unit 12

$\boxed{\text{Can problem}}$

$$\begin{cases} f(r) = 2\pi r h + 2\pi r^2 \\ V(r) = \pi r^2 h = 2\pi \end{cases}$$

i) Reduce number of variables

$$h = \frac{2}{r^2}$$

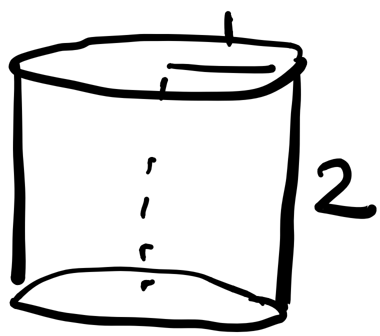
$$\boxed{f(r) = \frac{4\pi}{r} + 2\pi r^2}$$

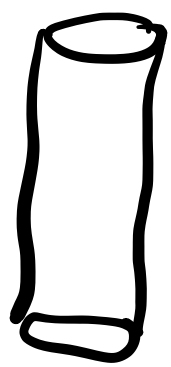
ii) $f'(r) = -\frac{\cancel{4\pi}}{r^2} + \cancel{4\pi} r = 0$

Solve for r : $r^3 = 1$, $r = 1$

$h = 2$

$\boxed{\text{200 PSI}}$





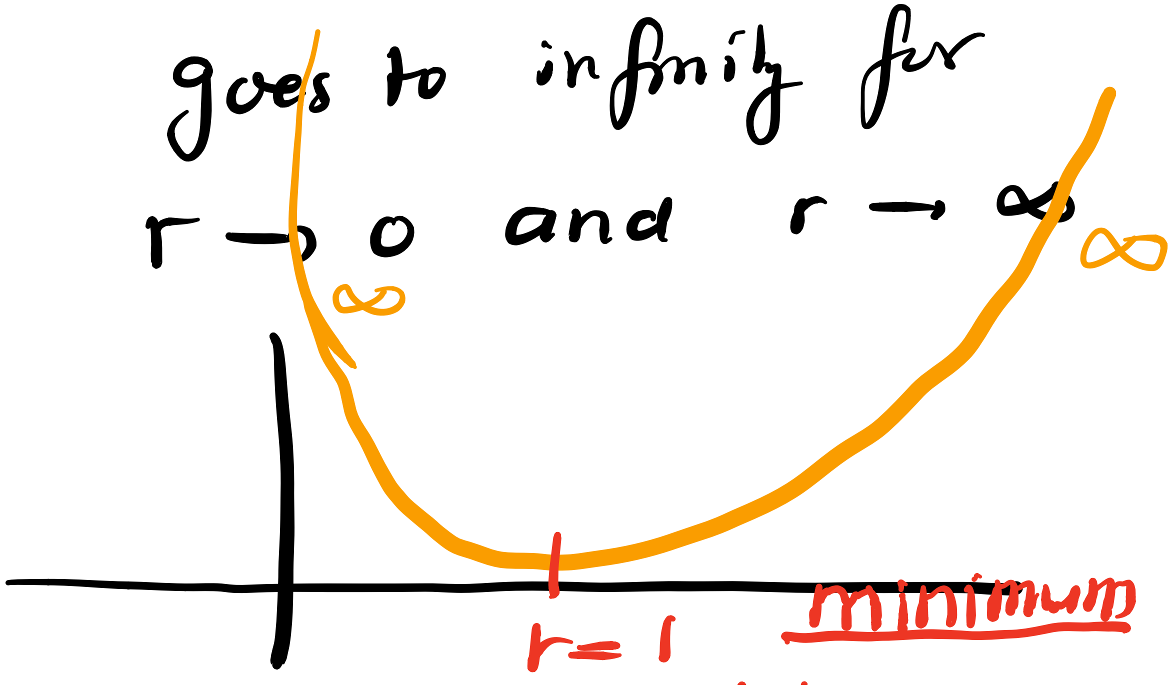
PIZZA $= \pi z^2 A$

iii) Look at the boundary!

