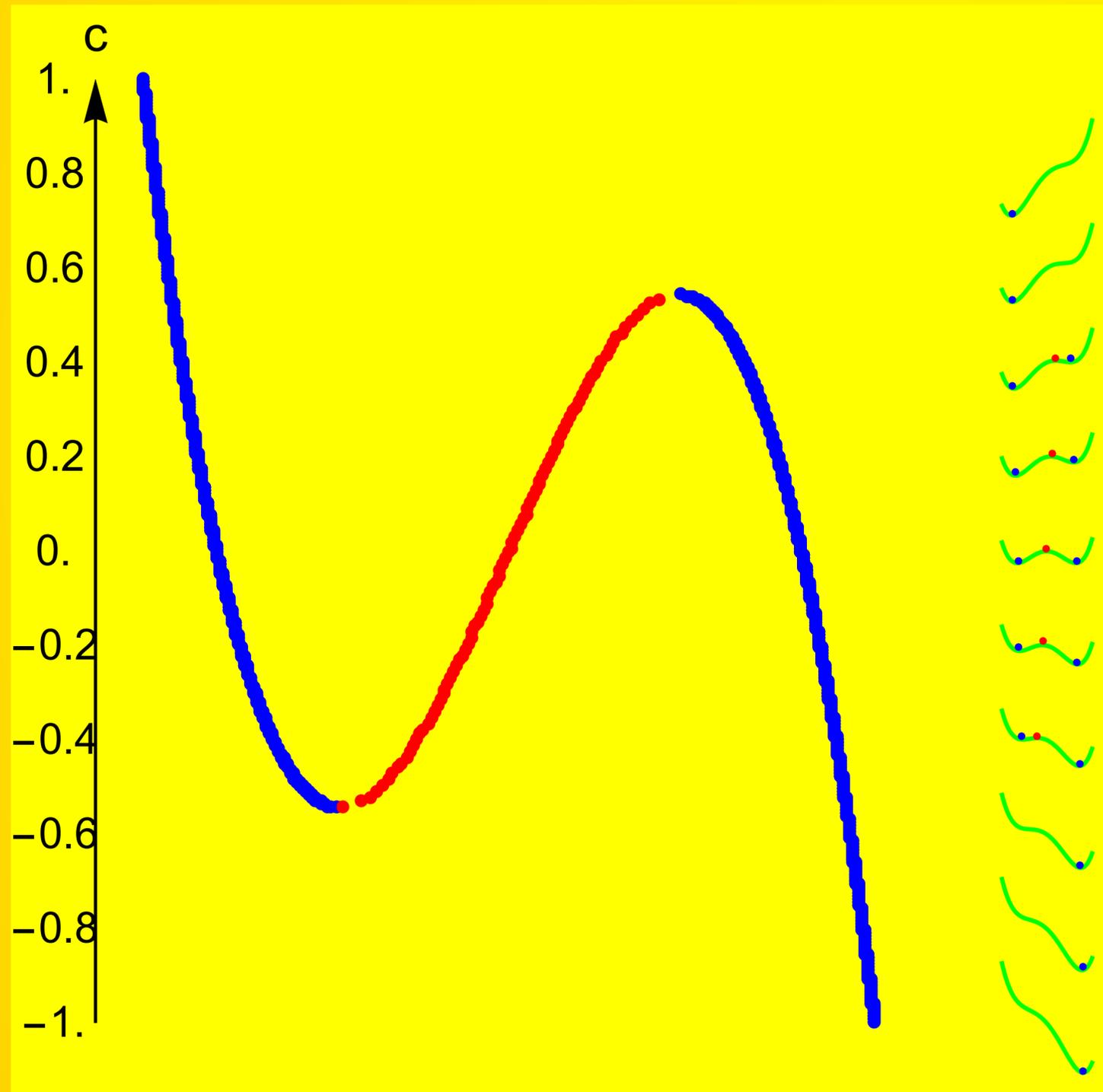


16

Catastrophes



minima are also called
stable equilibria

Catastrophe: c value,
where number of minima
changes.

$$f(x) = x^4 - x^2 + cx$$

About the exam.

Poll

What is this function



What is this function

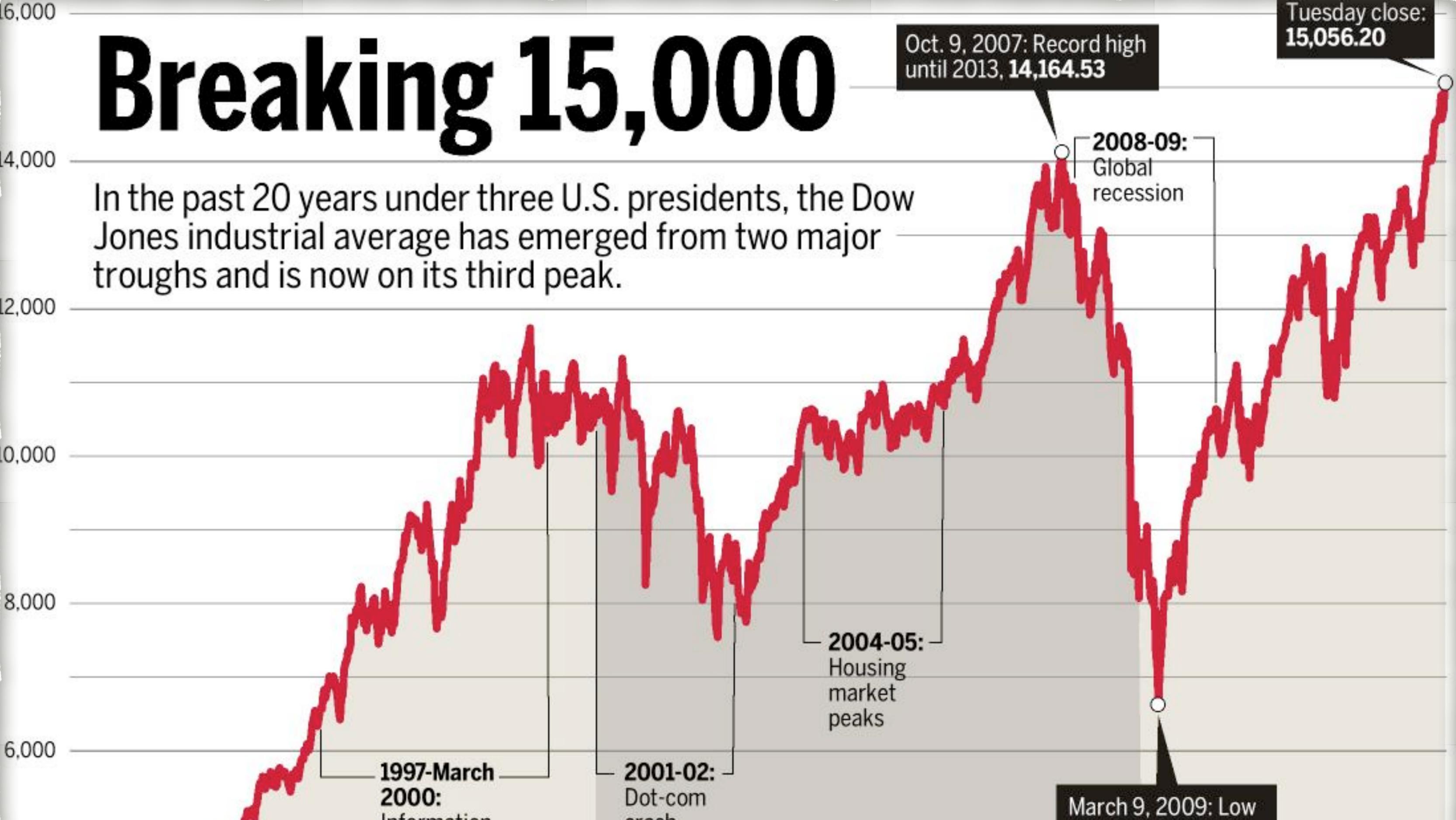


Breaking 15,000

Oct. 9, 2007: Record high until 2013, **14,164.53**

Tuesday close: **15,056.20**

In the past 20 years under three U.S. presidents, the Dow Jones industrial average has emerged from two major troughs and is now on its third peak.



