

17

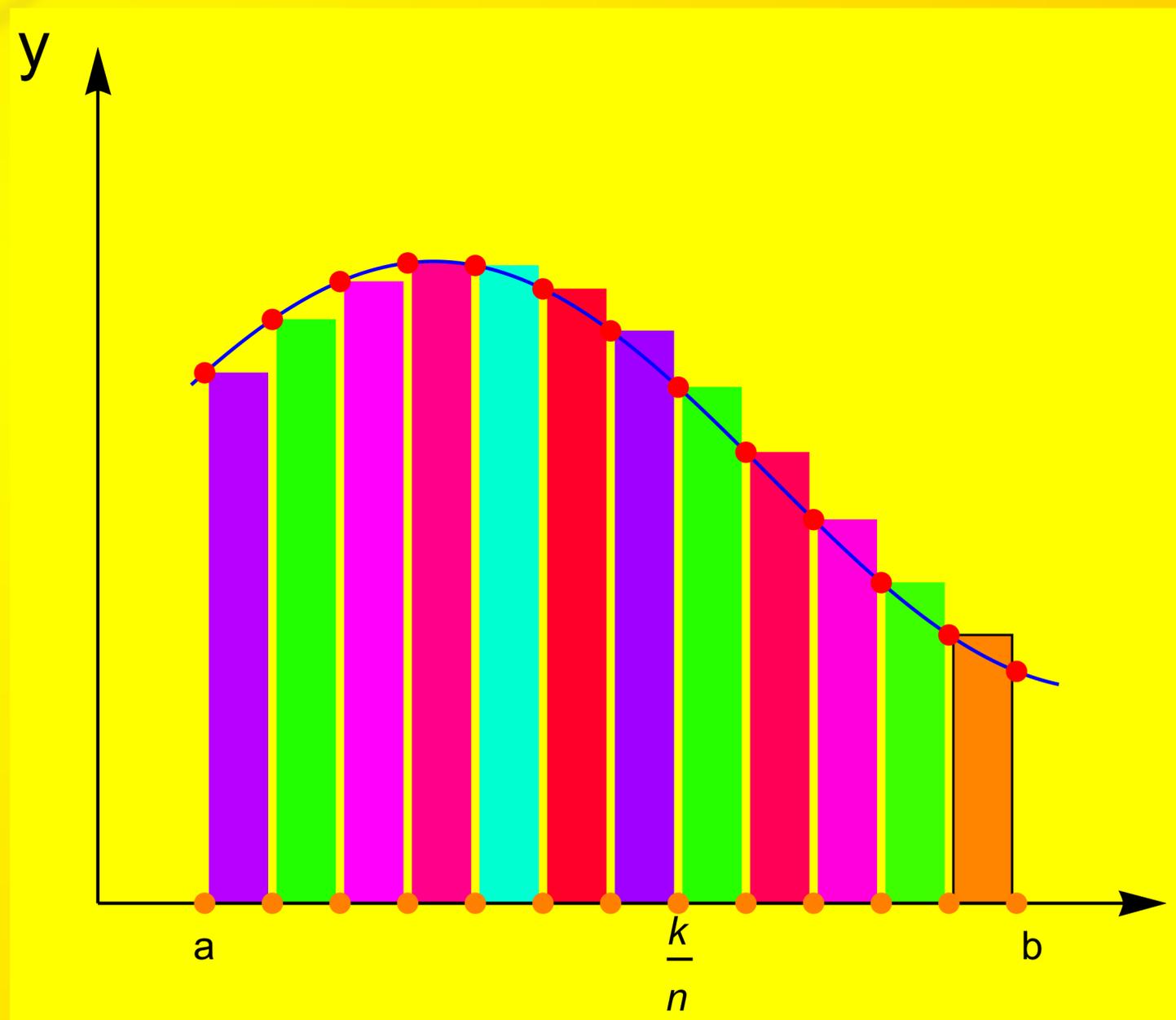
Integrals

Riemann Sum

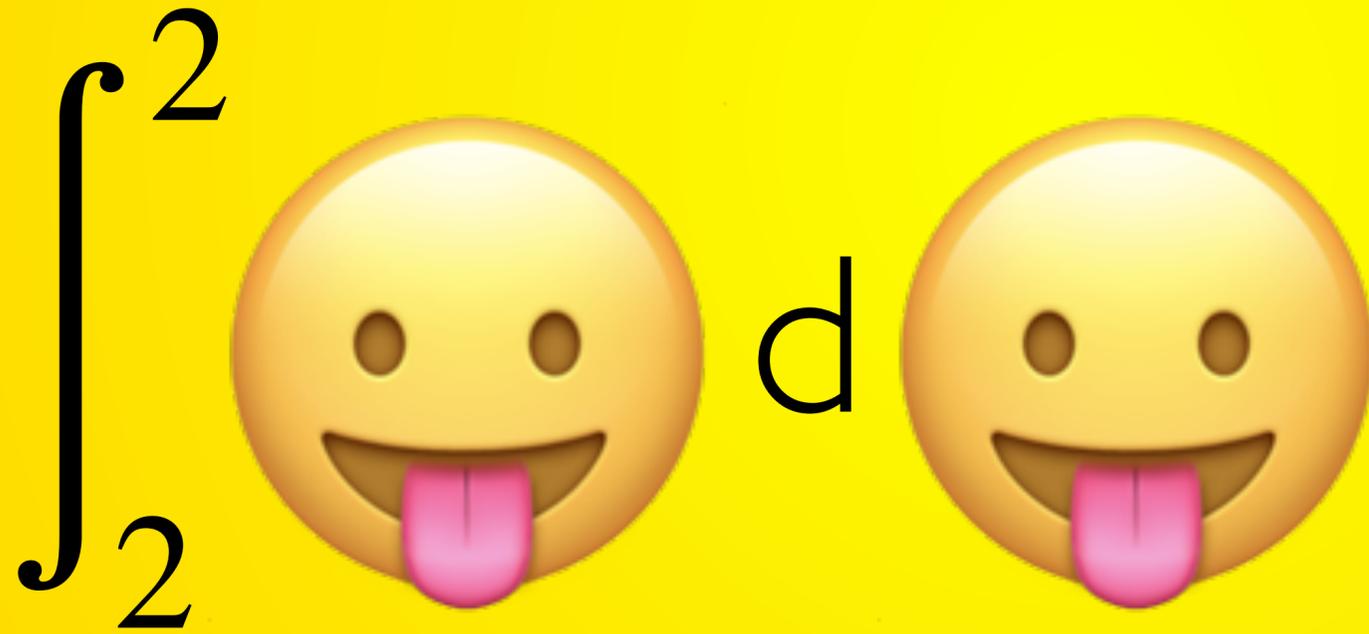
$$S_n = \frac{1}{n} \sum_{a \leq \frac{k}{n} < b} f\left(\frac{k}{n}\right)$$

