

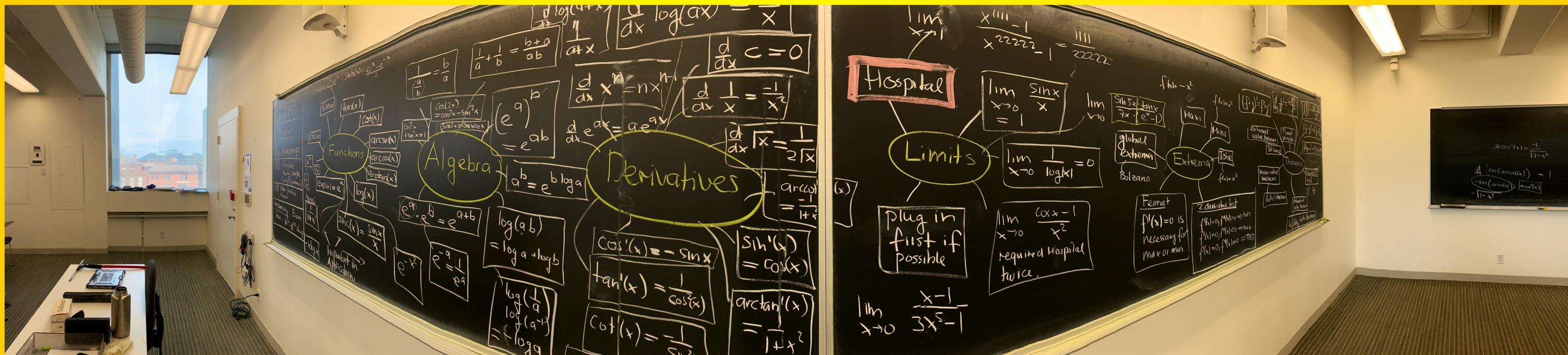
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# Review