1997-March
2000:
Information

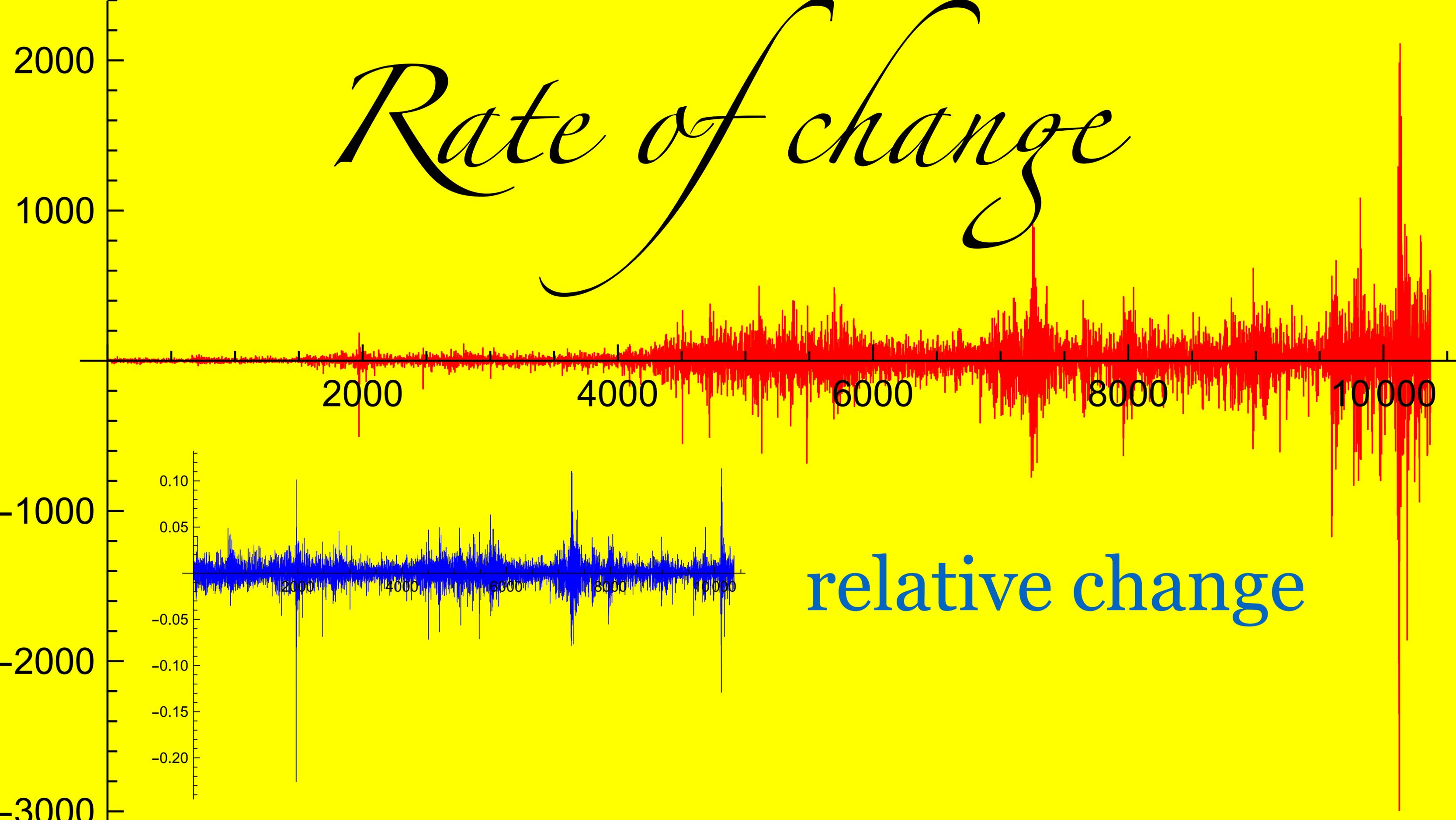
2001-02:
Dot-com
crash

2004-05:
Housing
market
peaks

March 9, 2009: Low

2008-09:
Global
recession

Rate of change



relative change

V. I. Arnold

Catastrophe Theory

Second, Revised and Expanded Edition

1 Singularities, Bifurcations, and Catastrophes

The first information on catastrophe theory appeared in the western press about ten years ago. In magazines like “Newsweek” there were reports of a revolution in mathematics, comparable perhaps to Newton’s invention of the differential and integral calculus. It was claimed that the new science, catastrophe theory, was much more valuable to mankind than mathematical analysis: while Newtonian theory only considers smooth, continuous processes, catastrophe theory provides a universal method for the study of all jump transitions, discontinuities, and sudden qualitative changes. There appeared hundreds of scientific and popular science publications in which catastrophe theory was applied to such diverse targets as, for instance, the study of heart beat, geometrical and physical optics, embryology, linguistics, experimental psychology, economics, hydrodynamics, geology, and the theory of elementary particles. Among the published works on catastrophe theory are studies of the stability of ships, models for the activity of the brain and mental disorders, for prison uprisings, for the behaviour of investors on the stock exchange, for the influence of alcohol on drivers and for censorship policy with respect to erotic literature.

In the early seventies catastrophe theory rapidly became a fashionable and widely publicized theory which by its all-embracing pretensions called to mind the pseudo-scientific theories of the past century.

The mathematical articles of the founder of catastrophe theory, René Thom, were reprinted as a pocket book – something that had not happened in mathematics since the introduction

of cybernetics, from which catastrophe theory derived many of its advertising techniques.

Shortly after the eulogies of catastrophe theory there appeared more sober critical works. Some of these also appeared in publications intended for a wide readership, under eloquent titles like – ‘The Emperor has no clothes’. Now we already have many articles devoted to the criticism of catastrophe theory. (See for instance John Guckenheimer’s survey article ‘The Catastrophe Controversy’ in 1978 and the parody on the criticism of the theory in 1979.)¹

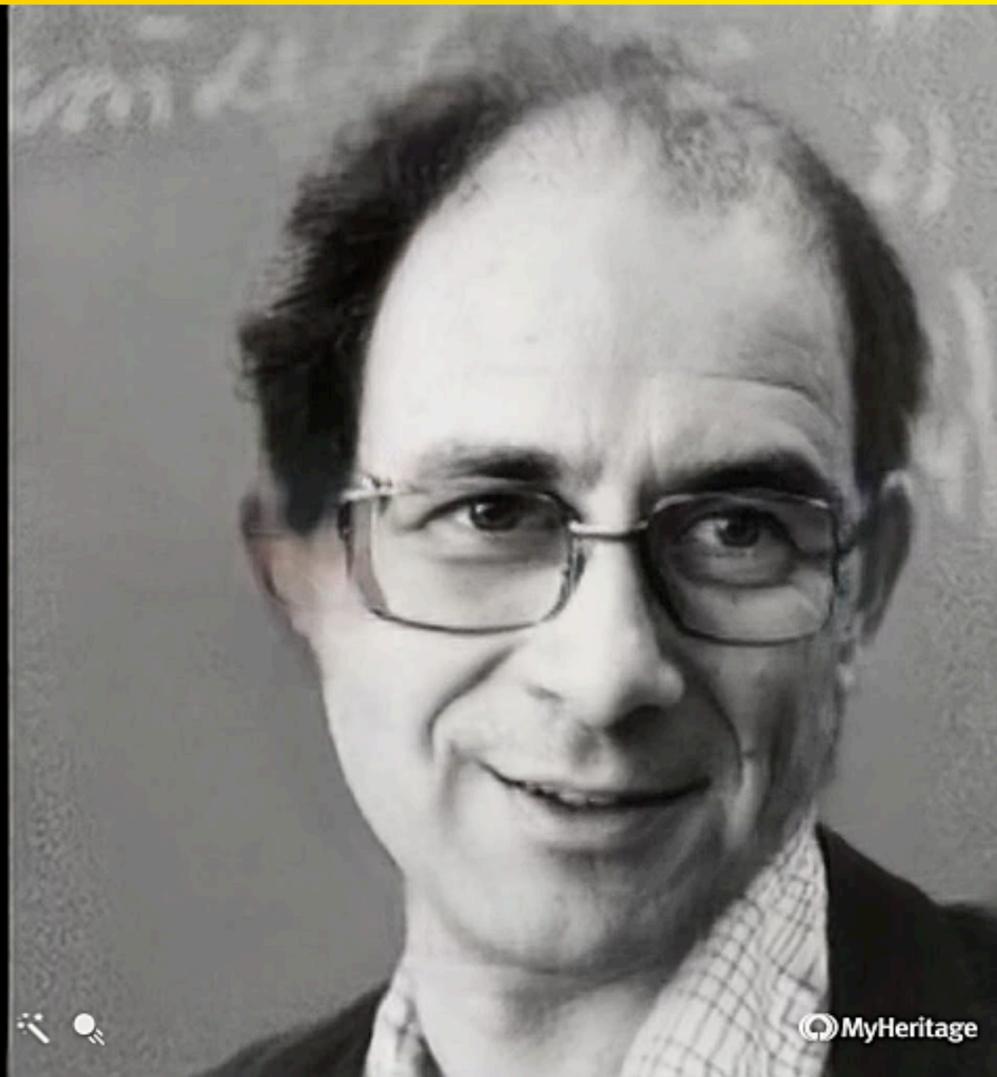
The origins of catastrophe theory lie in Whitney’s theory of singularities of smooth mappings and Poincaré and Andronov’s theory of bifurcations of dynamical systems.

Singularity theory is a far-reaching generalization of the study of functions at maximum and minimum points. In Whitney’s theory functions are replaced by mappings, i.e. collections of several functions of several variables.

The word *bifurcation* means *forking* and is used in a broad sense for designating all sorts of qualitative reorganizations or metamorphoses of various entities resulting from a change of the parameters on which they depend.

Catastrophes are abrupt changes arising as a sudden response of a system to a smooth change in external conditions. In order to understand what catastrophe theory is about one must first become acquainted with the elements of Whitney’s singularity theory.

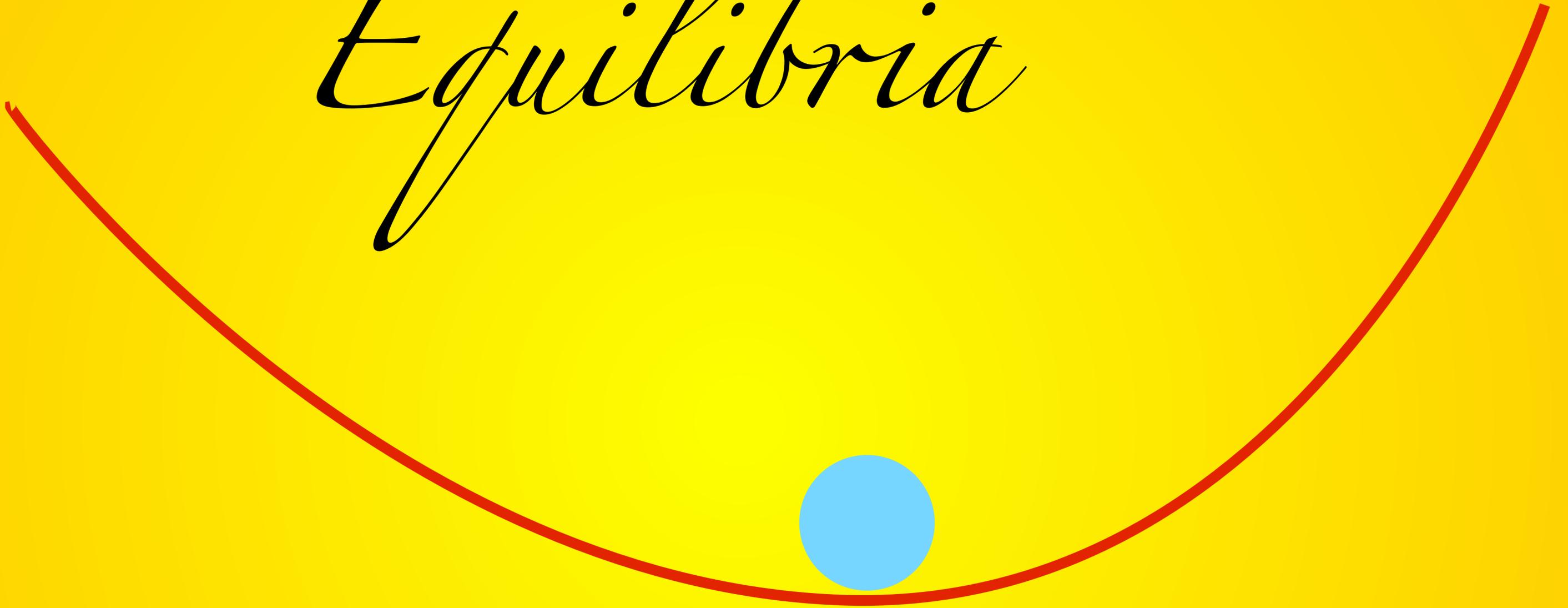
¹ Guckenheimer, J.: The Catastrophe Controversy. *The Mathematical Intelligencer* 1978, vol. 1, no. 1, pp. 15–20; Fussbudget, H. J. and Znarler, R. S., *Sagacity Theory: A Critique*, loc. cit. 1979, vol. 2, no. 1, pp. 56–59.