The problem makes sense for positive r only.

$$f(r) = \frac{4\pi}{r} + 2\pi r^2$$

goes to infinity for $r \to 0$ and $r \to \infty$

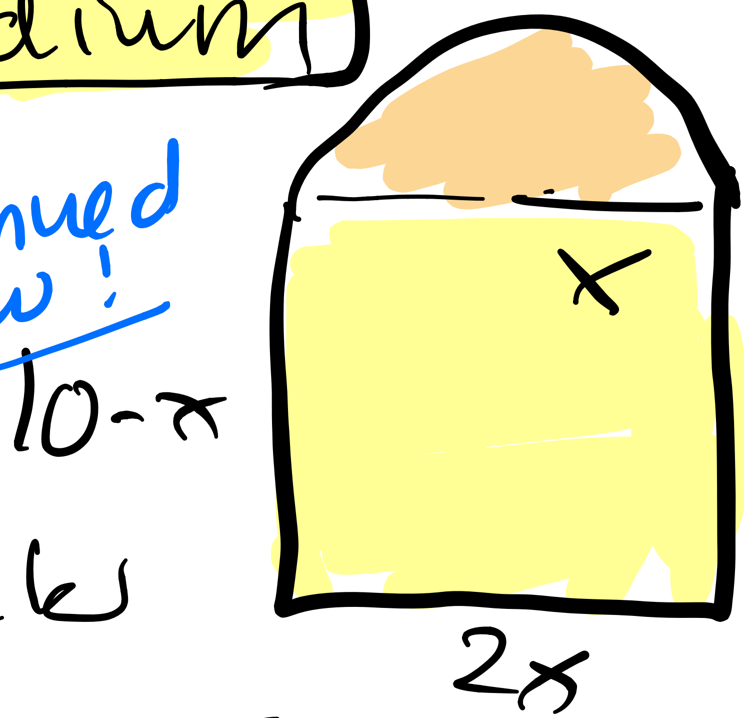


global min

no global max here!