converges for $n \rightarrow \infty$ to the definite integral

$$\int_a^b f(x) dx$$



Poll



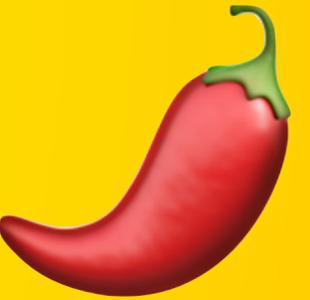
A

2

B

0

C



D

NA

PLAN

1) The Definition

2. Examples

3. Checks

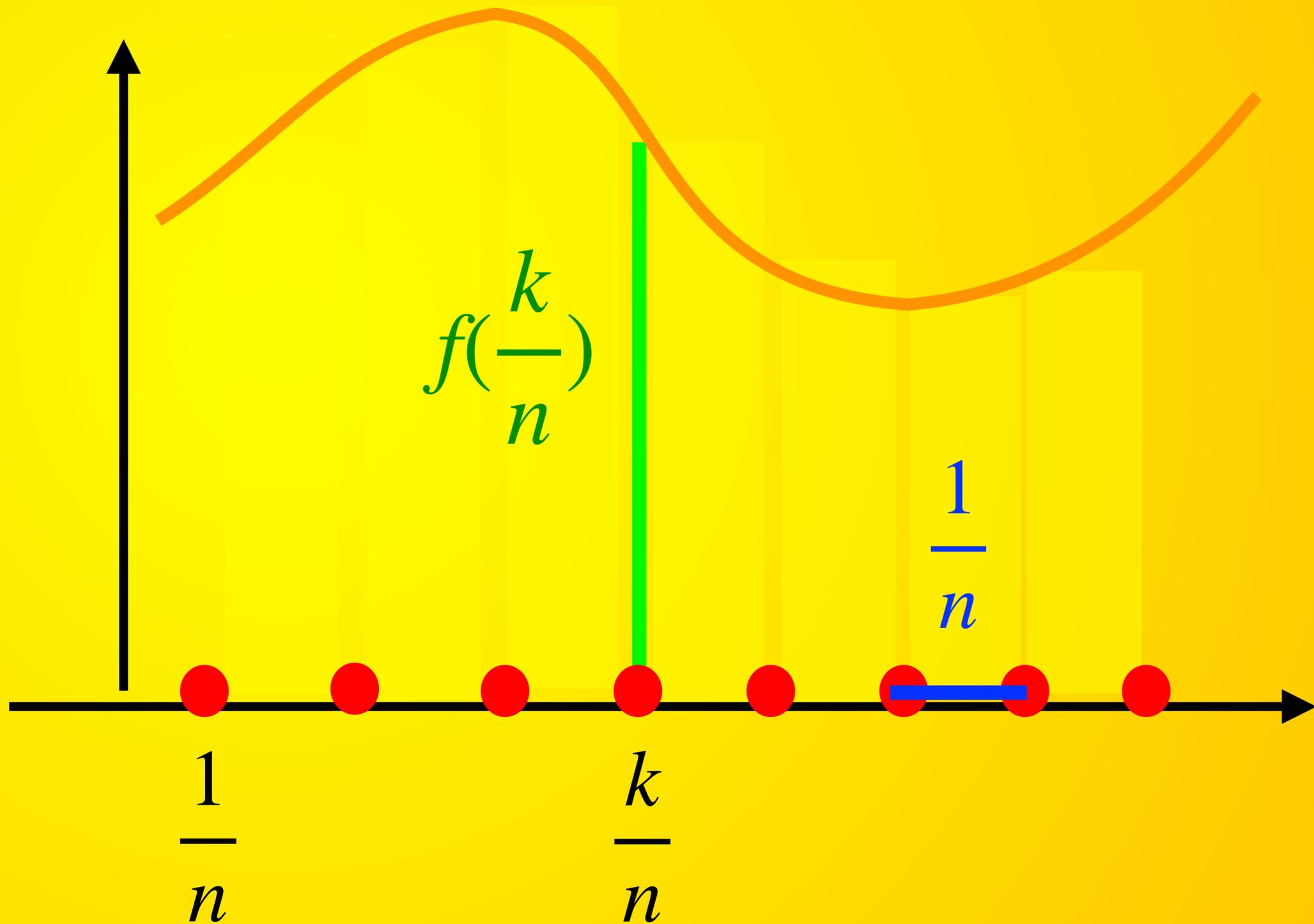
4. History

6. Monte Carlo method