Springer-Verlag

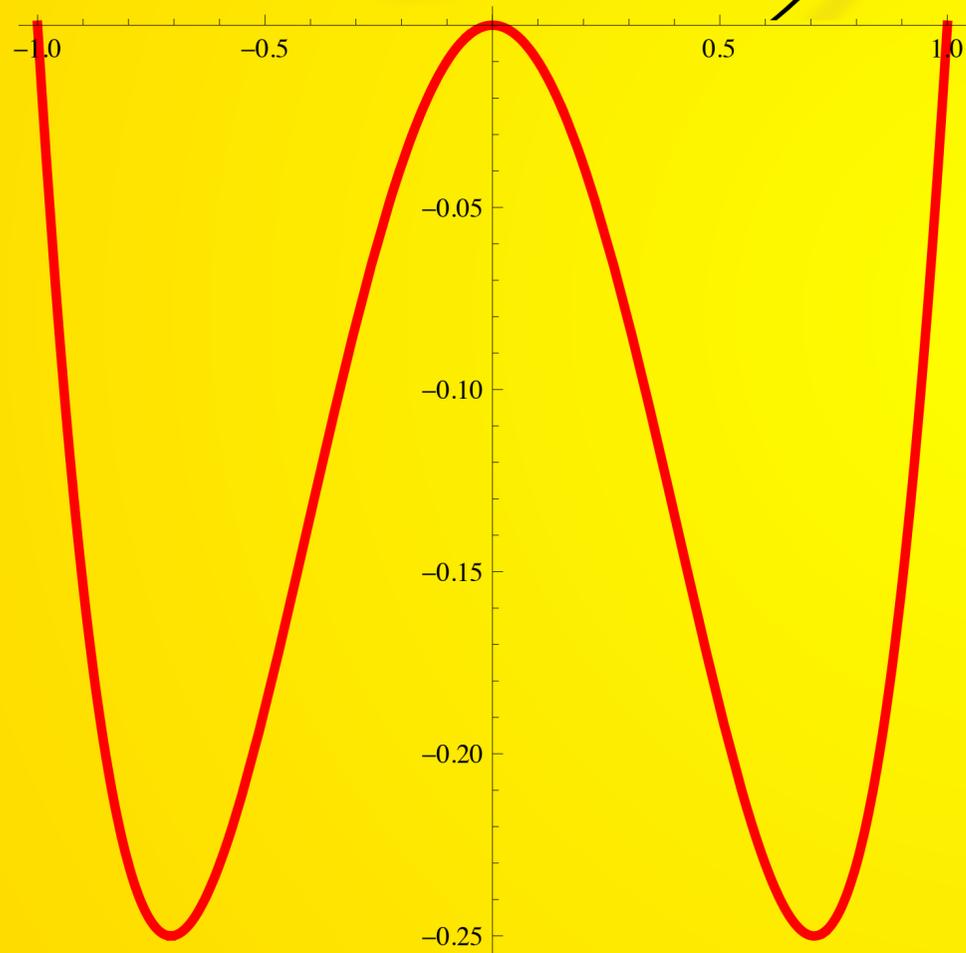
Berlin Heidelberg New York Tokyo

Equilibria

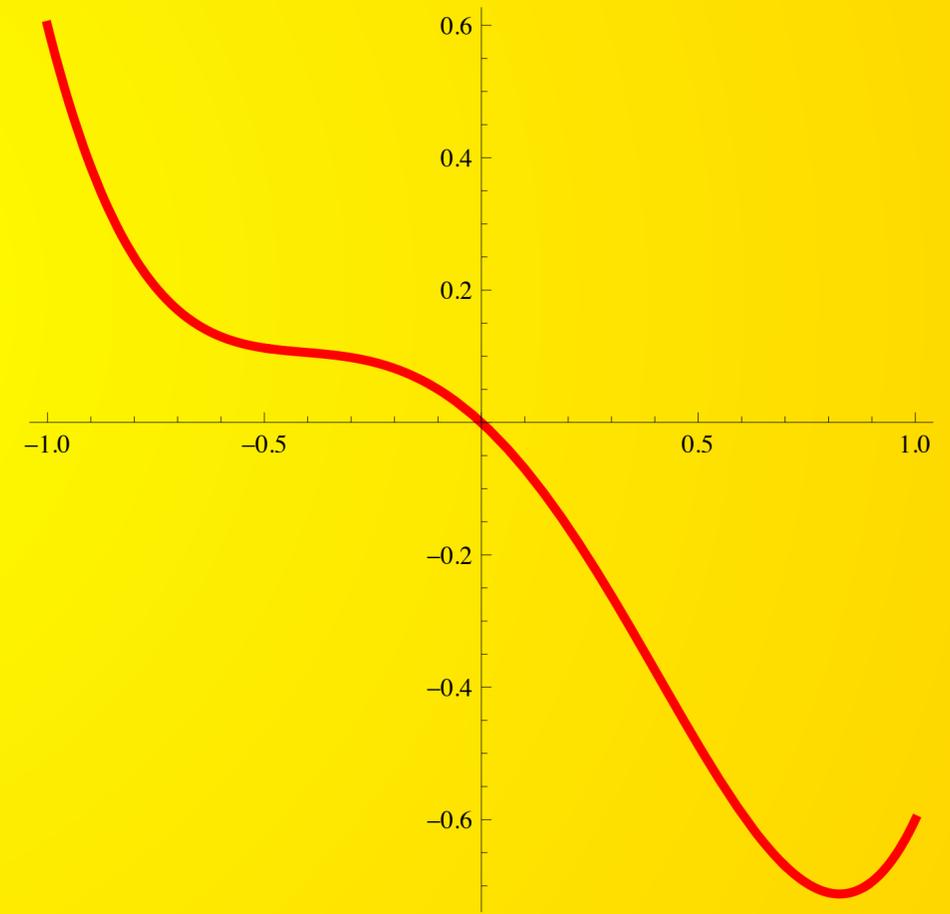


A local minimum is a stable equilibrium

Change of equilibria



$$x^4 - x^2$$

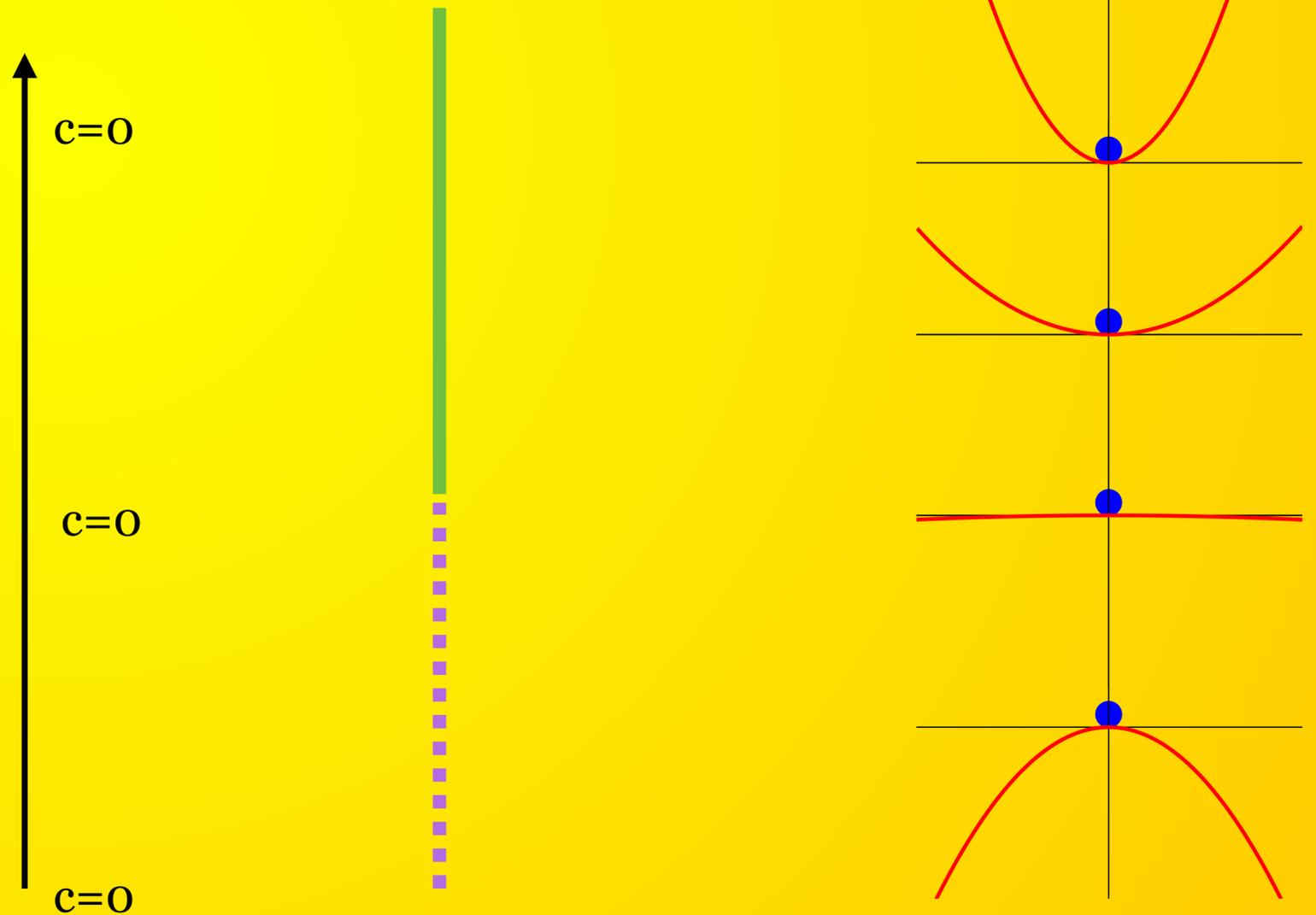


$$x^4 - x^2 - 0.6x$$

Catastrophes

Example: $f_c(x) = cx^2$

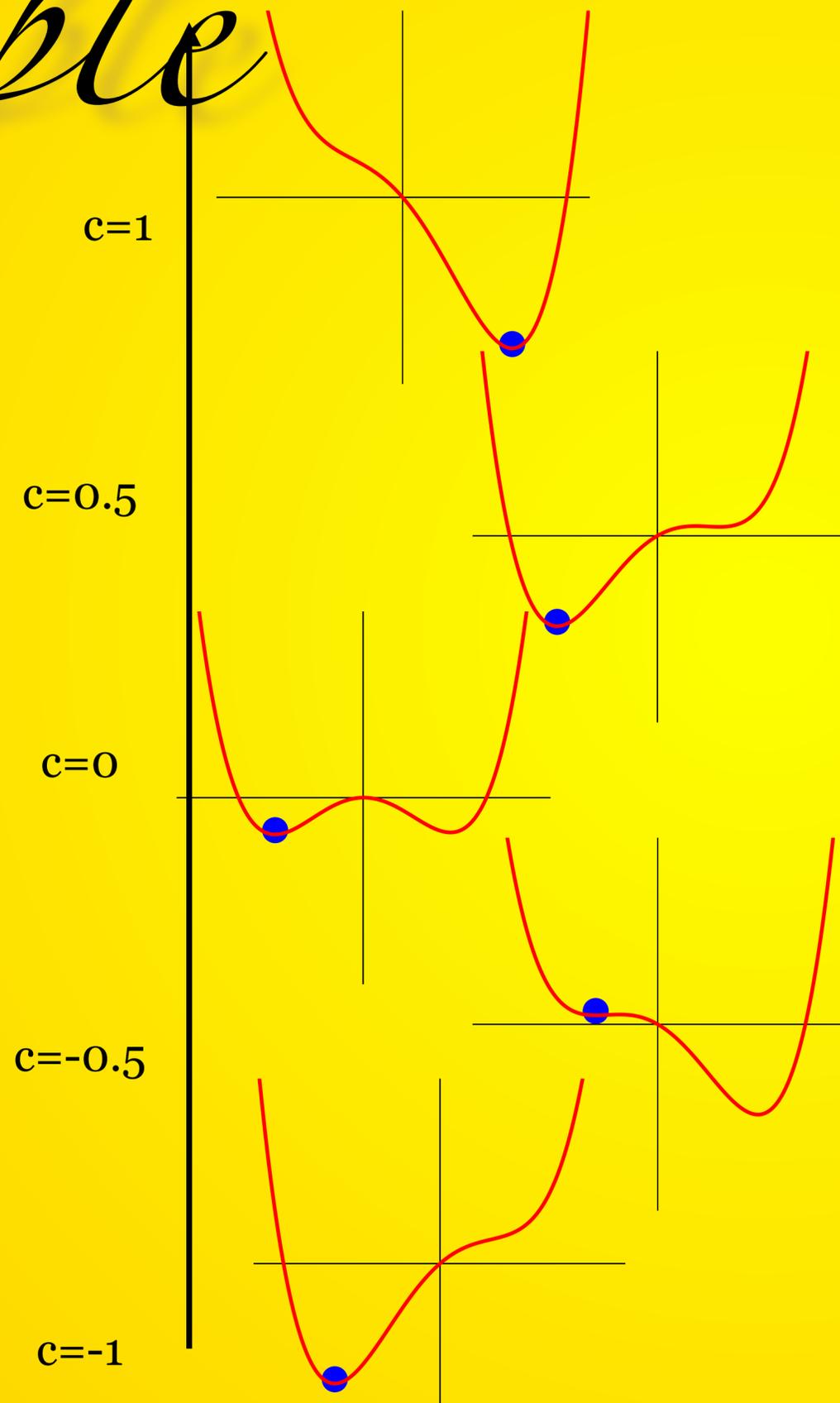
A parameter c , where the number of stable equilibria changes is called a catastrophe.

