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# Critical Points

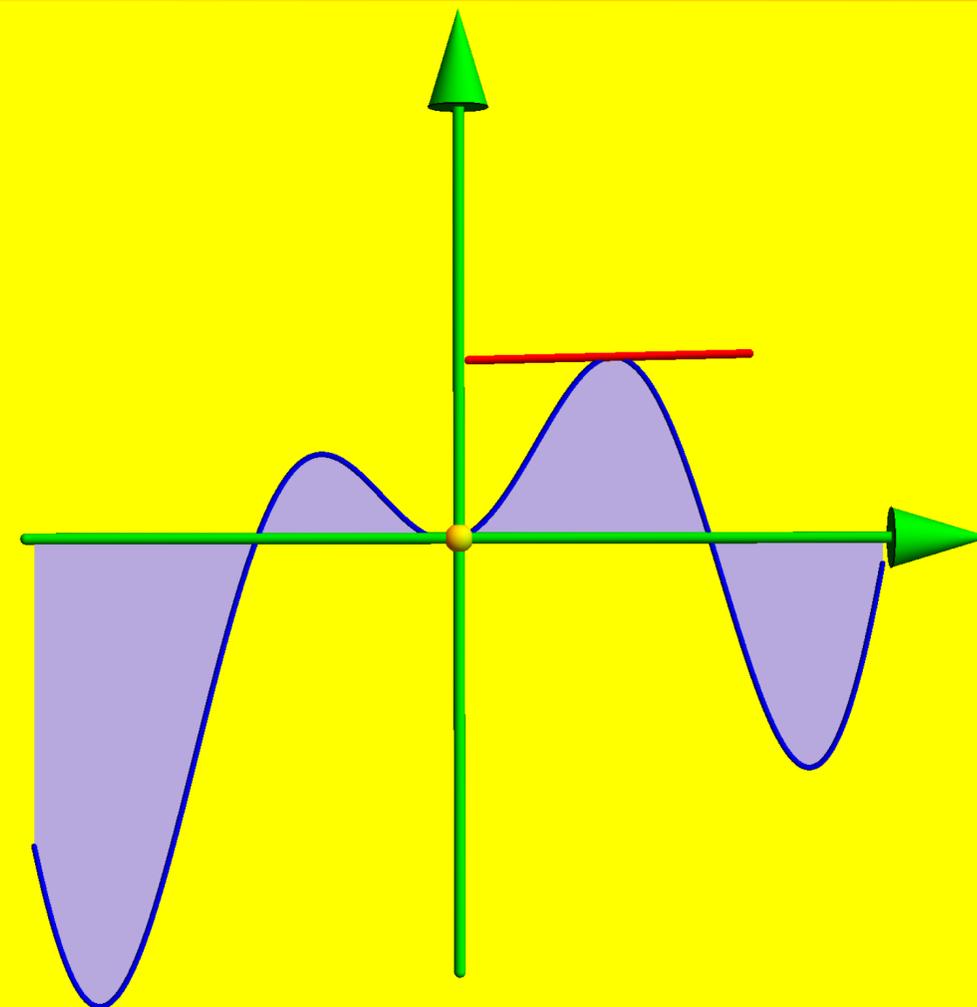
$$f'(x) = 0$$

Critical  
point

$$f'(x) = 0, f''(x) > 0 \Rightarrow \text{Min}$$

$$f'(x) = 0, f''(x) < 0 \Rightarrow \text{Max}$$

$$f'(x) = 0, f''(x) = 0 \Rightarrow \text{TBD}$$



# PLAN

1. Poll

2. critical points

3. Fermat Observation

4. Second Derivative test

5. Examples

6. Jam

# POLL

What is a critical point?

A: Never heard about it?