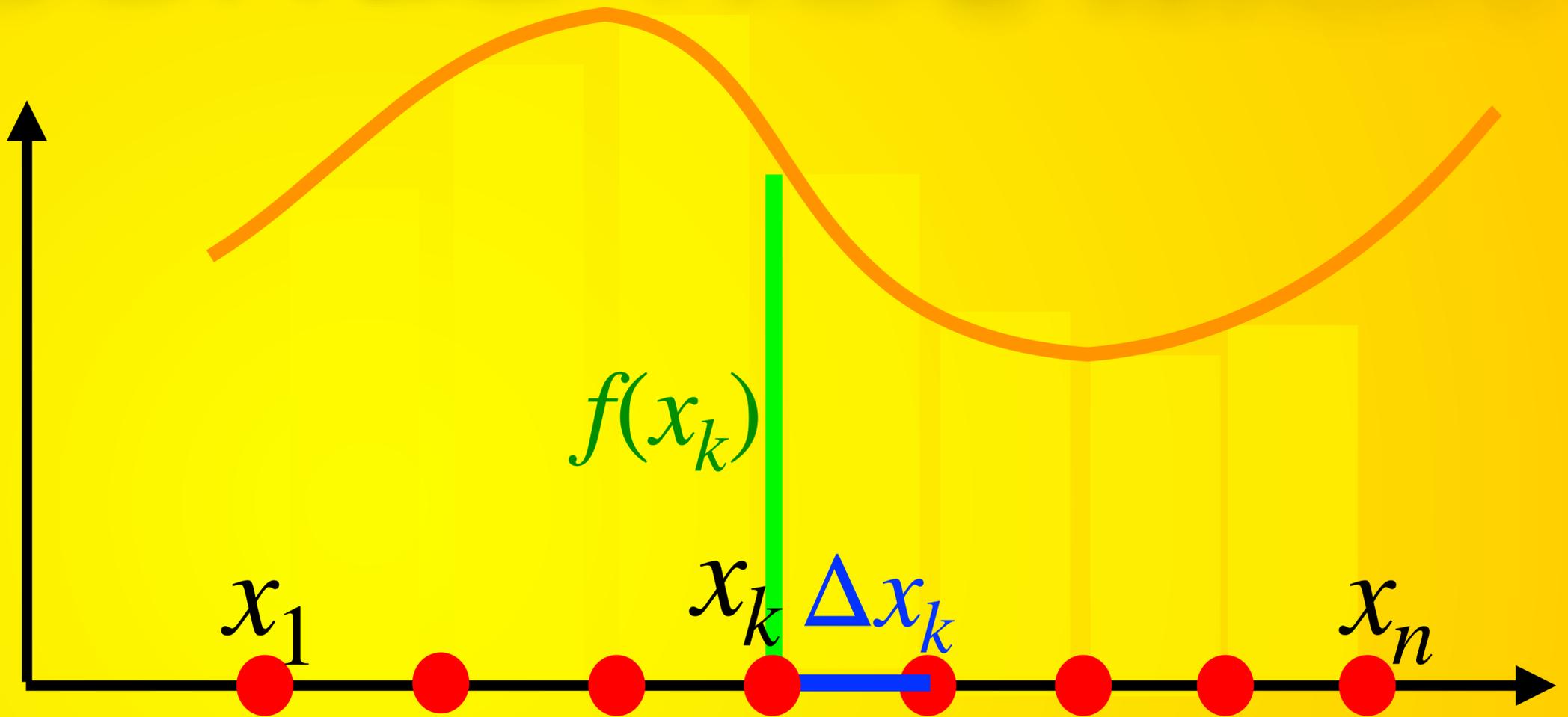
RIEMANN INTEGRAL

$$\sum_{a \leq \frac{k}{n} < b} f\left(\frac{k}{n}\right) \frac{1}{n}$$

$$\int_a^b f(x) dx$$



OLD FASHIONED NOTATION



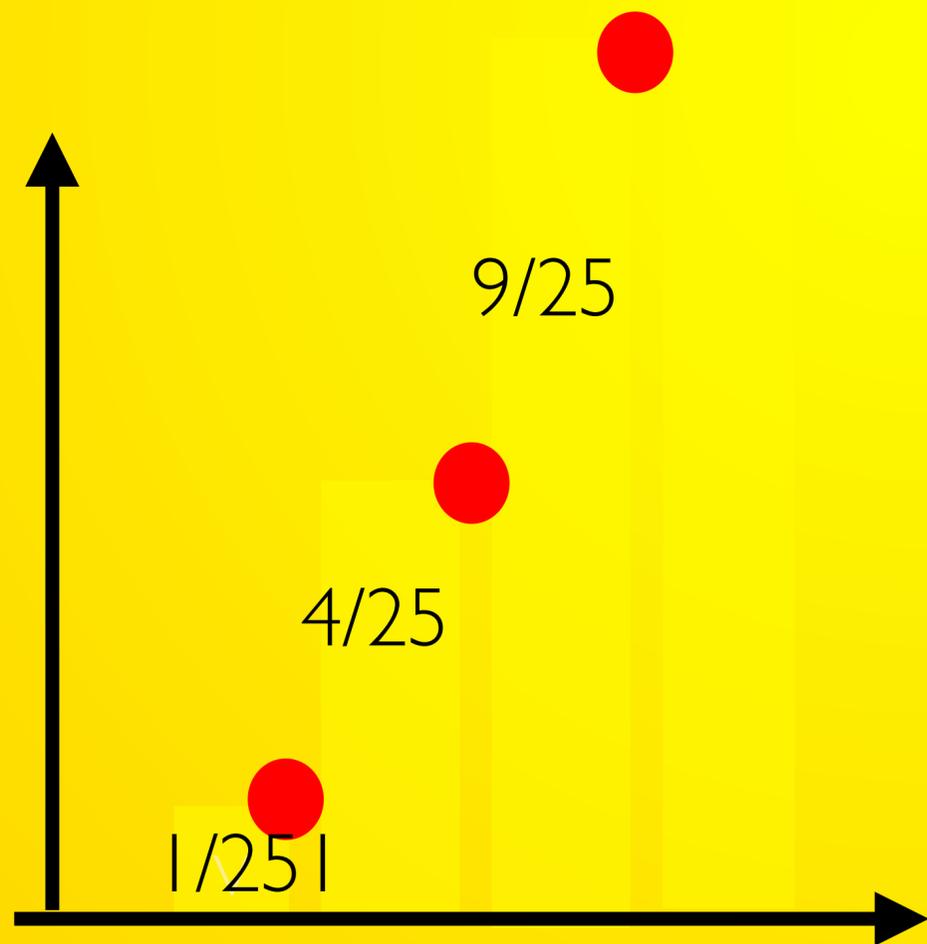
$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{a \leq x_k < b} f(x_k) \Delta x_k$$