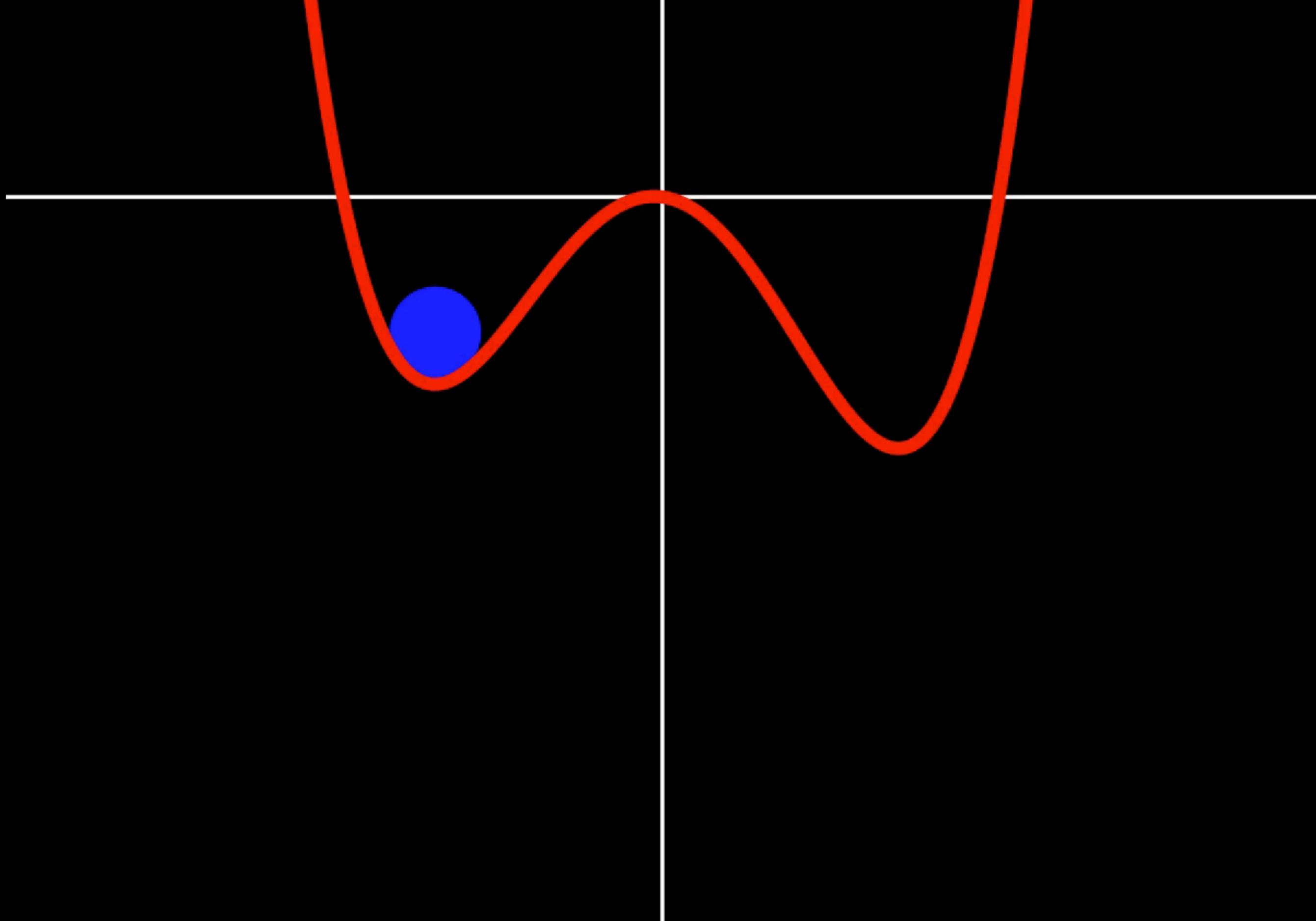


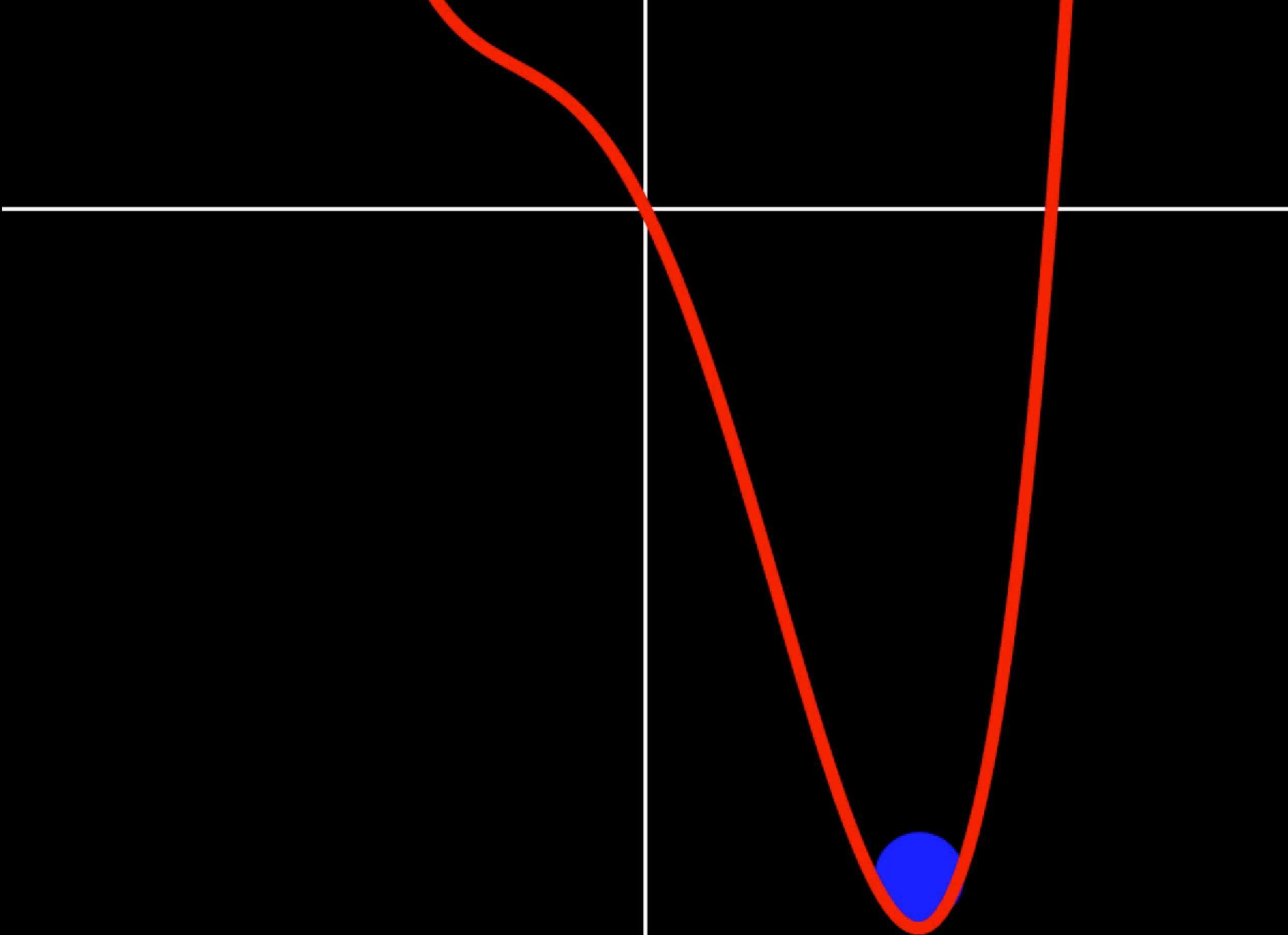
Example

$$f_c(x) = x^4 - x^2 - cx$$

draw the
bifurcation
diagram





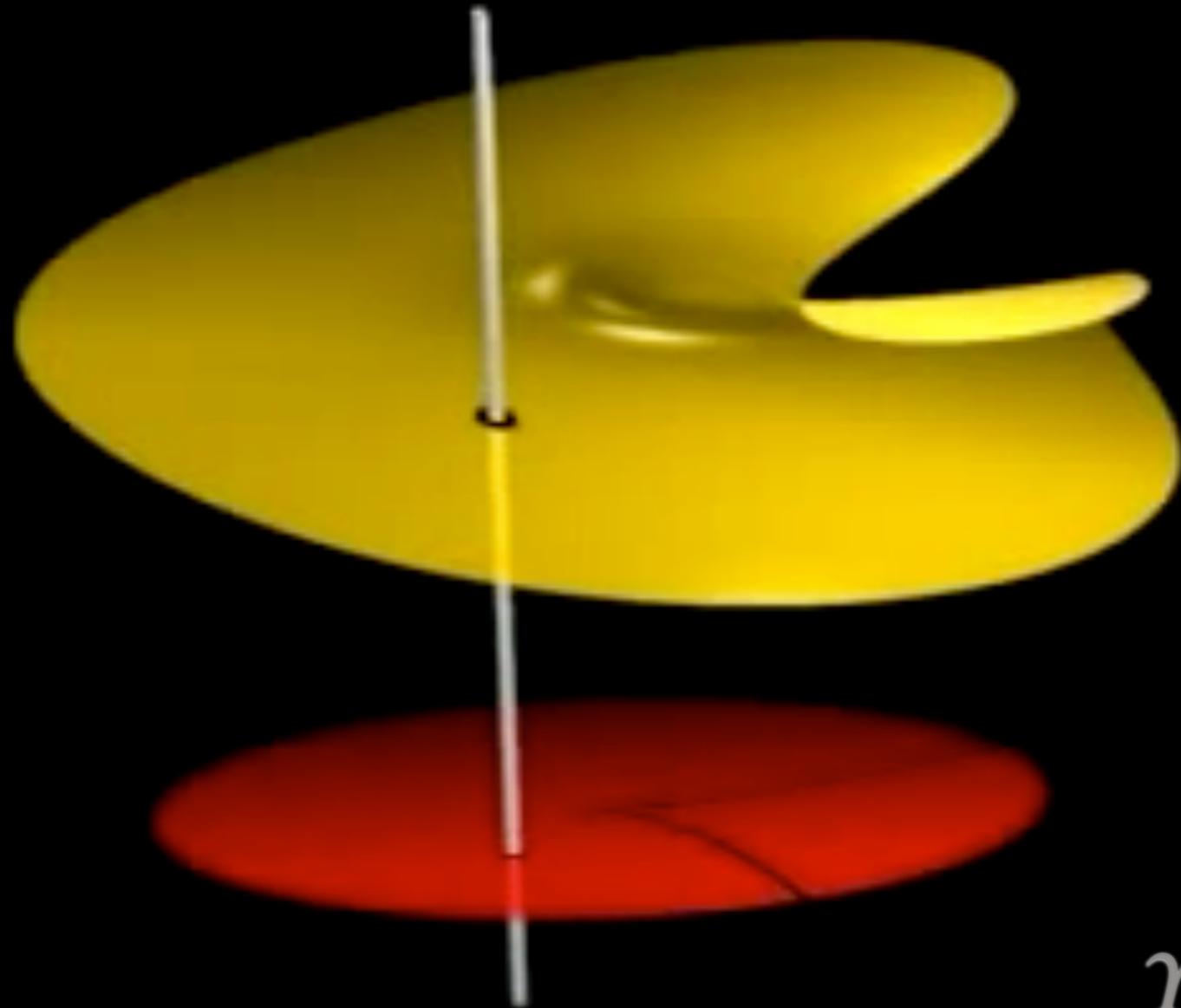