Stadium

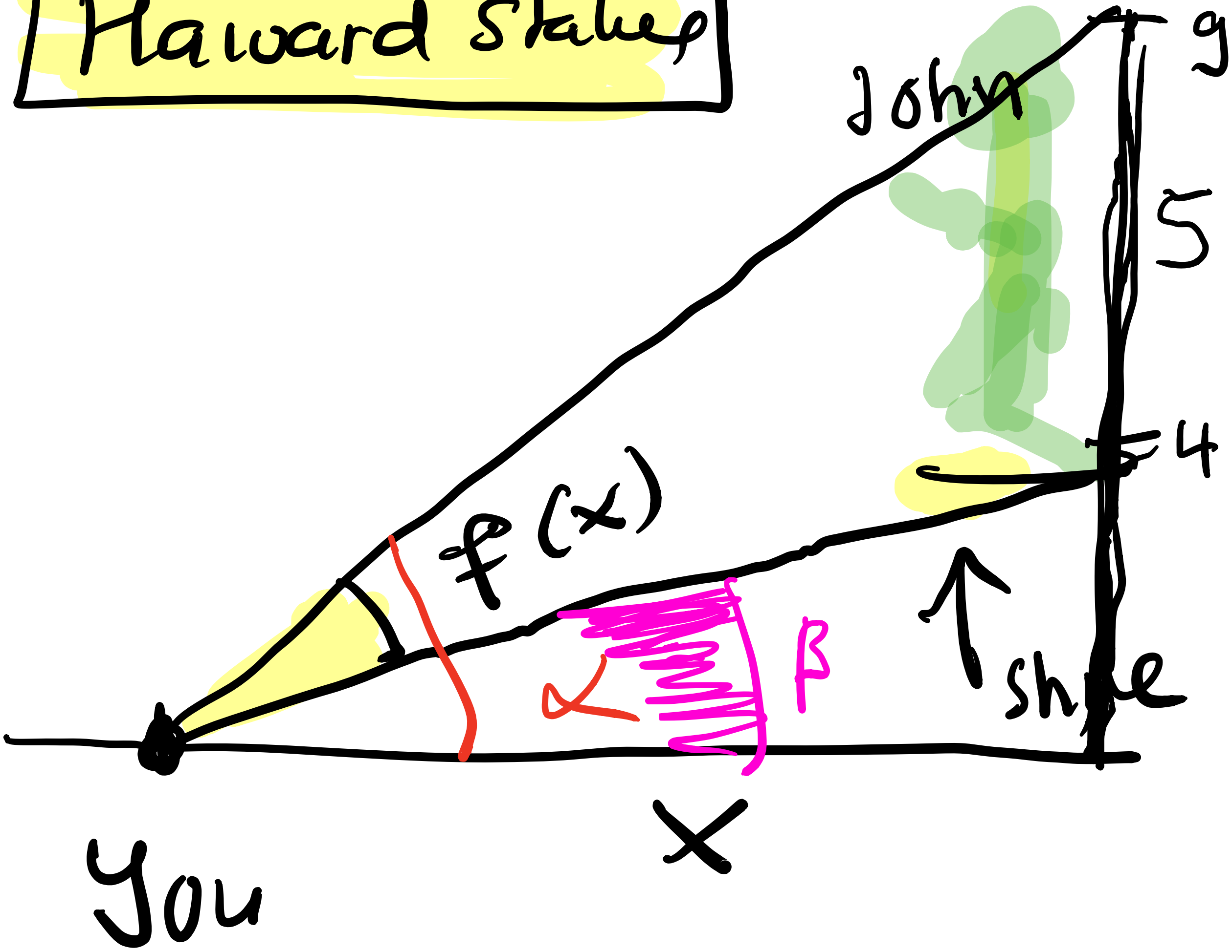
continued below!
#



→ lab

$$f(x) = (2-x)2x + rx^2$$

Haward Stakes



$$f(x) = \alpha(x) - \beta(x)$$

What is $\alpha(x)$?

$$\tan(\alpha) = 9/x,$$

$$\alpha = \arctan\left(\frac{9}{x}\right)$$

$$\beta = \arctan\left(\frac{4}{x}\right)$$

$$f(x) = \arctan\left(\frac{9}{x}\right) - \arctan\left(\frac{4}{x}\right)$$

<u>Remember!</u>

$$\arctan'(x) = \frac{1}{1+x^2}$$

$$f'(x) = \frac{\left(-\frac{9}{x^2}\right)}{1+\left(\frac{9}{x}\right)^2} + \frac{\left(-\frac{4}{x^2}\right)}{1+\left(\frac{4}{x}\right)^2}$$

