$= \frac{1}{4} + 2 + \frac{1}{4} = \boxed{2.5}$$

(c)



$x = 2y$

$\boxed{y = \frac{1}{2}x}$

$x = 1$

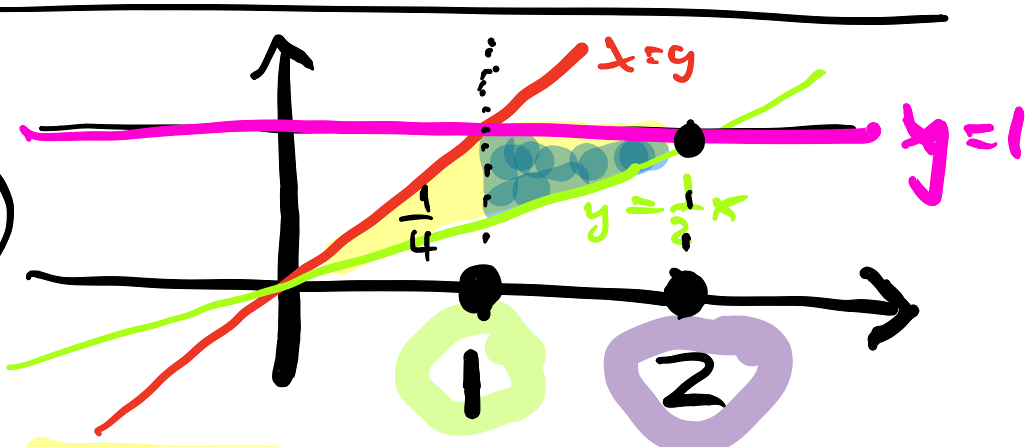
modified problem

read the problem well

$$\int_0^1 x - \frac{1}{2}x\,dx = \int_0^1 \frac{1}{2}x\,dx$$

$$= \frac{x^2}{4}\Big|_0^1 = \boxed{\frac{1}{4}}$$

Split up



$$\int_0^1 x - \frac{1}{2}x\,dx + \int_1^2 \left(1 - \frac{x}{2}\right) dx$$

$$= \frac{1}{4} + \frac{1}{4} = \boxed{\frac{1}{2}}$$

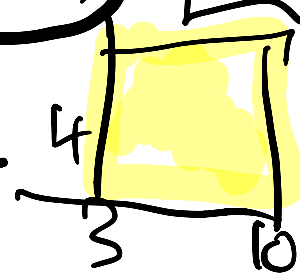
How did we find the intersection point.

$x=y$ intersect with $y=1$ gives $x=1$

$y=\frac{1}{2}x$ intersect with $y=1$ gives $x=2$

A $\displaystyle\int_3^{10} 4\,dx = 7\cdot 4 = \boxed{28}$

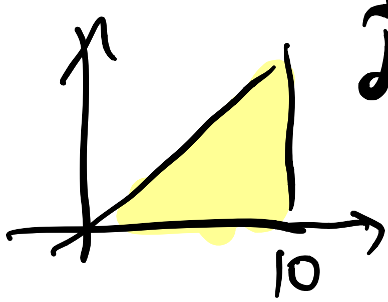
rectangle



$$\int_a^b c\,dx = c(b-a)$$

B

$$\int_0^{10} x\,dx = \boxed{50}$$



triangle

C

$$\int_0^1 \sqrt{1-x^2}\,dx$$

$$= \boxed{\frac{\pi}{4}}$$