Boat Tipping





Conflicts



Youtube

THE CATASTROPHE CUSP MODEL AND THE CONFLICT HELIX

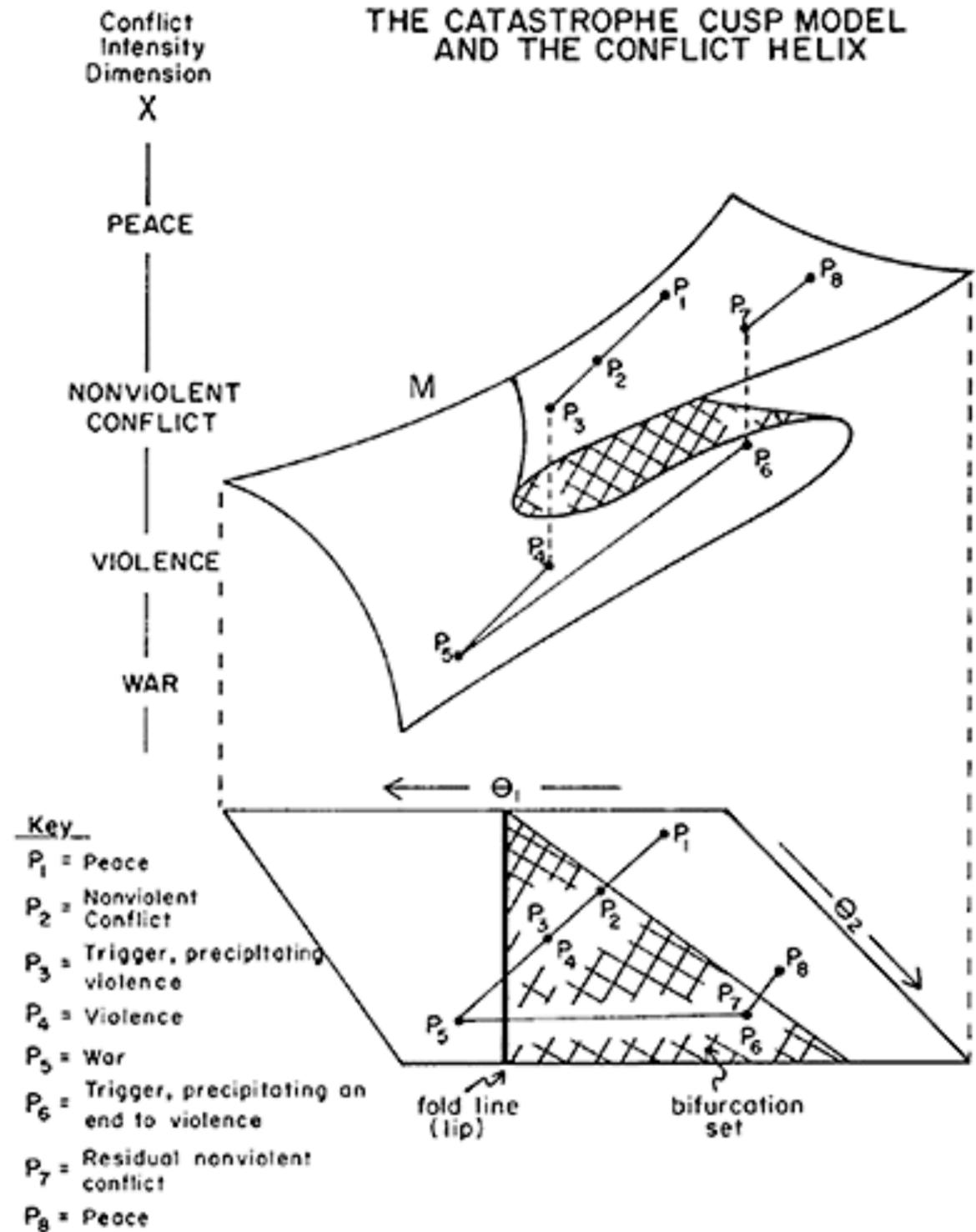
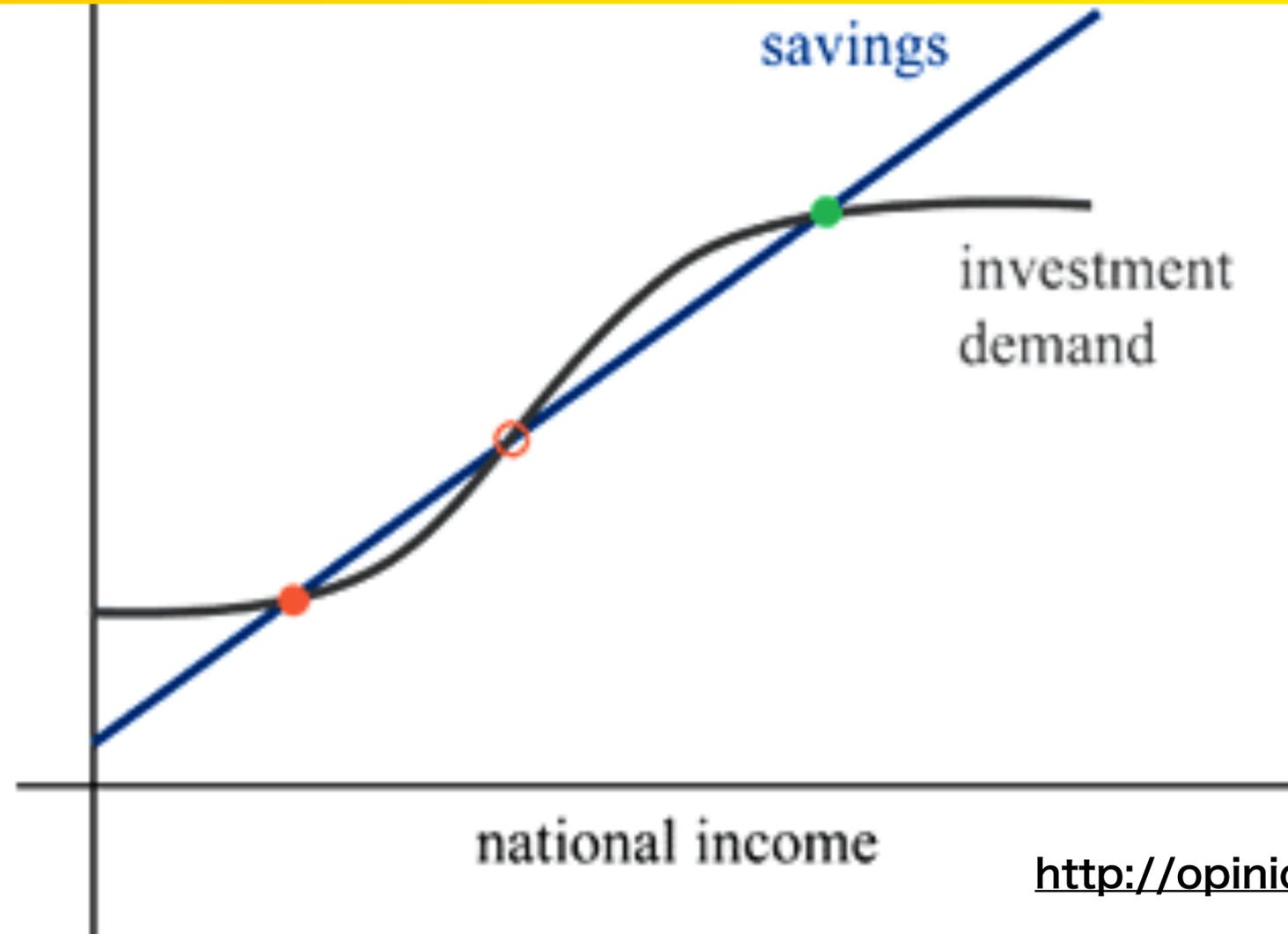