KEY IDENTITY

If there is F with $F'(x)=f(x)$, then

$$\int_a^b f(x) dx = F(b) - F(a)$$

$$\int_0^1 x^2 dx$$



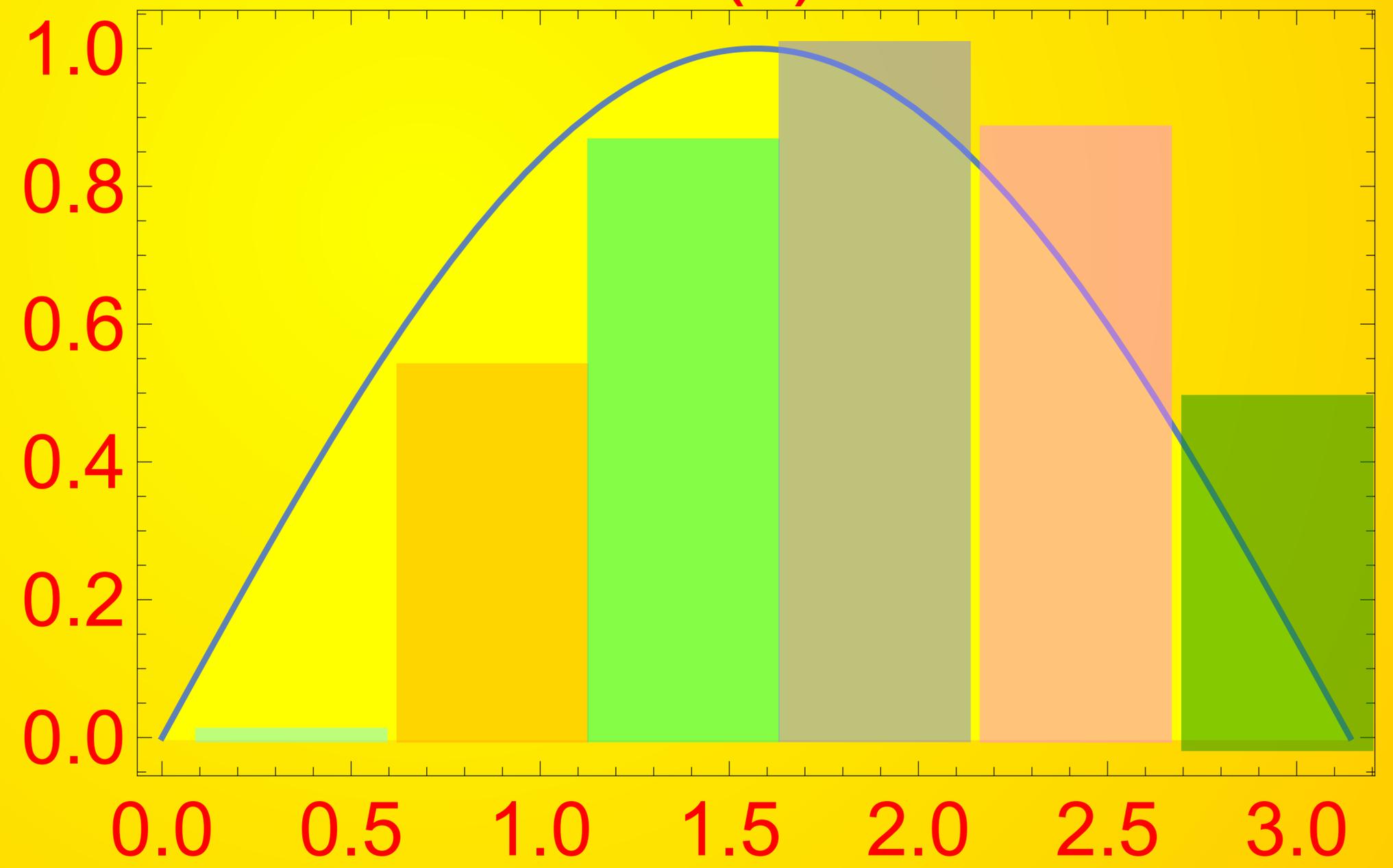
Problem 1

We develop
this slowly on
paper (ipad)