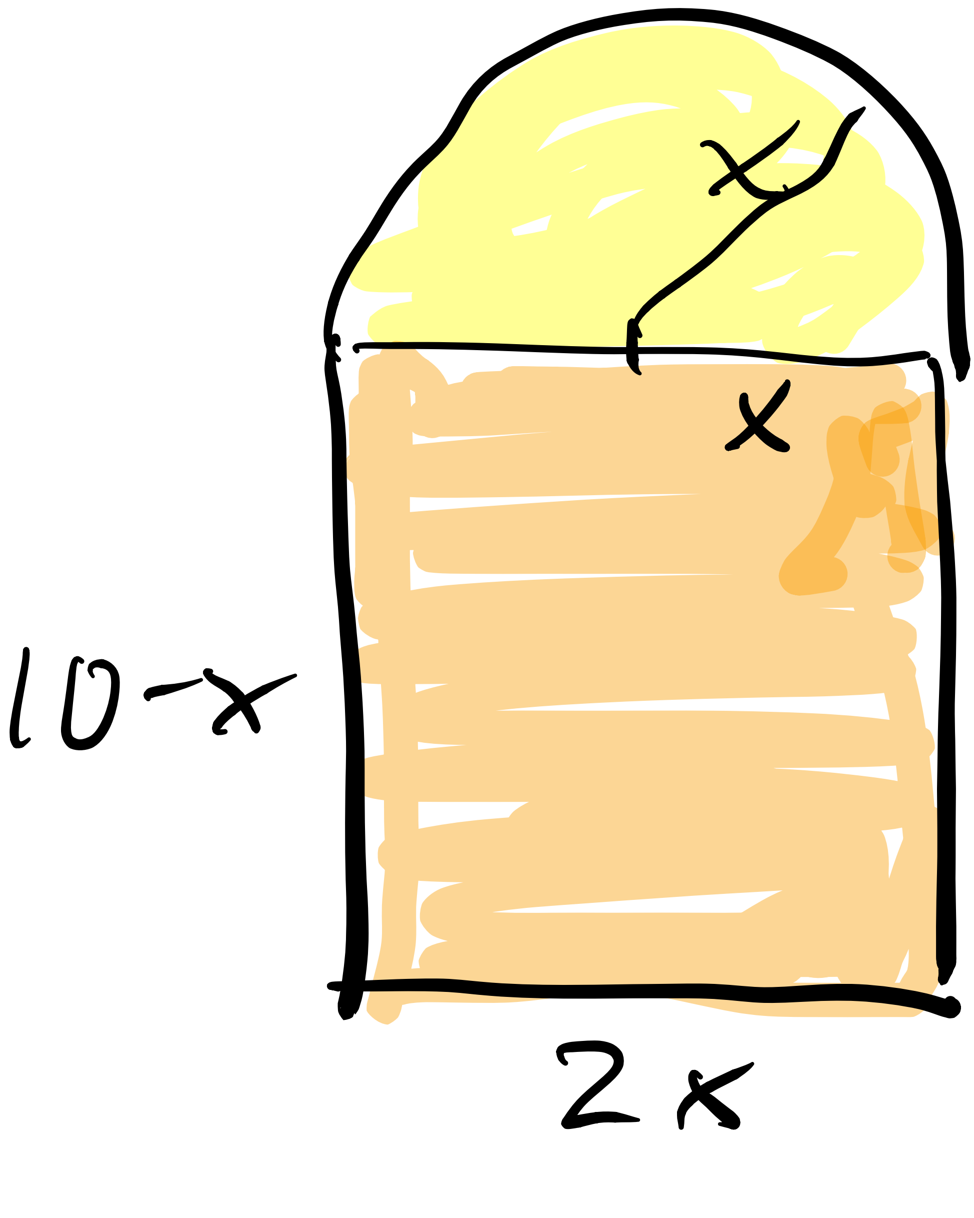
Simplification

$$\therefore = \frac{5(x^2-36)}{(1296+97x+x^4)}$$

$x = 6$ is the maximum

continuation.

Back to stadium



$$A(x) = \frac{\pi x^2}{2} + (10 - x)2x$$

$$= x^2\left(\frac{\pi}{2} - 2\right) + 20x$$

$$= \frac{x^2}{2}(\pi - 4) + 20x$$

$$A'(x) = x(\pi - 4) + 20 = 0$$

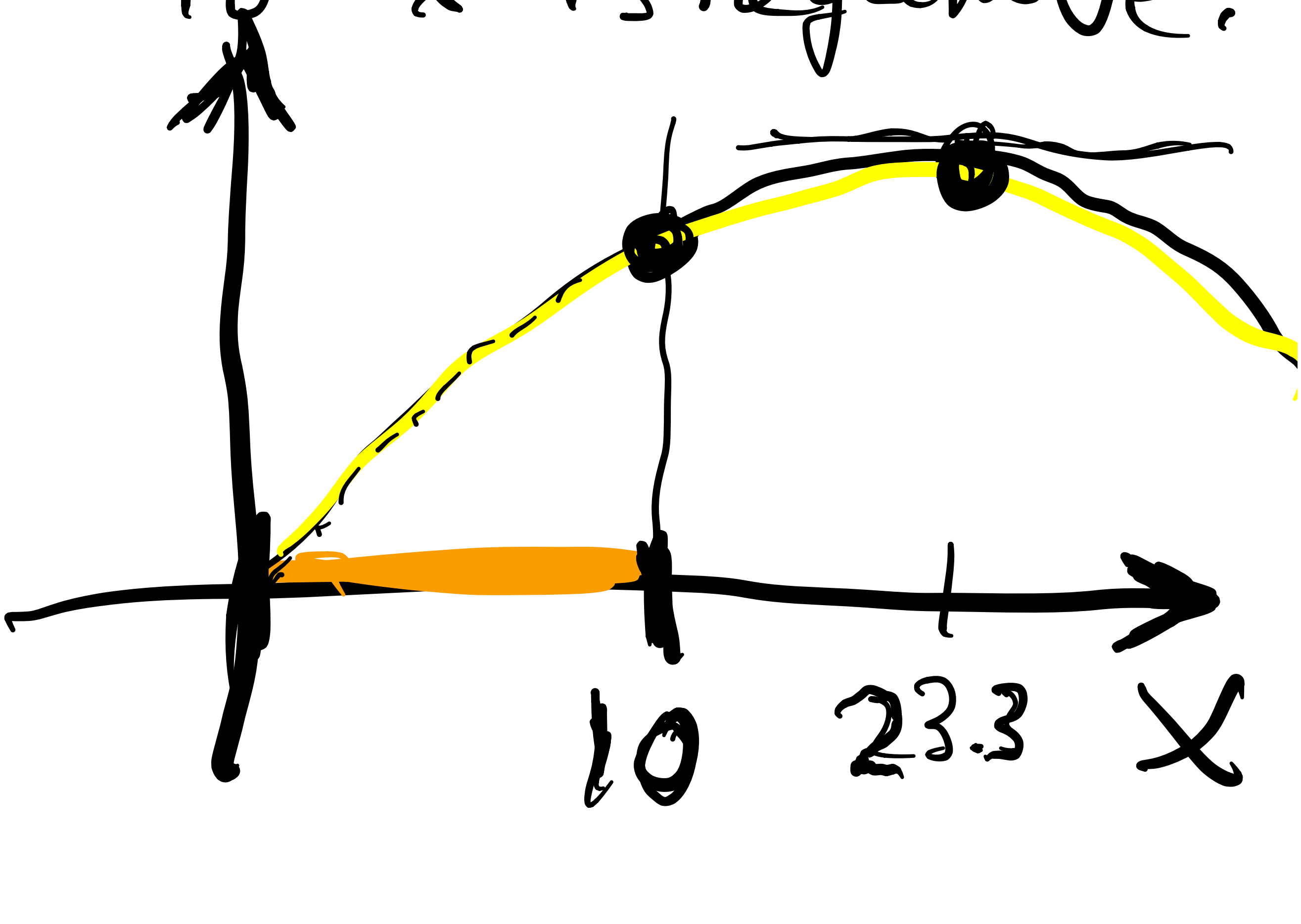
$$x = -\frac{20}{\pi - 4} = \frac{20}{4 - \pi}$$