FIG. 3

Economics



<http://opinionator.blogs.nytimes.com/2012/10/08/dangerous-intersection>

Depending on how the line and curve are situated, one, two or three intersections can occur. The upper intersection (the green dot) represents a strong economy with high levels of national income. The lower equilibrium (solid red dot) depicts an economy stuck in the doldrums. The middle equilibrium (open red circle) turns out to be unstable and acts like a watershed; when economic conditions are near it, they drift away toward one of the other two equilibriums. The amount of investment doesn't depend only on national income but also on how much investment has already been accumulated. At some point enough is enough. During the housing boom, for example, increases in income fueled the demand for housing investment. But as the stock of housing rose, the demand for investment dropped. In the model this drags the S-shaped investment curve down. It's like the straw being added to the camel's back.

Perception



Perception

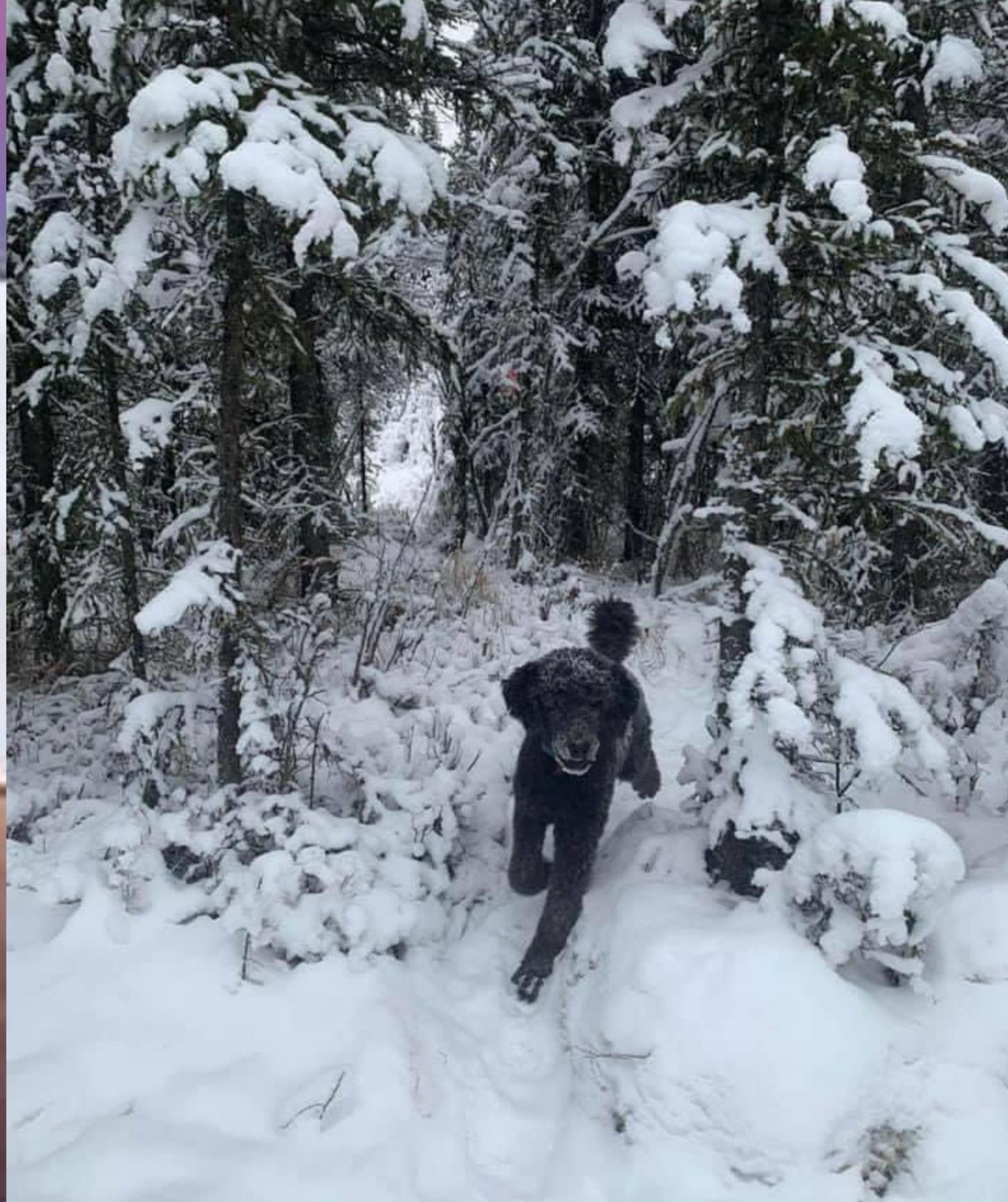


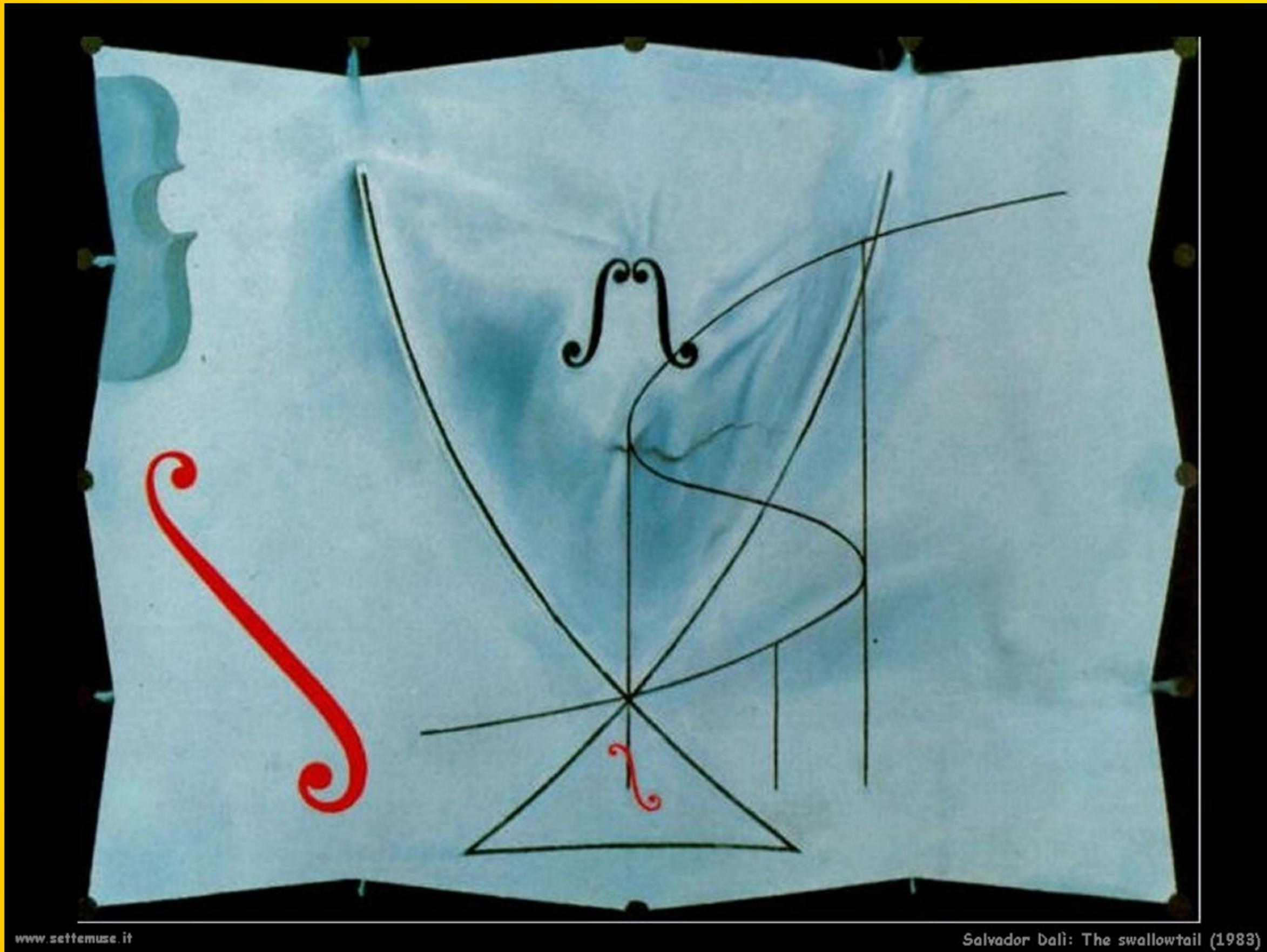
Perception





Do you see the face? Or an Eskimo?