$$\int_0^{\pi} \sin(x) dx$$

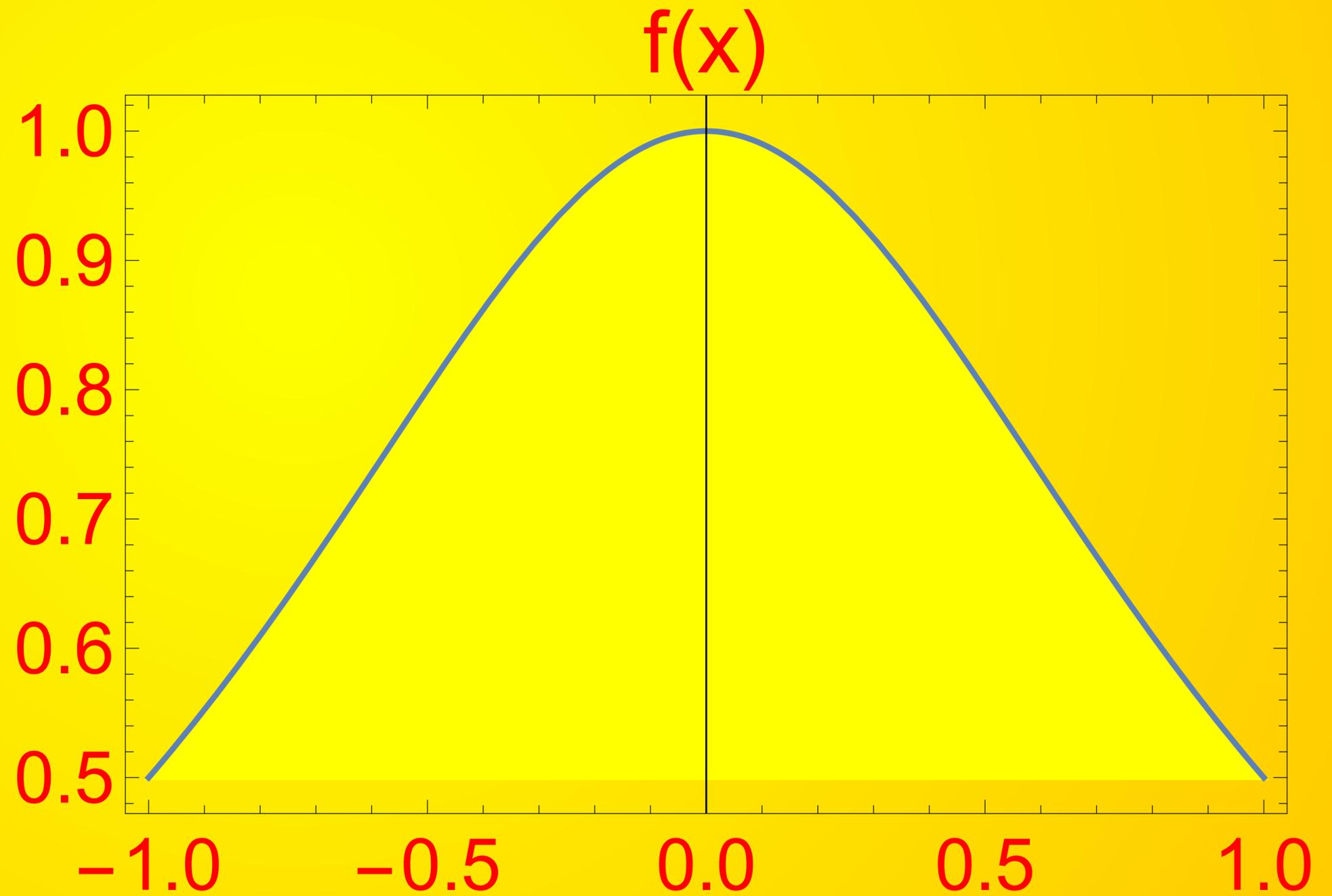
Problem 2

$f(x) = \sin(x)$



$$\int_{-1}^1 \frac{1}{1+x^2} dx$$

Problem 3



Jam

A

Visualize the identity $\int_a^b 1 dx = b - a$

What is the limit $n \rightarrow \infty$

B

$$\frac{1}{n} \sum_{k=1}^n e^{k/n}$$

C