B: A point where  $f'(x)=0$

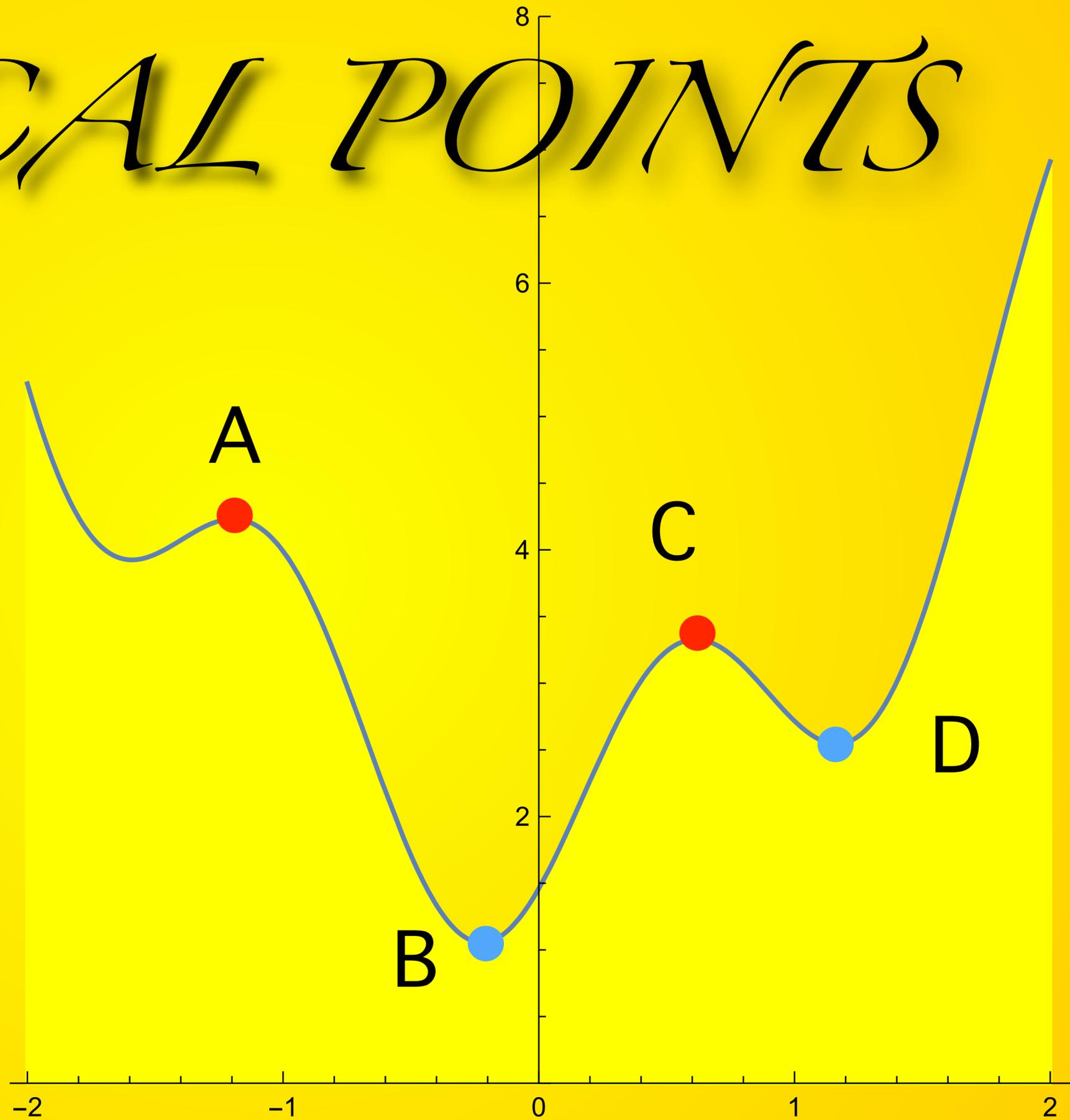
C: A maximum or minimum

D: A point where  $f'(x)=0$   
or a singular point.