Art

Catastrophically creative: Salvador Dalí and Maths

By [Silvia Benvenuti](#) on 20/05/2017



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TWEET



SHARE



SHARE

2 COMMENTS

May 14th, 2017 was the last day of the exhibition [Dalí Experience](#), at Palazzo Belloni in Bologna. A fantastic experience for maths lovers!

By Silvia Benvenuti

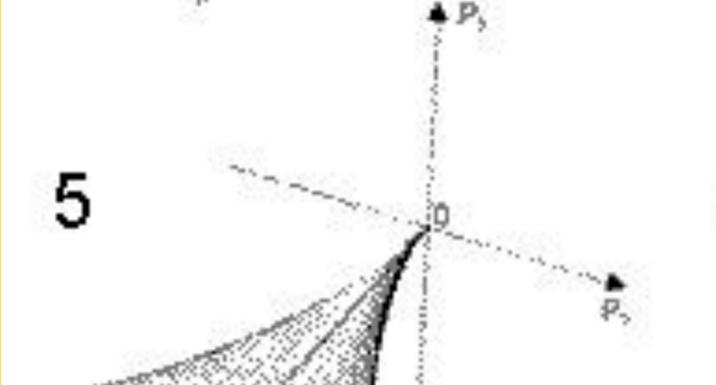
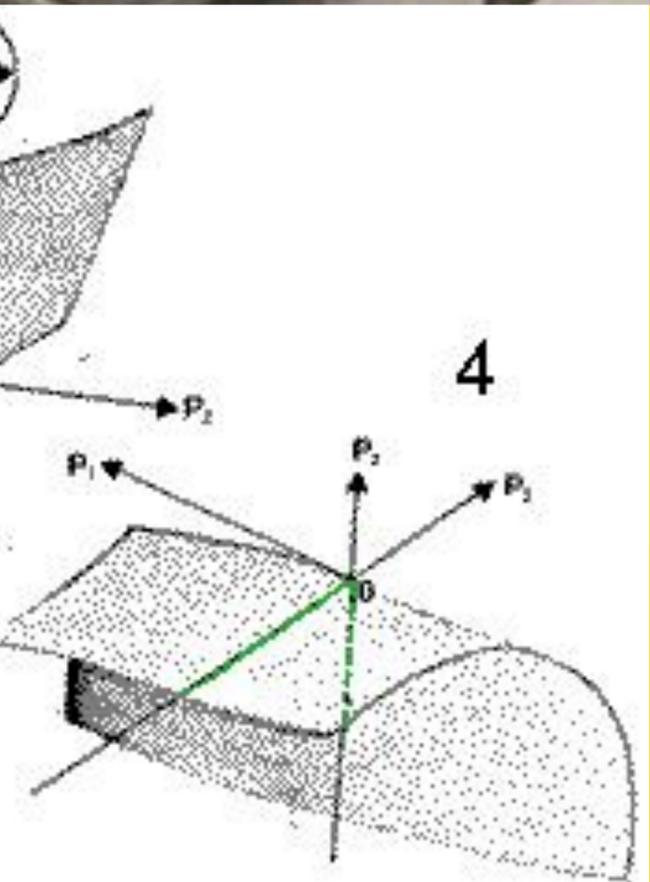
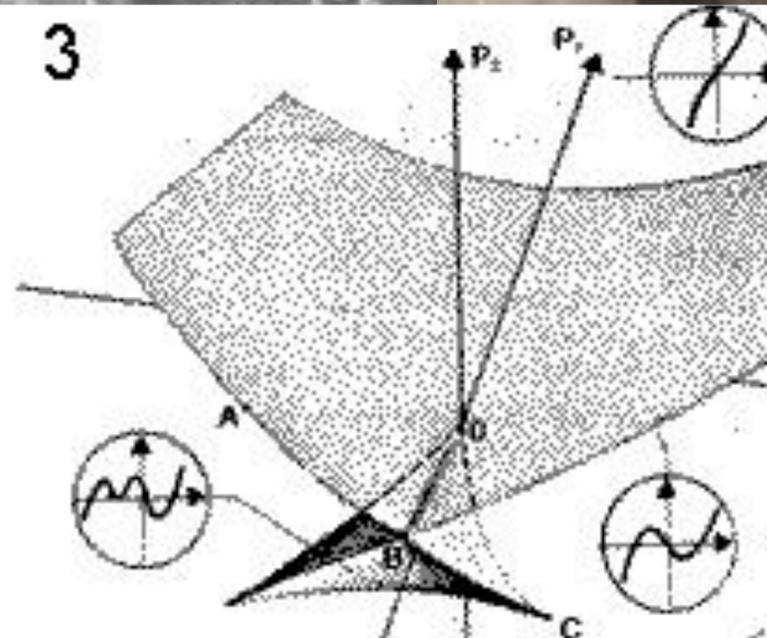
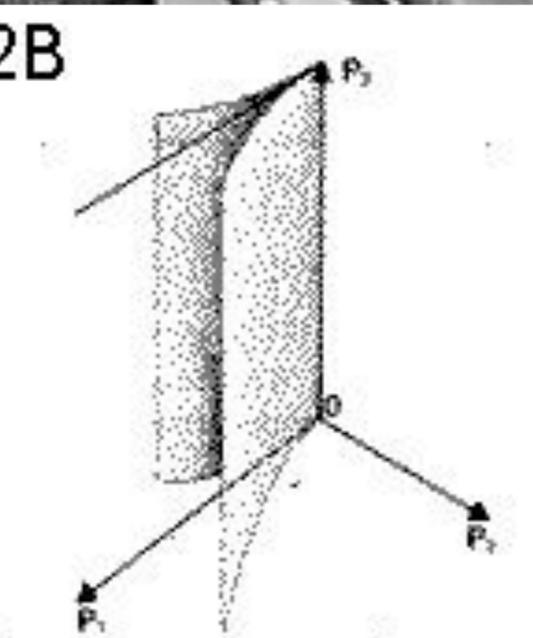
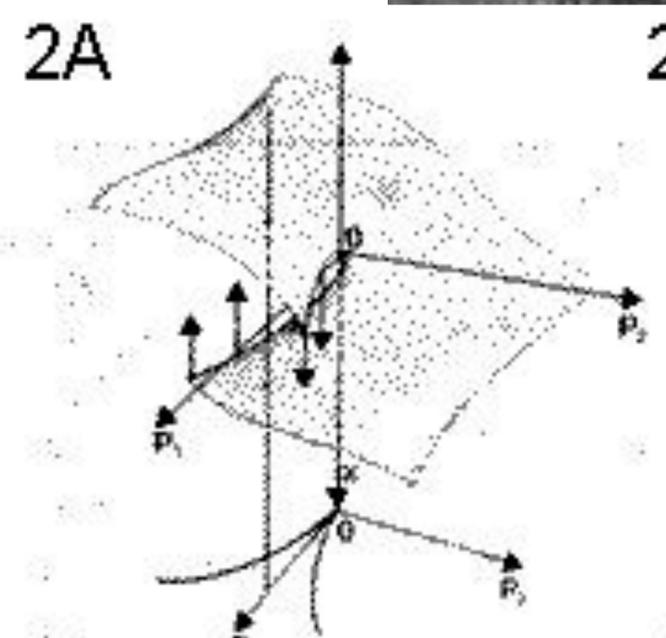
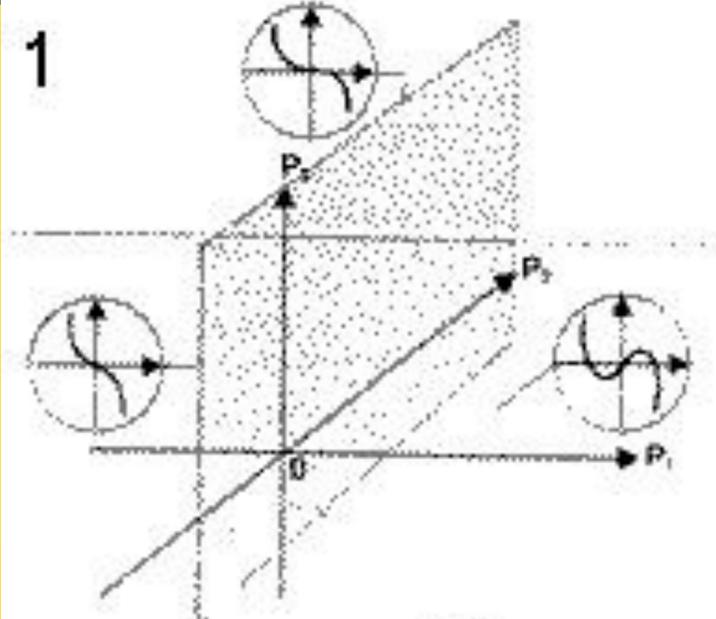
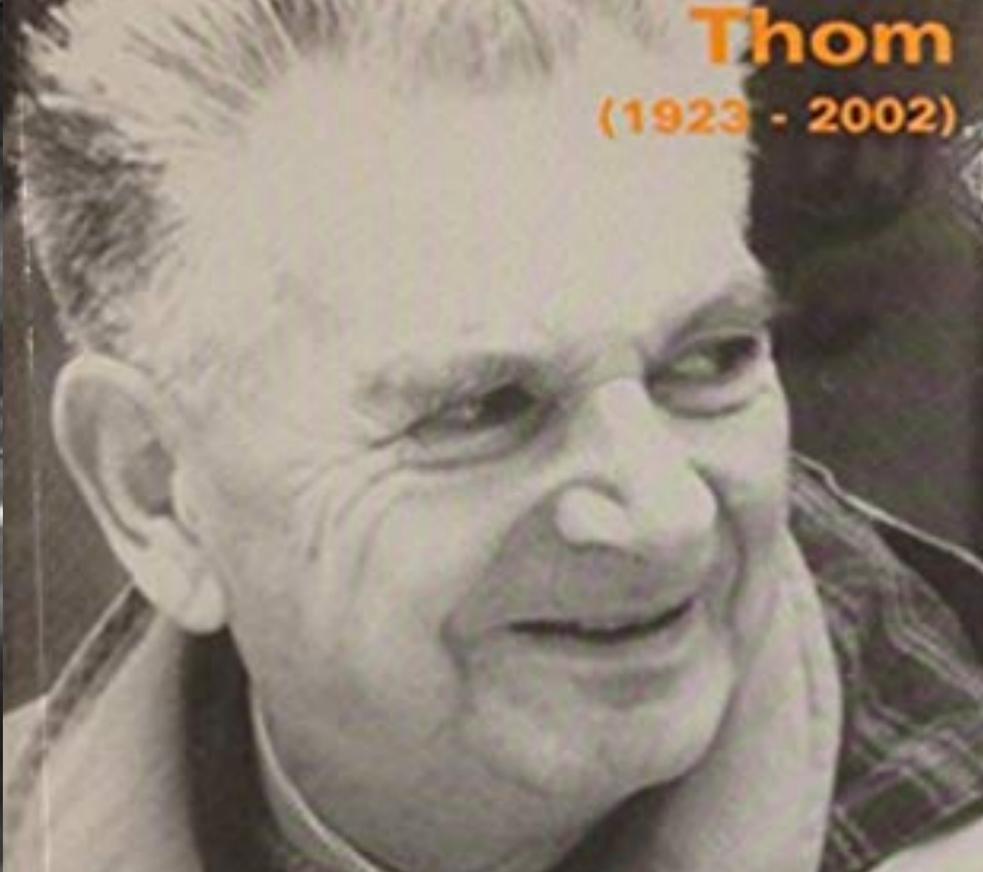
Maths?!