Functions

Derivatives

Algebra



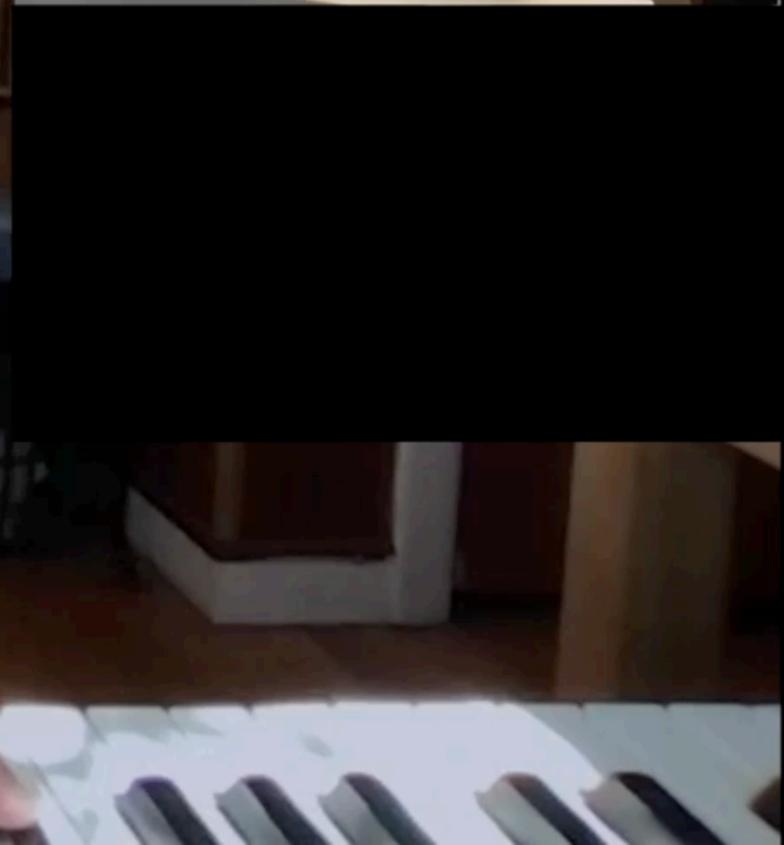
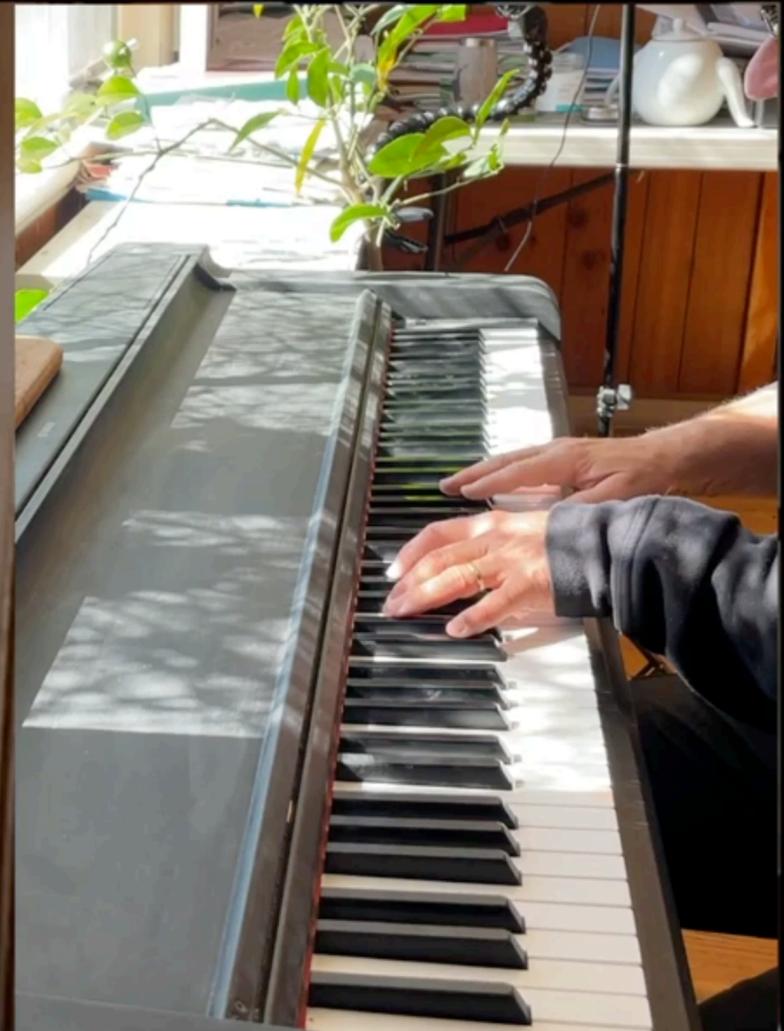
Limits

Extrema

Theorems

I made quite in the past some  
music videos for exam reviews.

Here is a new one:



$\sinh(x) = \frac{e^x - e^{-x}}{2}$ 

 $\frac{1}{\frac{1}{a}} = a$ 

 $\frac{1}{\frac{1}{a} + \frac{1}{b}} = \frac{ab}{b+a}$ 

 $\frac{d}{dx} \log(a+x) = \frac{1}{a+x}$ 

 $\frac{d}{dx} \log(a)$

**Algebra**

- $\frac{1}{\frac{1}{a}} = a$
- $\frac{1}{\frac{1}{a} + \frac{1}{b}} = \frac{ab}{b+a}$
- $\frac{d}{dx} \log(ax) = \frac{1}{x}$
- $\frac{d}{dx} c = 0$
- $\frac{d}{dx} x^n = nx^{n-1}$
- $\frac{d}{dx} \frac{1}{x} = -\frac{1}{x^2}$
- $\frac{d}{dx} \sqrt{x} = \frac{1}{2\sqrt{x}}$
- $\frac{d}{dx} e^{ax} = ae^{ax}$
- $\frac{d}{dx} \sqrt{x} = \frac{1}{2\sqrt{x}}$
- $\frac{d}{dx} \frac{1}{1+x} = -\frac{1}{(1+x)^2}$
- $(e^a)^b = e^{ab}$
- $a^b = e^{b \log a}$
- $e^a \cdot e^b = e^{a+b}$
- $e^{-a} = \frac{1}{e^a}$
- $\log(ab) = \log a + \log b$
- $\log\left(\frac{1}{a}\right) = -\log a$
- $\log(a^{-1}) = -\log a$
- $\log\left(\frac{1}{a^b}\right) = -\log a^b$
- $\log(a^{-b}) = -\log a^b$