11~4 4~11

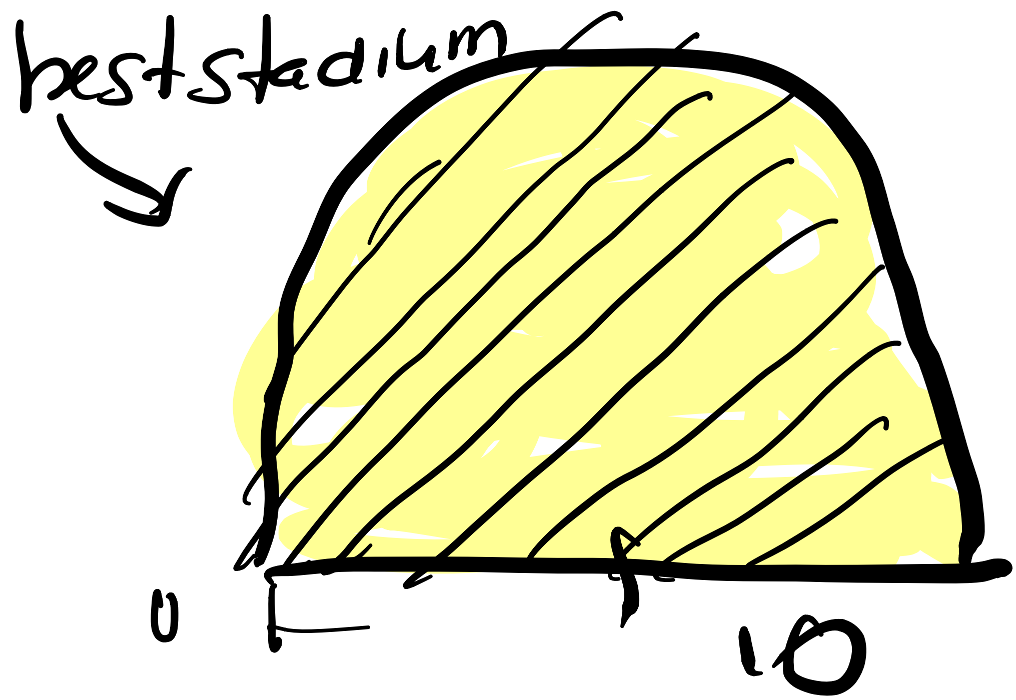
$= 23.3$

Does this make sense? No!

$10 - x$ is negative!



On the interval
given [0, 10]
the maximum
is $\boxed{x = 10}$



In the stadium example, we saw that the maximum can appear on the boundary.

Jam problem

$\longrightarrow$ website.