Maths, yes.

(And art, of course!)

If you are wondering what the hell Salvador Dalí, one of the most subversive and revolutionary artists of his time, has to do with maths, just take a look at his mustache in one of the photographs

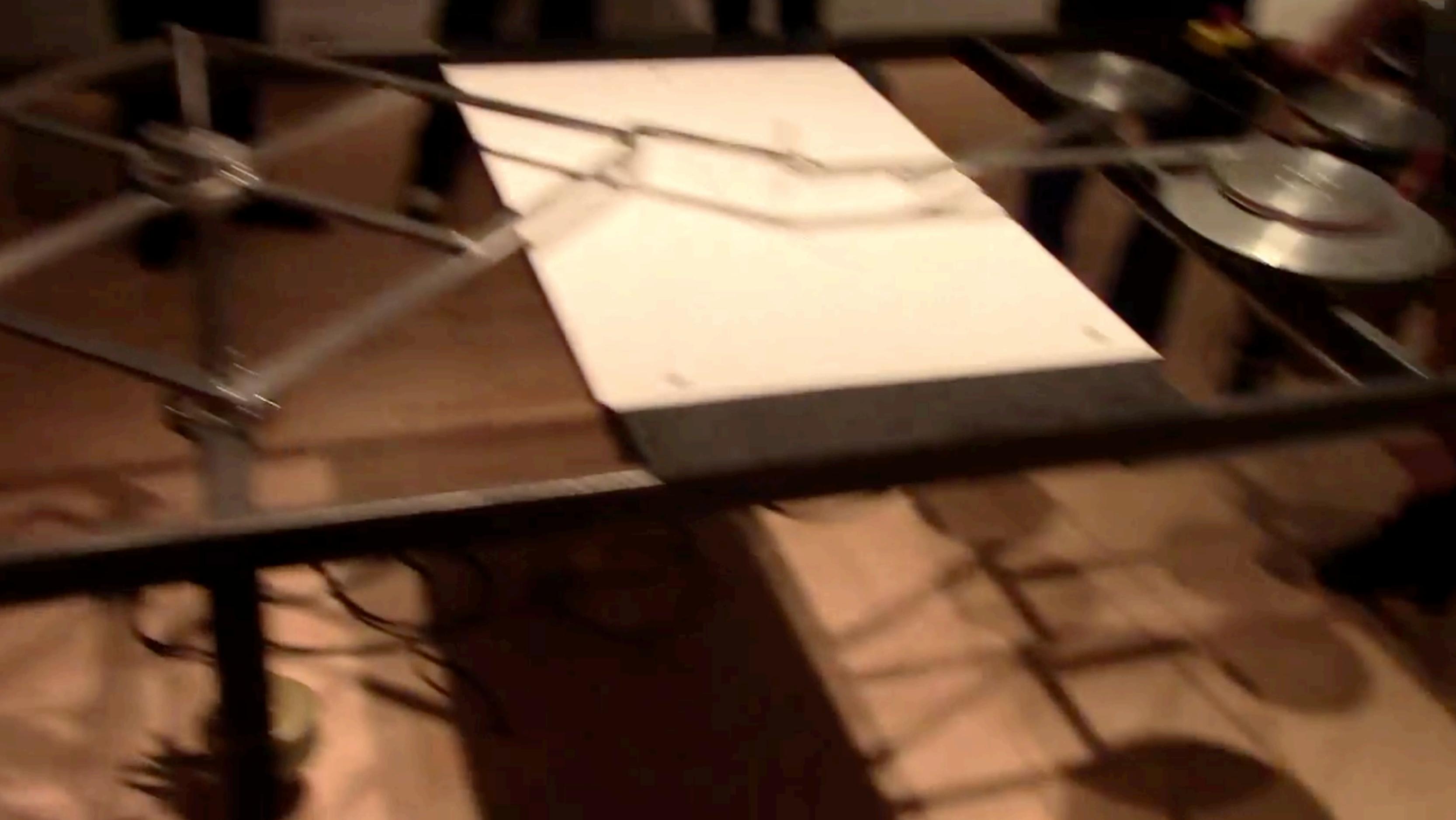








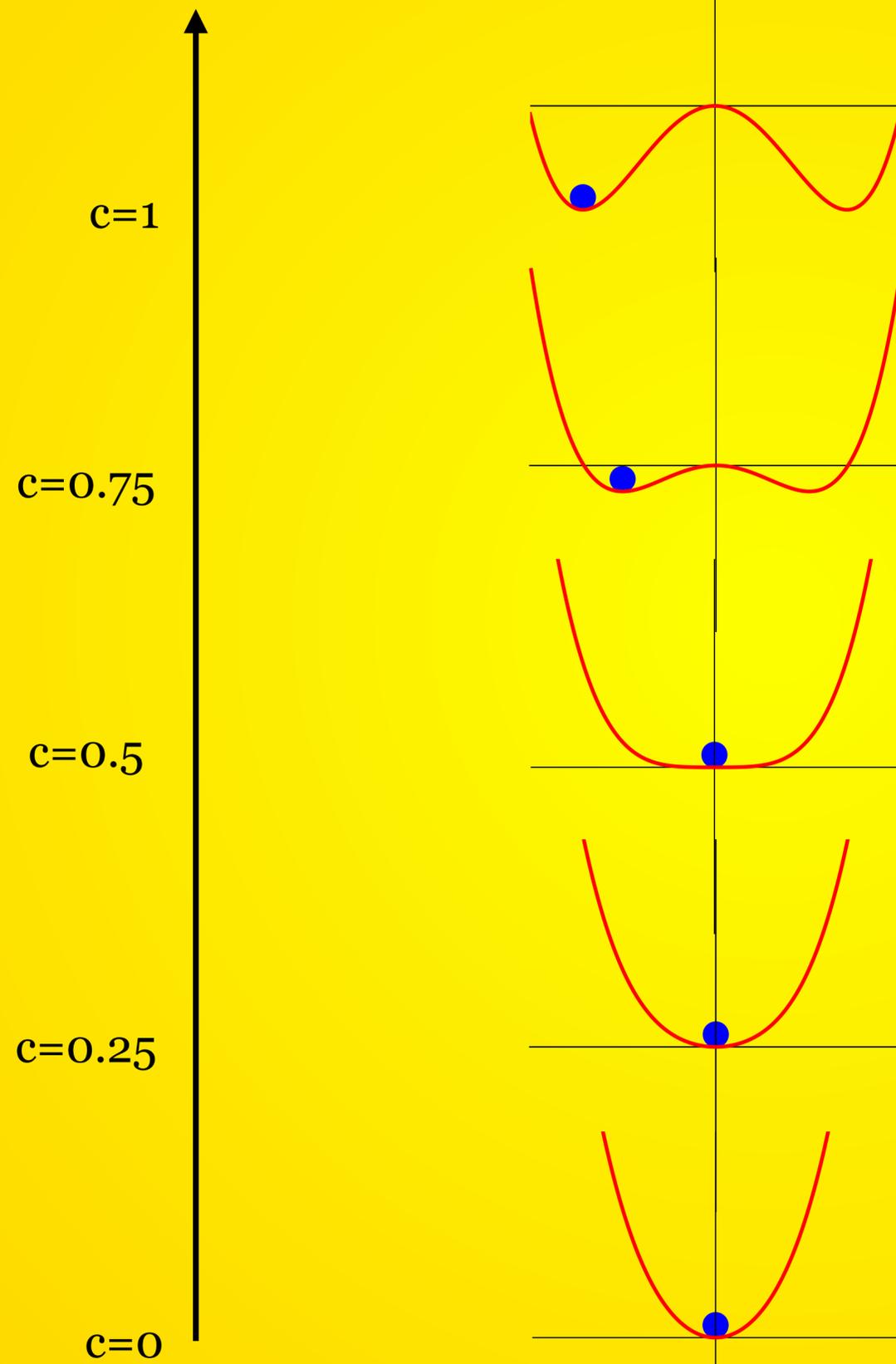
Zeeman



Jam

$$f_c(x) = x^4 - cx^2$$

draw the
bifurcation
diagram



The End