Compute $\int_0^{\pi/4} \frac{1}{\cos^2(x)} dx$

PROPERTIES

$$\int_a^b 1 dx = b - a$$

$$\int_a^b f(x) + g(x) dx = \int_a^b f(x) dx + \int_a^b g(x) dx$$

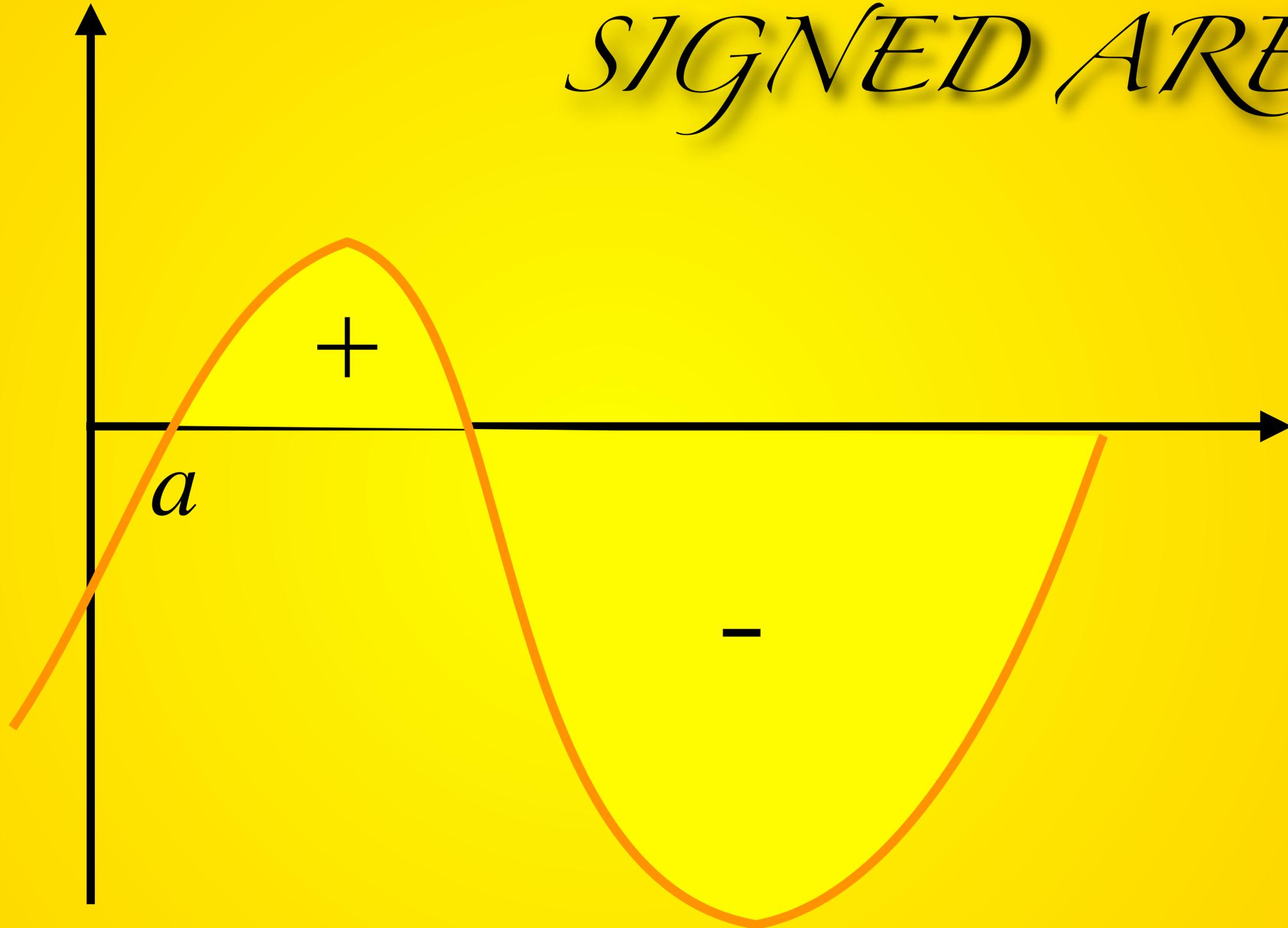
$$\int_a^b cf(x) dx = c \int_a^b f(x) dx$$

$$\int_a^b f(x) dx + \int_b^c f(x) dx = \int_a^c f(x) dx$$

Can you
visualize
this?