**Derivatives**

- $\sin'(x) = \cos(x)$
- $\cos'(x) = -\sin(x)$
- $\tan'(x) = \frac{1}{\cos^2(x)}$
- $\cot'(x) = -\frac{1}{\sin^2(x)}$
- $\arcsin'(x) = \frac{1}{\sqrt{1-x^2}}$
- $\arccos'(x) = -\frac{1}{\sqrt{1-x^2}}$
- $\arctan'(x) = \frac{1}{1+x^2}$
- $\operatorname{arccot}'(x) = -\frac{1}{1+x^2}$
- $\operatorname{arcsinh}'(x) = \frac{1}{\sqrt{1+x^2}}$
- $\operatorname{arcosh}'(x) = \frac{1}{\sqrt{x^2-1}}$

**Functions**

- $\sin(x)$
- $\cos(x)$
- $\tan(x)$
- $\cot(x)$
- $\operatorname{arcsin}(x)$
- $\operatorname{arccos}(x)$
- $\operatorname{arctan}(x)$
- $\operatorname{arcsinh}(x)$
- $\operatorname{arcosh}(x)$
- $\exp(x) = e^x$
- $\log(x)$
- $\sinh(x) = \frac{e^x - e^{-x}}{2}$
- $\cosh(x) = \frac{e^x + e^{-x}}{2}$
- $\sin(2x) = 2\sin x \cos x$
- $\cos(2x) = \cos^2 x - \sin^2 x$
- $\cos^2 x + \sin^2 x = 1$

**Hospital**

**Limits**

- $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$
- $\lim_{x \rightarrow 0} \frac{1}{\log|x|} = 0$
- $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x^2}$  required Hospital twice.
- $\lim_{x \rightarrow 0} \frac{x-1}{3x^5-1}$
- $\lim_{x \rightarrow 1} \frac{x^{111} - 1}{x^{2222} - 1} = \frac{111}{2222}$
- $\lim_{x \rightarrow 0} \frac{\sin 5x \cdot \tan x}{7x \cdot (e^x - 1)}$
- $\lim_{x \rightarrow 0} \frac{f(x) - g(x)}{h(x) - k(x)}$

**Extrema**

- Max
- Min
- global extrema
- Bolzano
- Fermat
- $f'(x) = 0$  is necessary for max or min
- 2. derivative test
- $f'(x) = 0, f''(x) > 0 \Rightarrow \text{min}$
- $f'(x) = 0, f''(x) < 0 \Rightarrow \text{max}$
- $f'(x) = 0, f''(x) = 0 \Rightarrow \text{TSD}$

**Other**

- Mean value theorem
- Rolle's theorem
- Intermediate value theorem
- Continuity
- Derivatives
- Integration

$\sinh(x) = \frac{\sin x}{x}$ 

 $e^{-a} = \frac{1}{e^a}$ 

 $\log a + \log b$ 

 $\cos'(x) = -\sin x$ 

 $\tan'(x) = \frac{1}{\cos^2(x)}$

# Mind Map

sin

cos

Functions

Derivatives

Algebra

Limits

Extrema

Theorems

Lets build one together!

Algebra

# Functions

Limits

Extrema

# Theorems

*A: True False*

- General Tips

Examples, low hanging fruits, gut feeling

What do we test?

Concepts, Understanding, Insight,  
Knowledge

# Examples

T/F  $x \sin(1/x)$  is an indefinite form at 0.

T/F  $\log(1000x) + x^{1000}$  has a root!

T/F  $x/\sin(x)$  is continuous everywhere

# *B: Extrema*

## - General Tips

Read the problem carefully.

What do we test

Technique, Theorem, local versus global

# Example

analyze  $f(x) = x^2 e^x$  Analyze with the  
2. derivative test

where are the local and global maxima?

