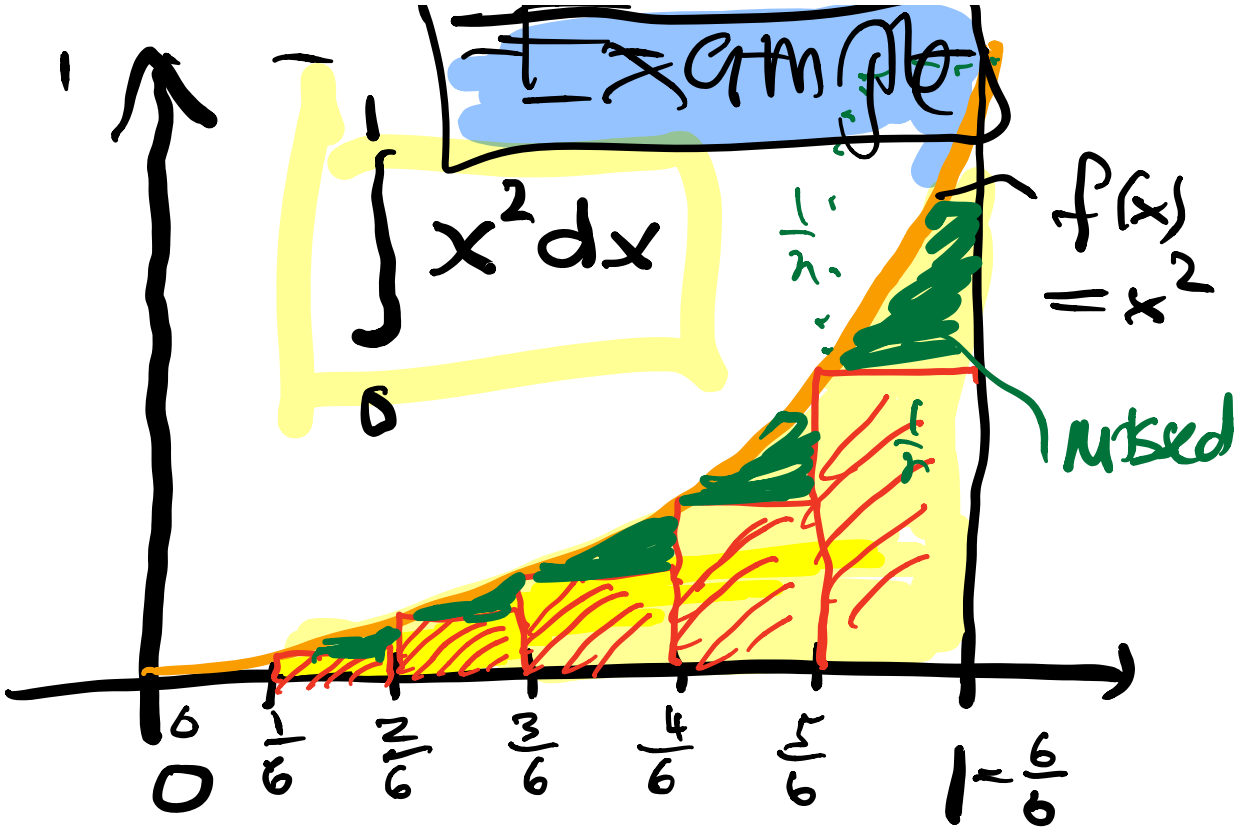


$$\sum_{a \le \frac{k}{n} < b} f\left(\frac{k}{n}\right) \frac{1}{n}$$

error $\le \frac{M}{n}$

Break out

$$\int_a^b f(x)\,dx$$

Nice

$$\frac{1}{6}\left(\left(\frac{0}{6}\right)^2+\left(\frac{1}{6}\right)^2+\left(\frac{2}{6}\right)^2+\left(\frac{3}{6}\right)^2+\left(\frac{4}{6}\right)^2+\left(\frac{5}{6}\right)^2\right)$$

$$\frac{1}{6^3}\left(0^2+1^2+2^2+3^2+4^2+5^2\right)=\frac{55}{216}$$

$$0+1^2+2^2+\ldots(n-1)^2=\frac{n^3}{3}-\frac{n^2}{2}+\frac{n}{6}$$

$$\frac{6^3}{3}-\frac{6^2}{2}+\frac{6}{6}$$ → magic formula.

0.254...

Riemann sum for $n=6$

This is tough!