Checks

SIGNED AREA



Signed area under curve

Notation

$$\int_1^6$$

| d



A

1

B

5

C



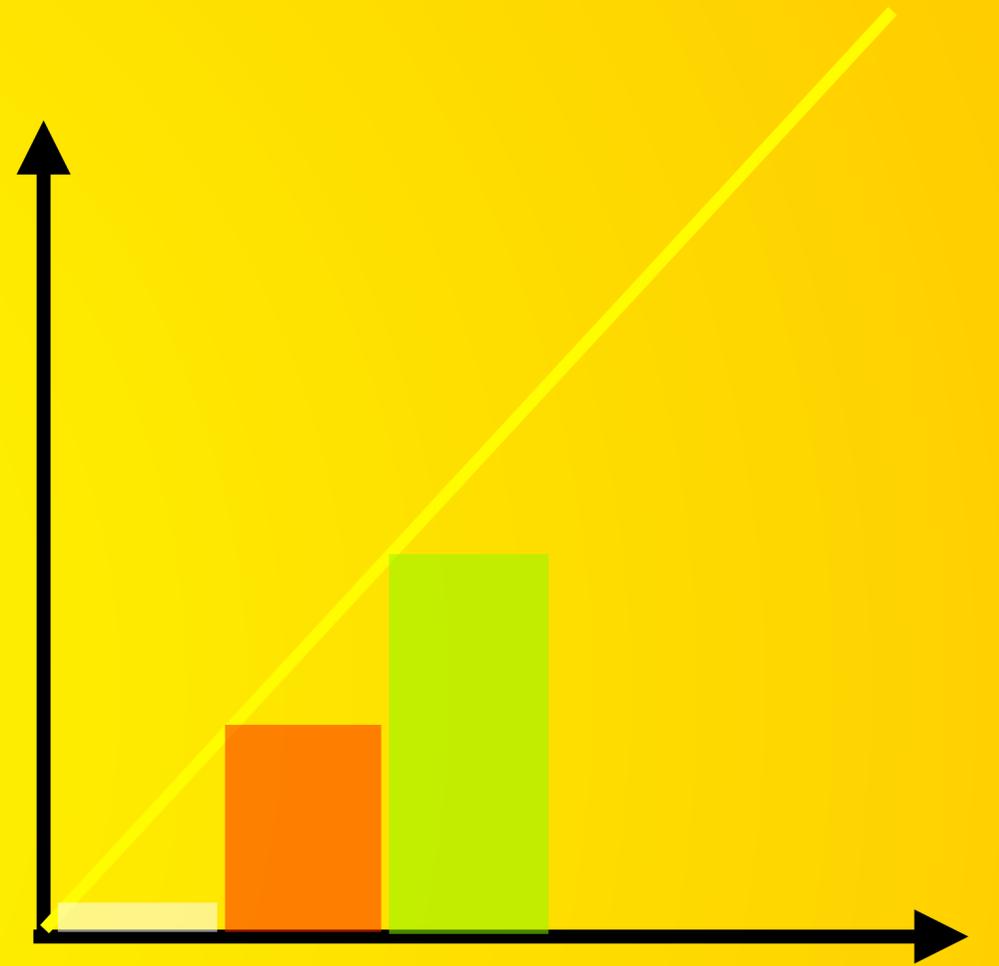
D

7

What is the value?

Sums are hard

$$\int_0^{99} x dx$$



$$S = 0 + 1 + 2 + 3 + \dots + 99$$



MEASURING THE WORLD 2012

SUM OF SQUARES

$$1 + 4 = 5$$

$$1 + 4 + 9 = 14$$

$$1 + 4 + 9 + 16 = 30$$

$$1 + 4 + 9 + 16 + 25 = 55$$

....

$$\int_0^n x^2 dx$$

```
data = Table[Sum[k^2, {k, n}], {n, 1, 10}]  
f = Chop[Fit[data, {1, x, x^2, x^3}, x]]
```