# *C: Limits*

## - General Tips

First check whether indefinite form

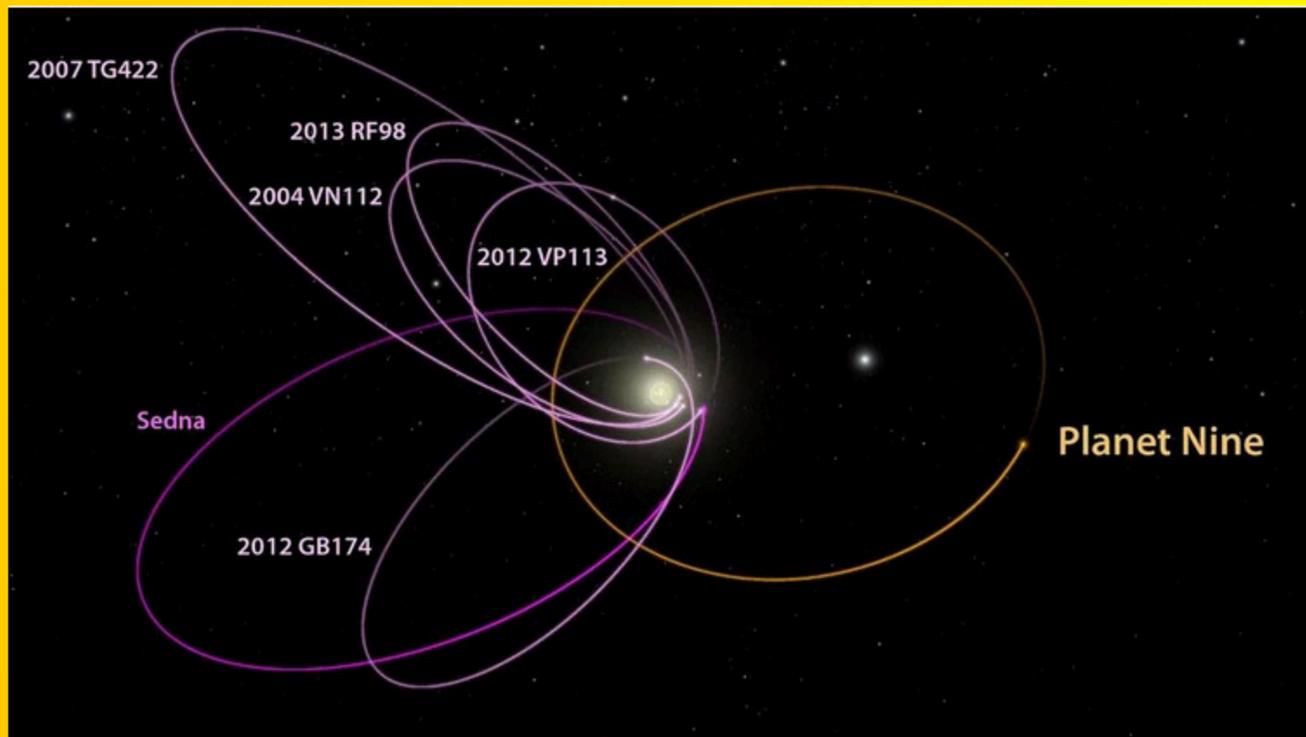
What do we test?