# CRITICAL POINTS

$x$  is a critical point of  $f$  if

$$f'(x) = 0$$



# EXAMPLES

Find the critical points of

A  $x^3 - 3x$

B  $x^2 + 1$

C  $\exp(x)$

D  $3x + 1$

no critical points where  $f, f'$  are defined

E  $1/x$

F  $|x|$

# MAX AND MIN

- local min

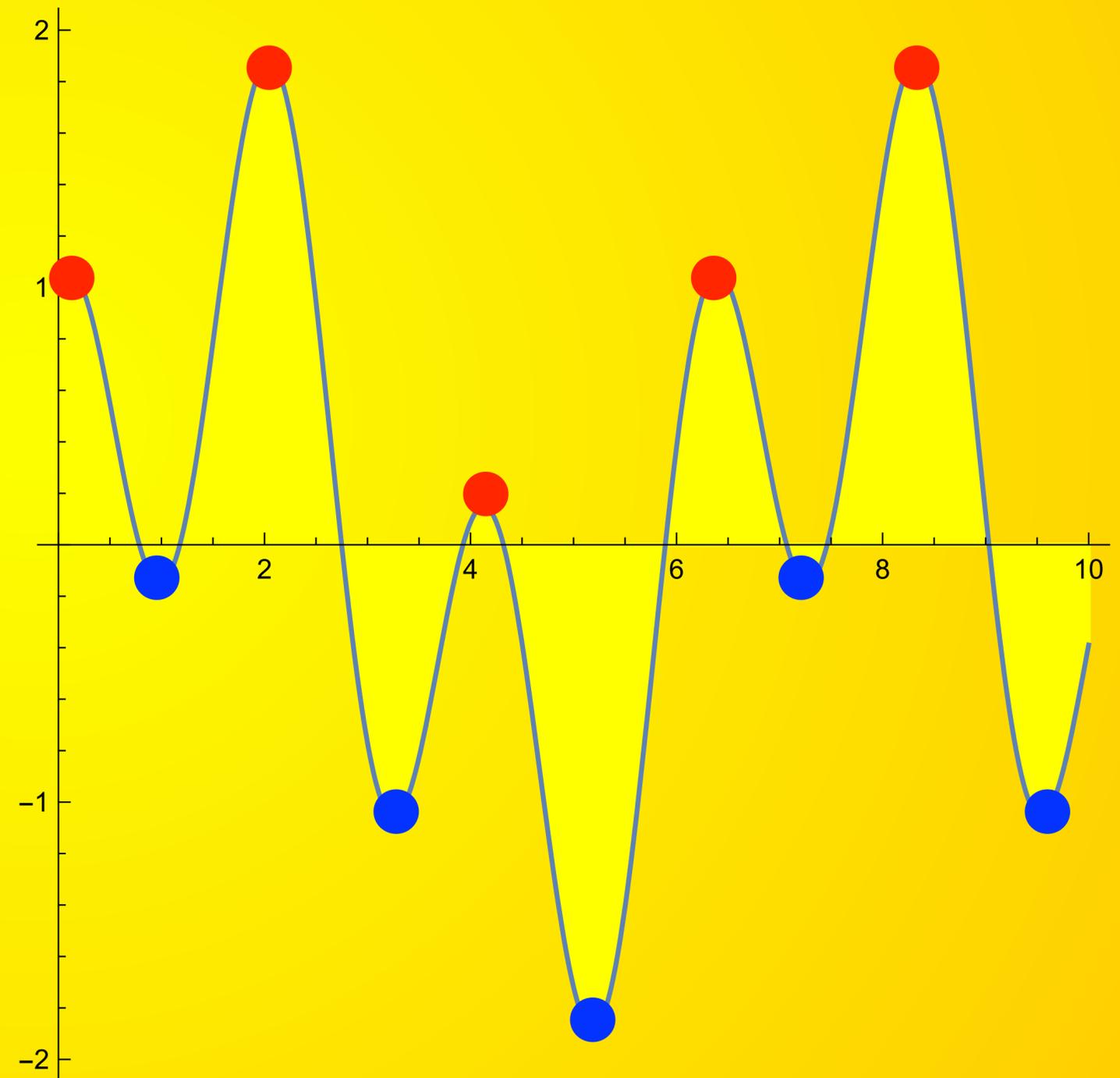
$$f(x) \leq f(y)$$

for  $y$  near  $x$

- local max

$$f(x) \geq f(y)$$

for  $y$  near  $x$



# FERMAT

If  $x$  is local max then  $f'(x) = 0$

Argument of Fermat:

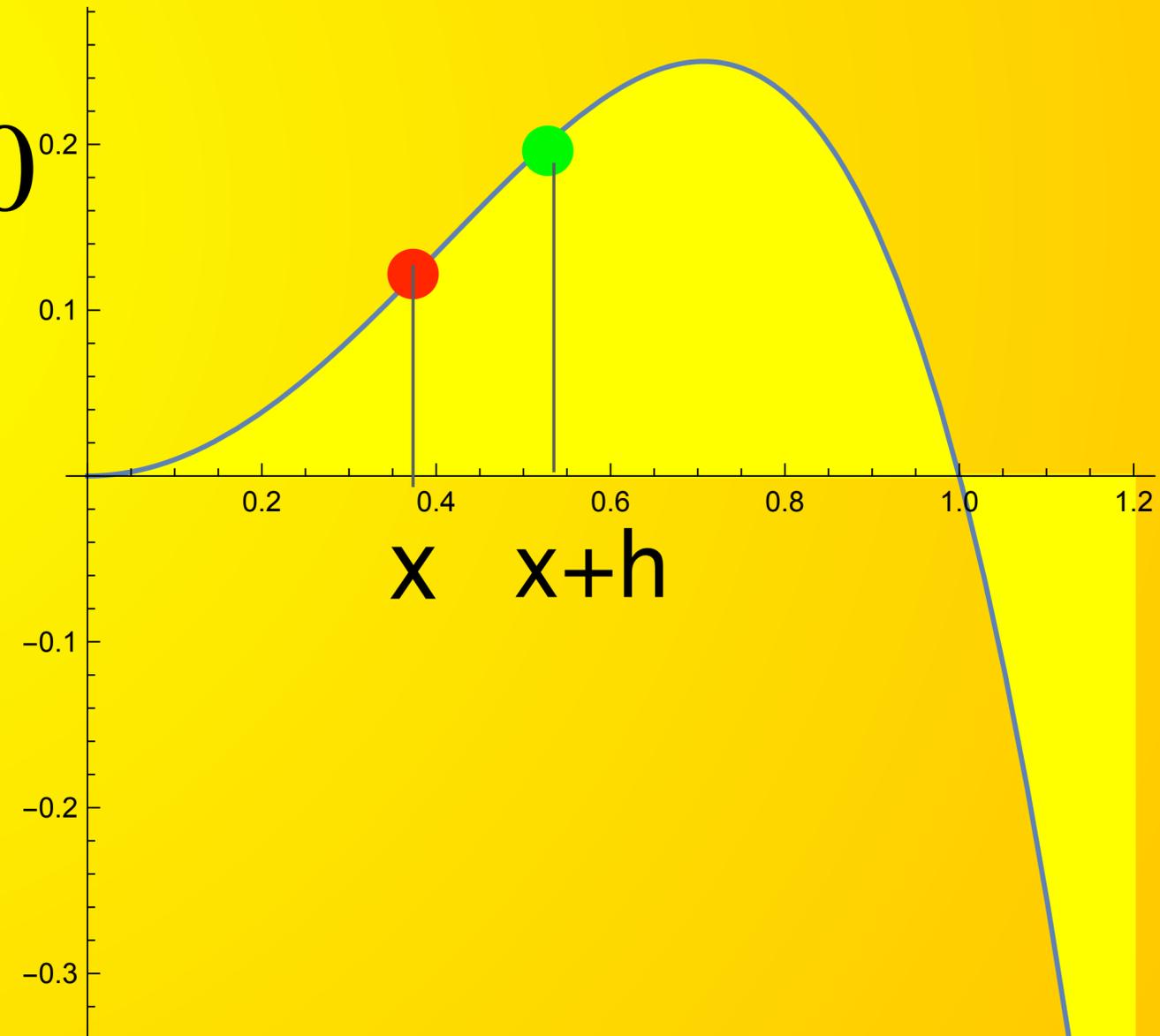
assume  $x$  is max and  $f'(x) = c > 0$