Calculus is much more conceptual

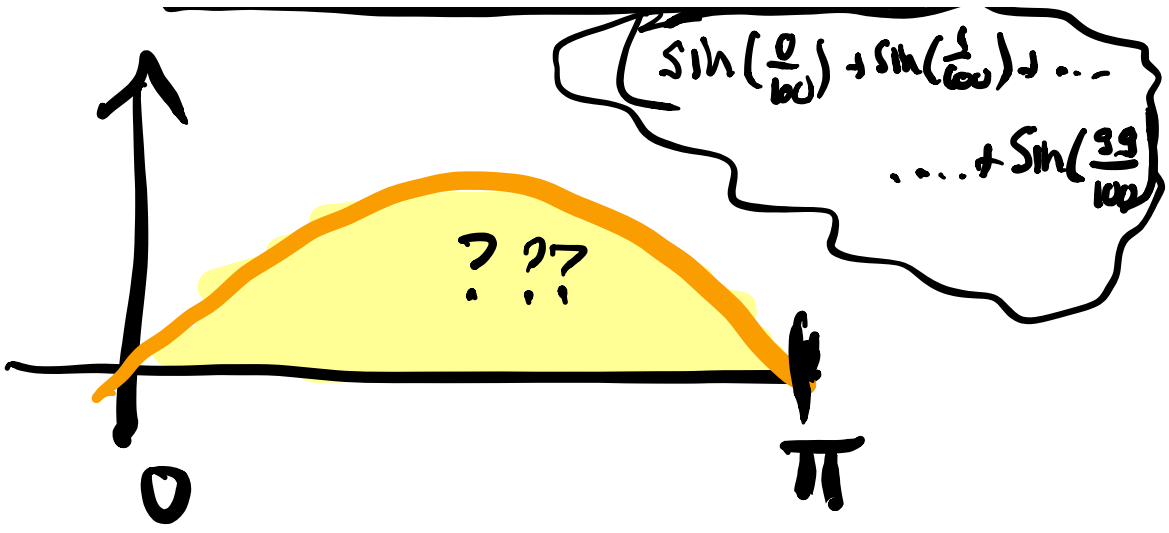
$$\int_0^1 x^2\,dx$$

$$= \frac{1}{3}x^3\Big|_0^1 = \frac{1}{3}\left(1^3 - 0^3\right) = \boxed{\frac{1}{3}}$$

$F(x)$ $\quad F' = x^2$

Example 2

$$\int_0^{\pi} \sin(x)\,dx$$



$$F(x) = -\cos(x)$$

$$F'(x) = \sin x$$

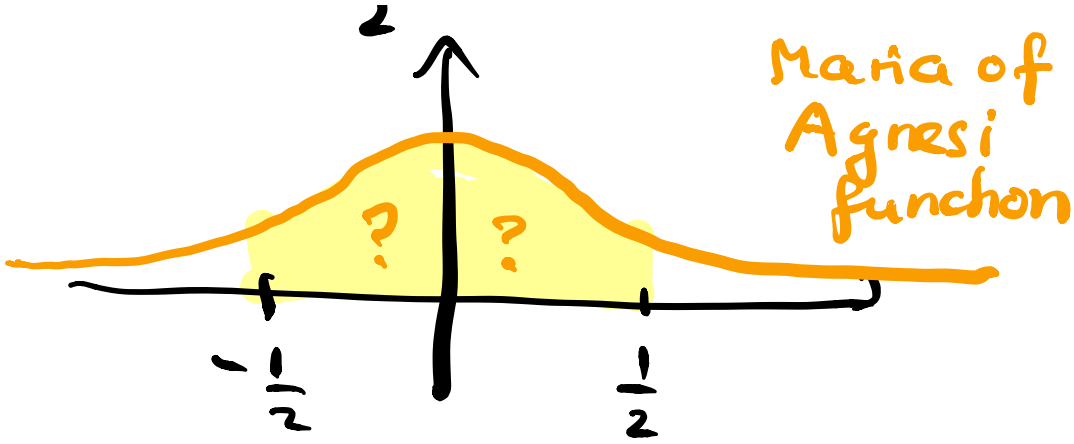
$$\int_0^{\pi} \sin x\,dx = -\cos x \Big|_0^{\pi}$$

$$= -\cos\pi + \cos 0$$

$$= 1 + 1 = \boxed{2}$$

Example 3

What is $\int_{-1}^{1/2} \frac{1}{1+x^2}\,dx$?

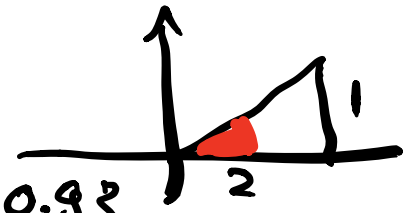


$$F(x) = \arctan(x)$$

$$F'(x) = \frac{1}{1+x^2}$$

$$\int_{-1/2}^{1/2} \frac{1}{1+x^2}\,dx = \arctan(x)\Big|_{-1/2}^{1/2}$$

$$\boxed{2\arctan\left(\tfrac{1}{2}\right)} \approx 0.93$$



arctan of $\frac{1}{2}$

Why not: $F(x) = \log(1+x^2)\frac{1}{2}$

$$F'(x) = \frac{1}{1+x^2}\frac{1}{2}\cdot 2x$$

(—› Substitution) $= \boxed{\frac{x}{1+x^2}}$?

Video hello choice of bonus.

$$\int_{-1}^{1} \frac{1}{1+x^2}\,dx = \arctan(x)\Big|_{-1}^{1}$$

$$= 2\arctan(1)$$

$$= 2\,\frac{\pi}{4} = \frac{\pi}{2}$$

Know derivatives to compute integrals.