Hospital, function knowledge, algebra

# Example

What is the limit

$$f(x) = x^2 \log(x)$$



## 'Any evidence for Planet Nine is gone': Scientists dispute probability of mystery planet

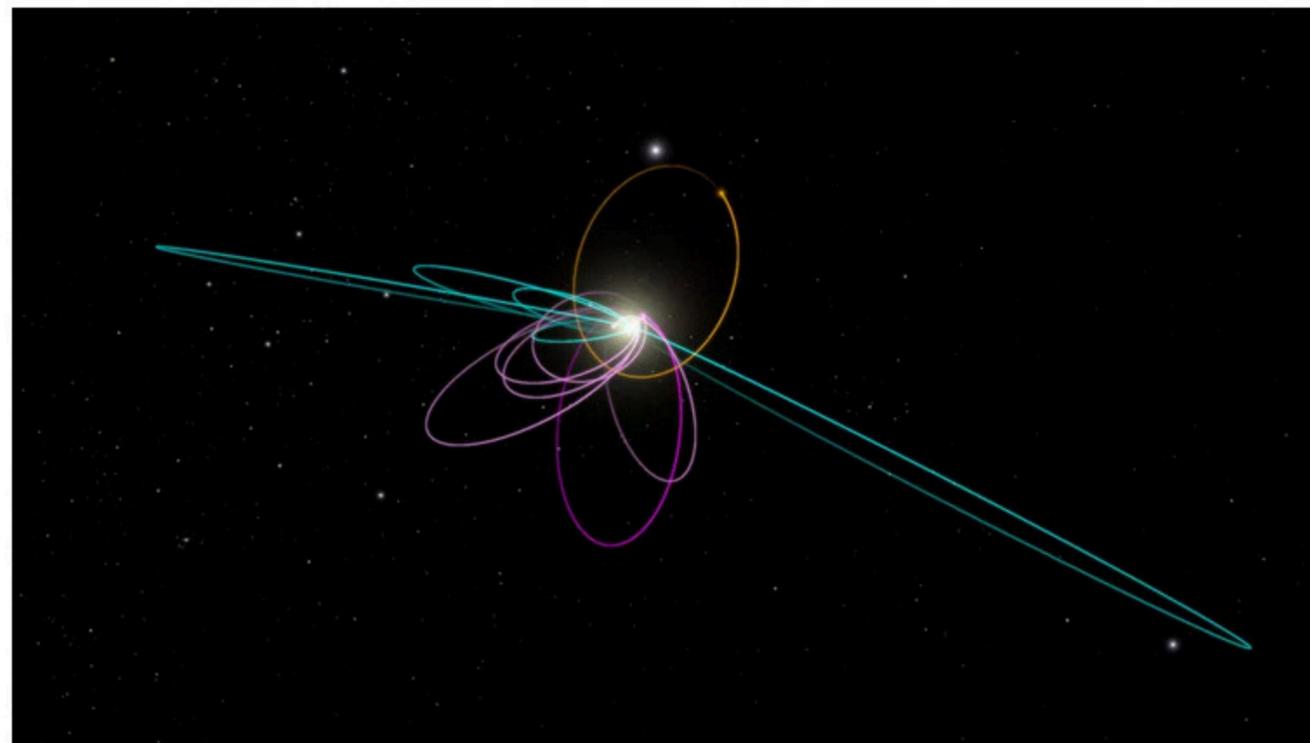
Is there a hidden ninth planet far from the sun affecting the orbits of distant space rocks? New research pours cold water on the theory.



Danel Van Boom Feb. 16, 2021 5:15 p.m. PT



▶ LISTEN - 02:59



Caltech

In the 1820s, French astronomer Alexis Bouvard speculated that Uranus' irregular orbit was being influenced by an eighth planet in our solar system, leading to the discovery of Neptune. In 2016, citing the unusual trajectory of planetoids in Neptune's orbit, two astronomers from the California Institute of Technology predicted there was another planet lurking in the solar system: Planet Nine.

### No Evidence for Orbital Clustering in the Extreme Trans-Neptunian Objects

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J. GARCÍA-BELLIDO<sup>8</sup>, D. GRUEN<sup>28,13,14</sup>, R. A. GRUENDL<sup>17,18</sup>, G. GUTIERREZ<sup>7</sup>, D. L. HOLLOWOOD<sup>24</sup>, K. HONSCHIED<sup>29,30</sup>,  
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R. MIQUEL<sup>38,19</sup>, R. MORGAN<sup>39</sup>, A. PALMESE<sup>7,35</sup>, F. PAZ-CHINCHÓN<sup>40,18</sup>, A. A. PLAZAS<sup>41</sup>, E. SANCHEZ<sup>23</sup>, V. SCARPINE<sup>7</sup>,  
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AND R.D. WILKINSON<sup>44</sup>  
(DES COLLABORATION)

'eb 2021

# What is the limit

$$f(x) = x \log^2(x)$$

**Figure 5.** Kernel density estimates of the mean  $(x, y)$  and  $(p, q)$  position of  $10^6$  samples of ETNOs drawn shown in Figure 4. The number of objects in each sample corresponds to the number of ETNOs detected by the survey. The contours represent the samples (the contours scale linearly, and darker contours are more densely populated), and the dots represent the mean position of the ETNOs detected by each survey.

unit hypercube, given by

$$\mathcal{P}_{joint} = P \sum_{k=0}^N (-1)^k \frac{\log(P)^k}{k!}$$

where  $P \equiv \prod_k \mathcal{P}_k$ . In our case,  $k \in \{\text{DES}, \text{OSSOS}, \text{ST}\}$ . Using Equation 4, we calculate the joint probability to be 24%.