Then  $[f(x+h)-f(x)]/h > c/2$

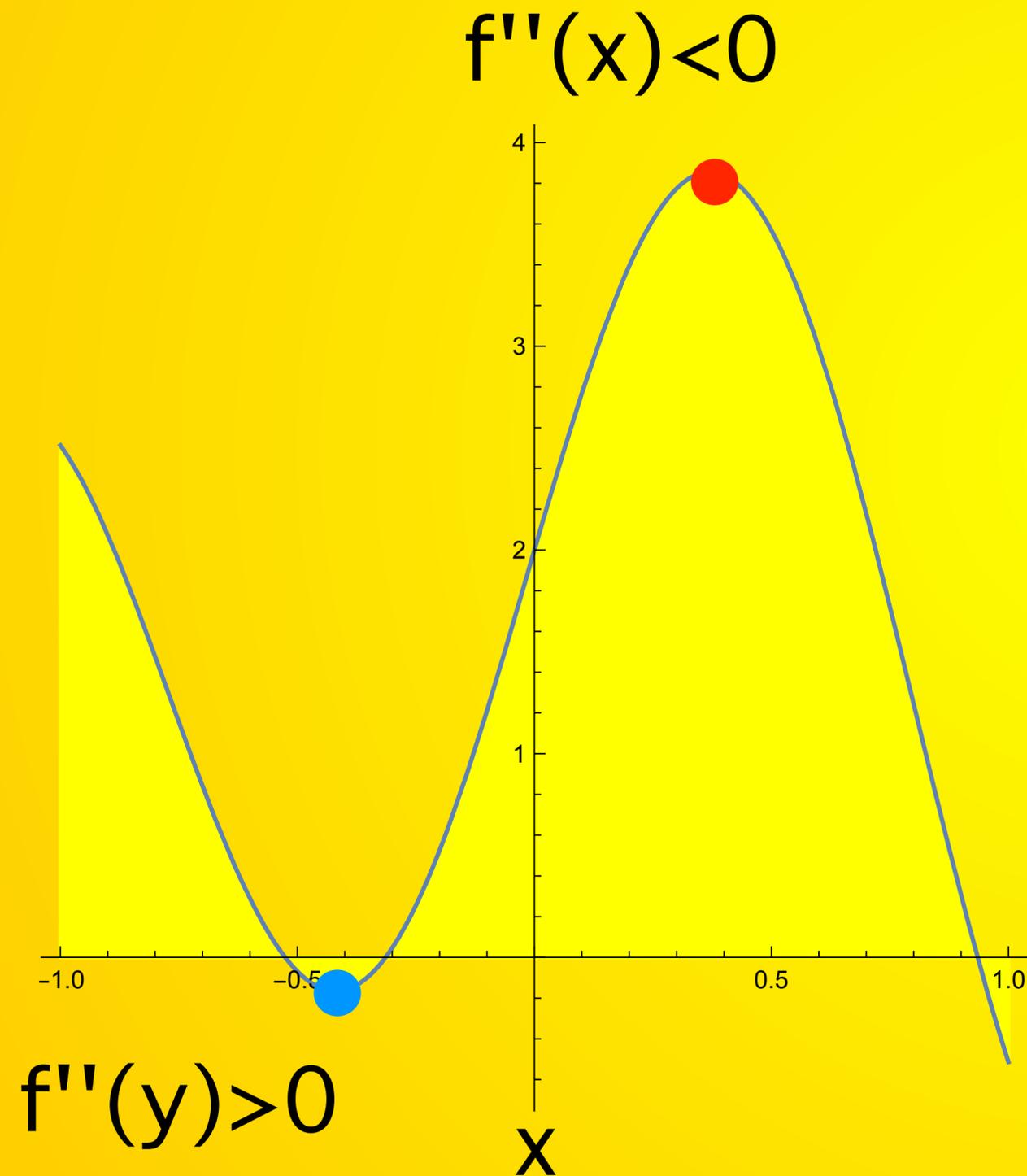
for small  $h$  and so

$f(x+h) > f(x) + c h/2$ .

Contradiction



# SECOND DERIVATIVE TEST



If  $f'(x) = 0$  and  $f''(x) < 0$  then  $x$  is a local maximum

If  $f'(x) = 0$  and  $f''(x) > 0$  then  $x$  is a local minimum

# *DISCUSSION*

Why is it true?