FROM FIRST GRADE

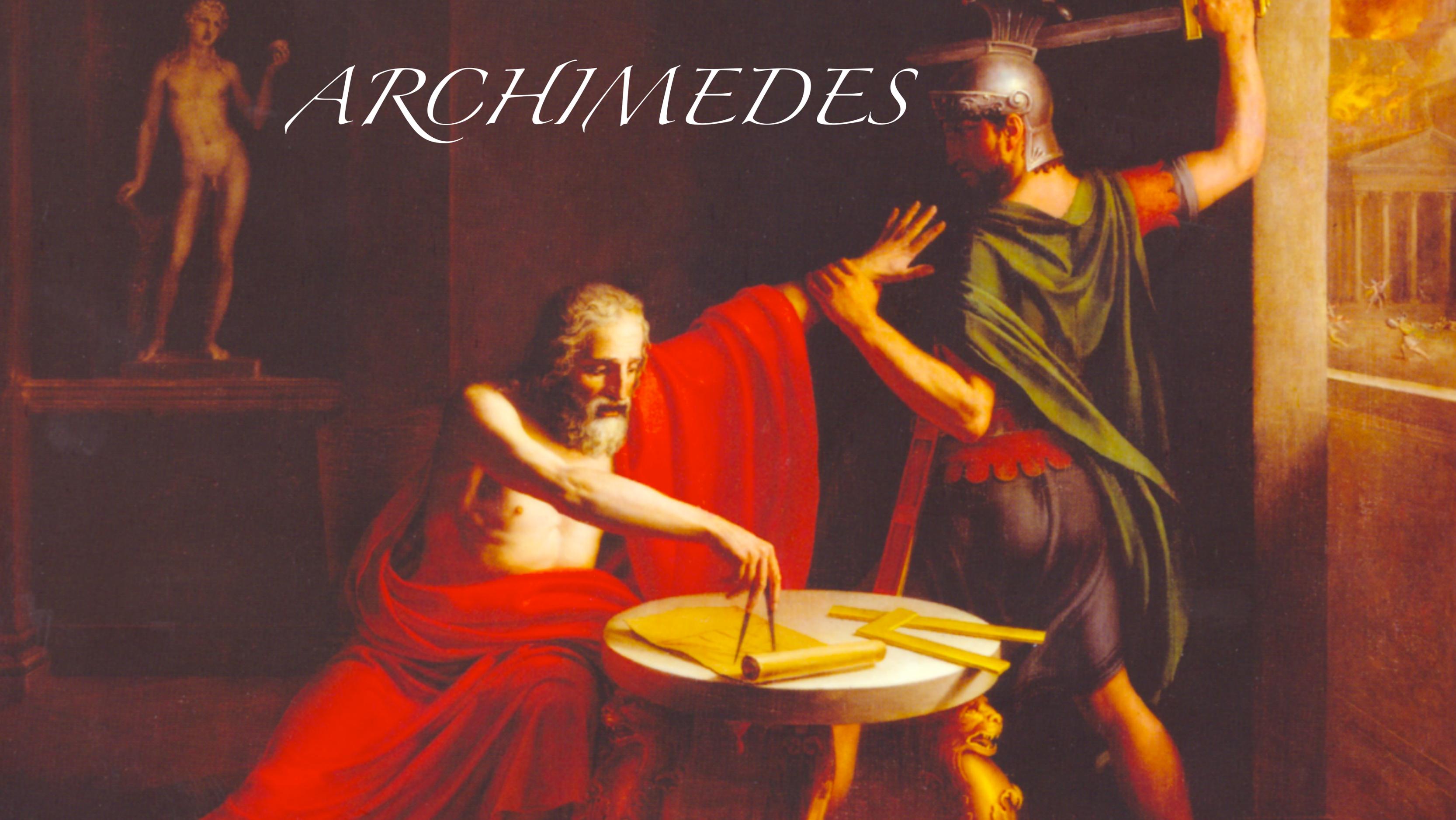


TO HIGH SCHOOL



History

ARCHIMEDES



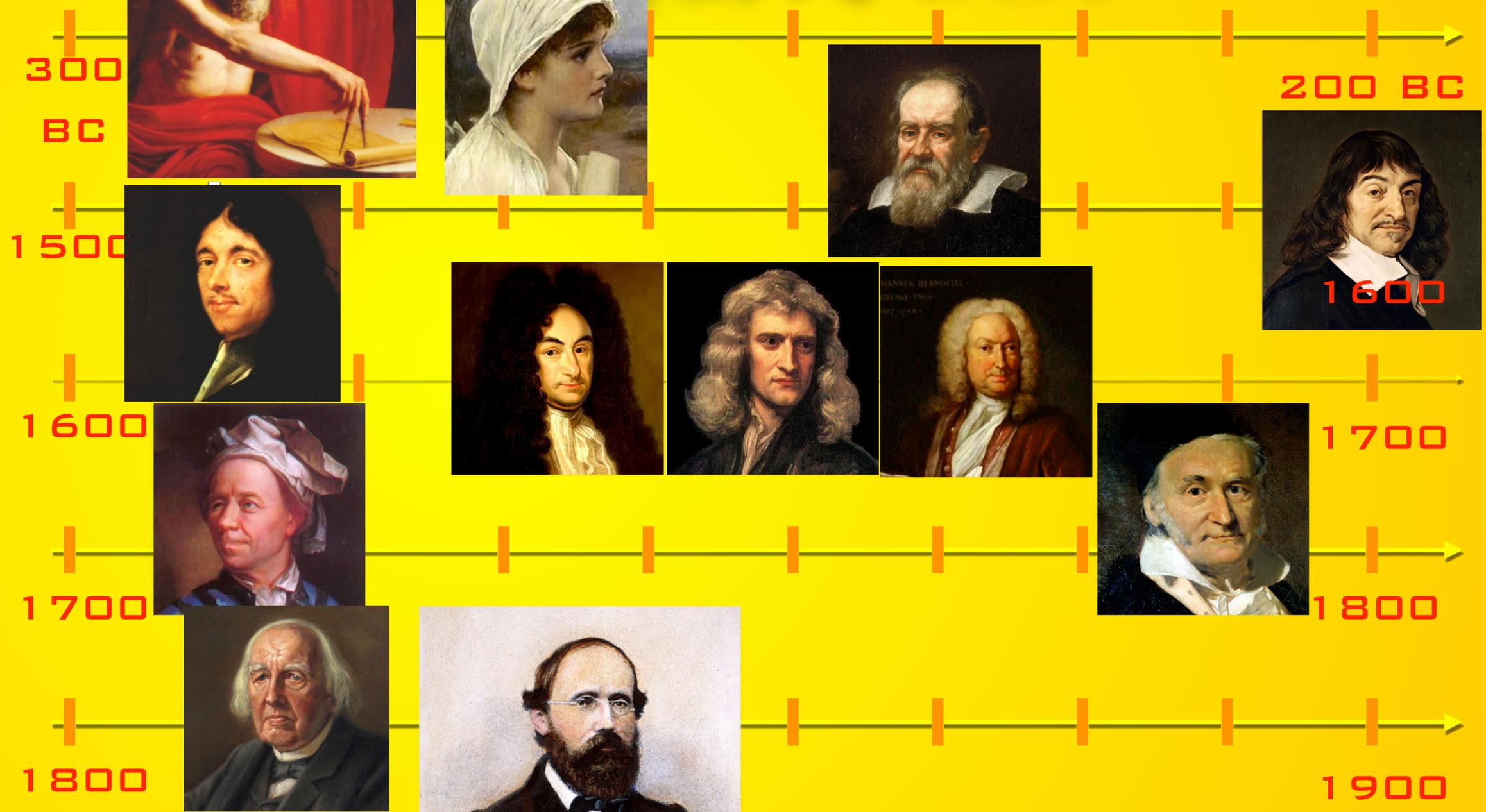


The volume of a sphere is



FROM ARCHIMEDES TO RIEMANN

2000
years of
developm
ent



Riemann

MONTÈ CARLO DEMO

The end