# *D: Matching*

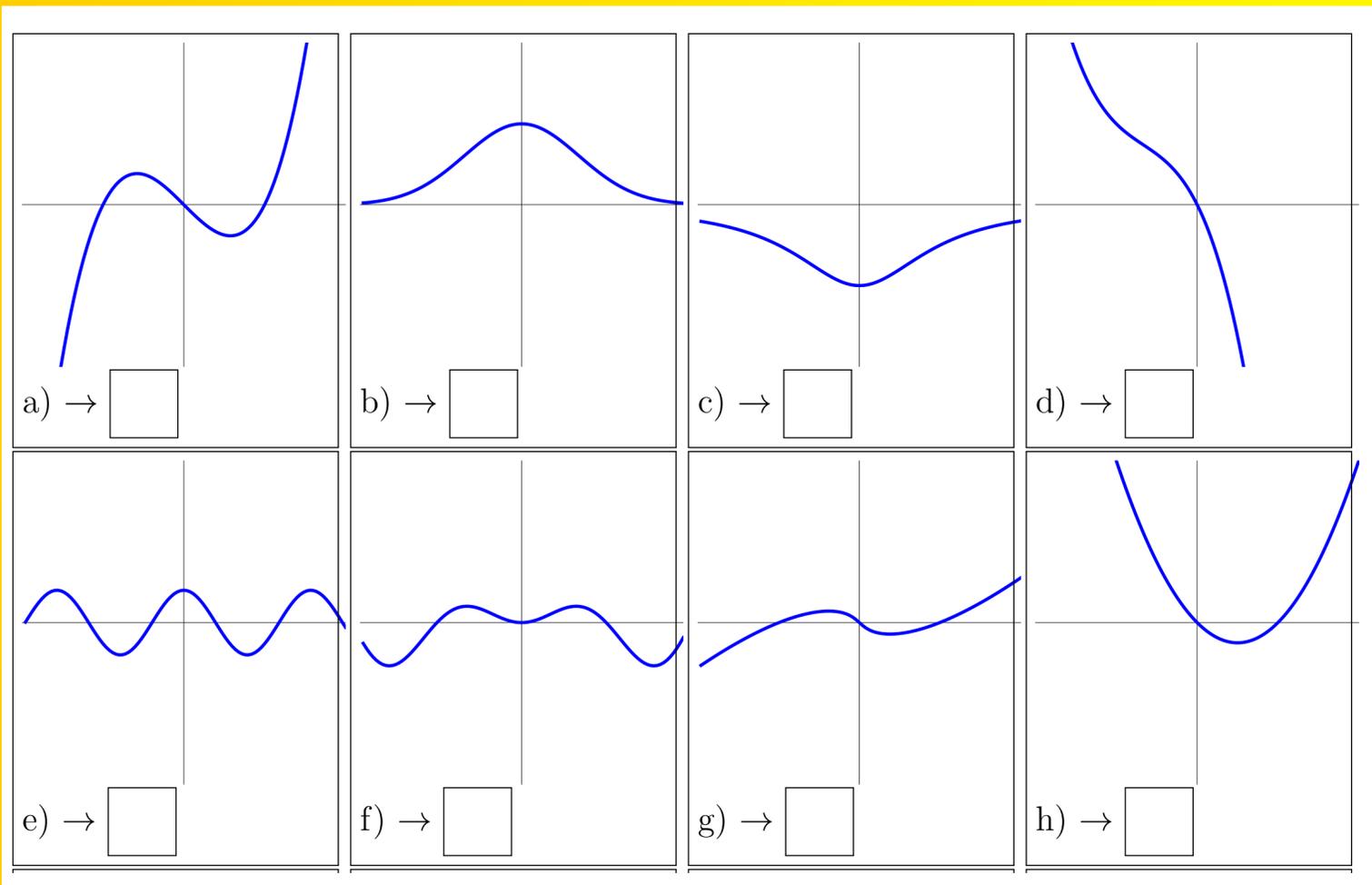
- General Tips

First easy cases

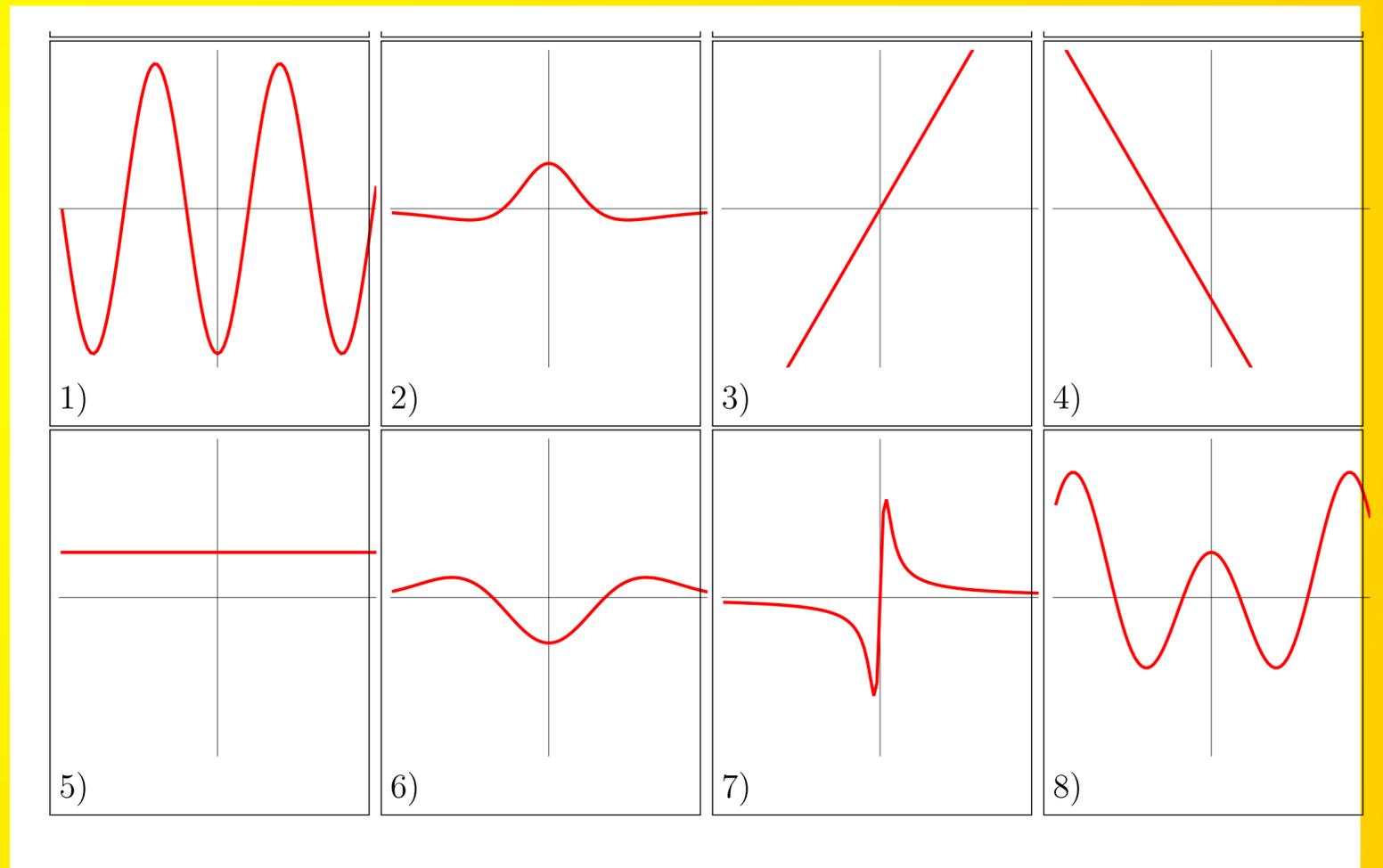
What do we test?

Understanding of basic definitions.

# Example (Practice 6)



**f**



**f'**

# *E: Techniques*

- General Tips

Just know them

What do we test?

Which part to use where? Algorithms

*The End*