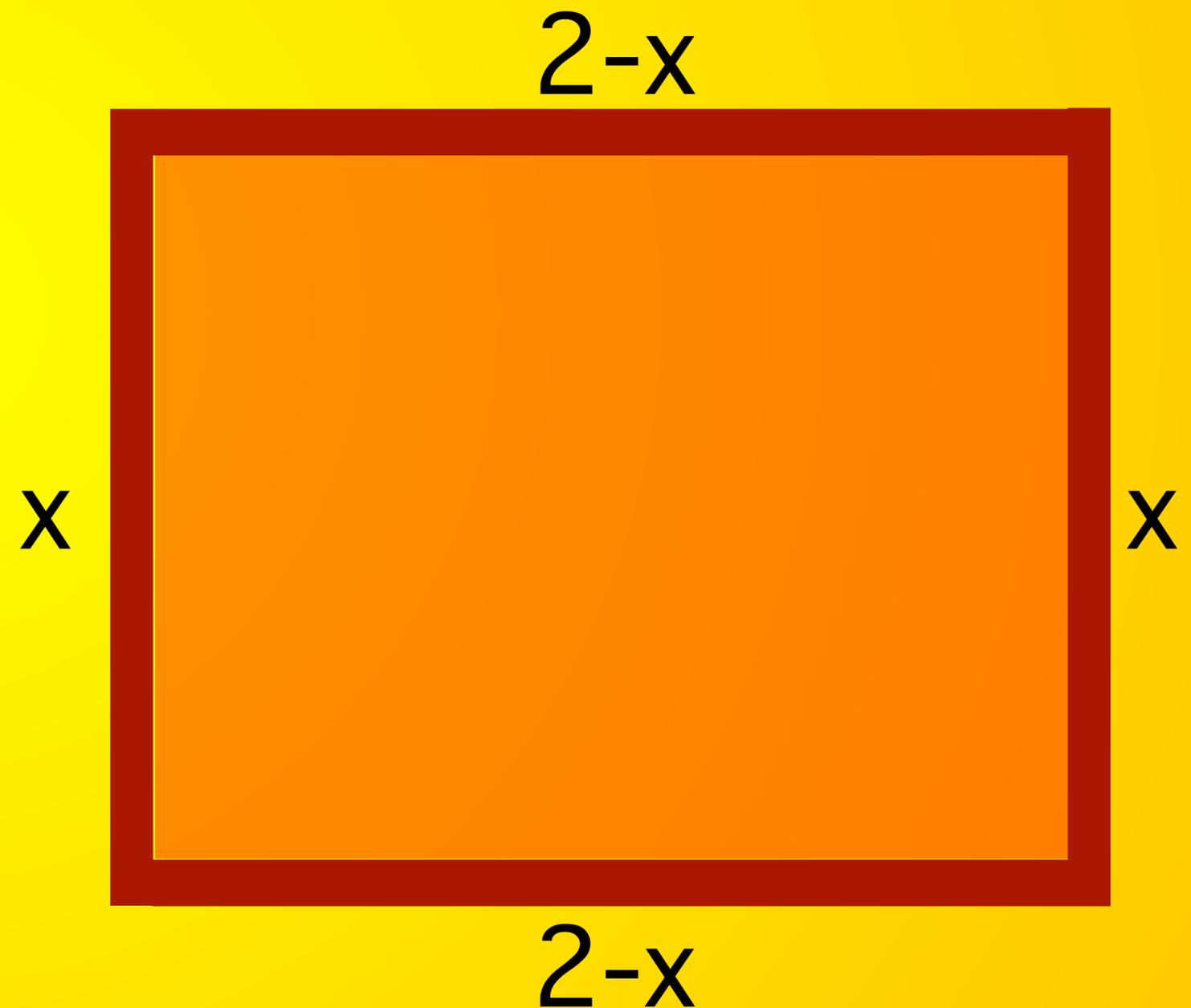
How can we remember  
it?

Why is it relevant?

....

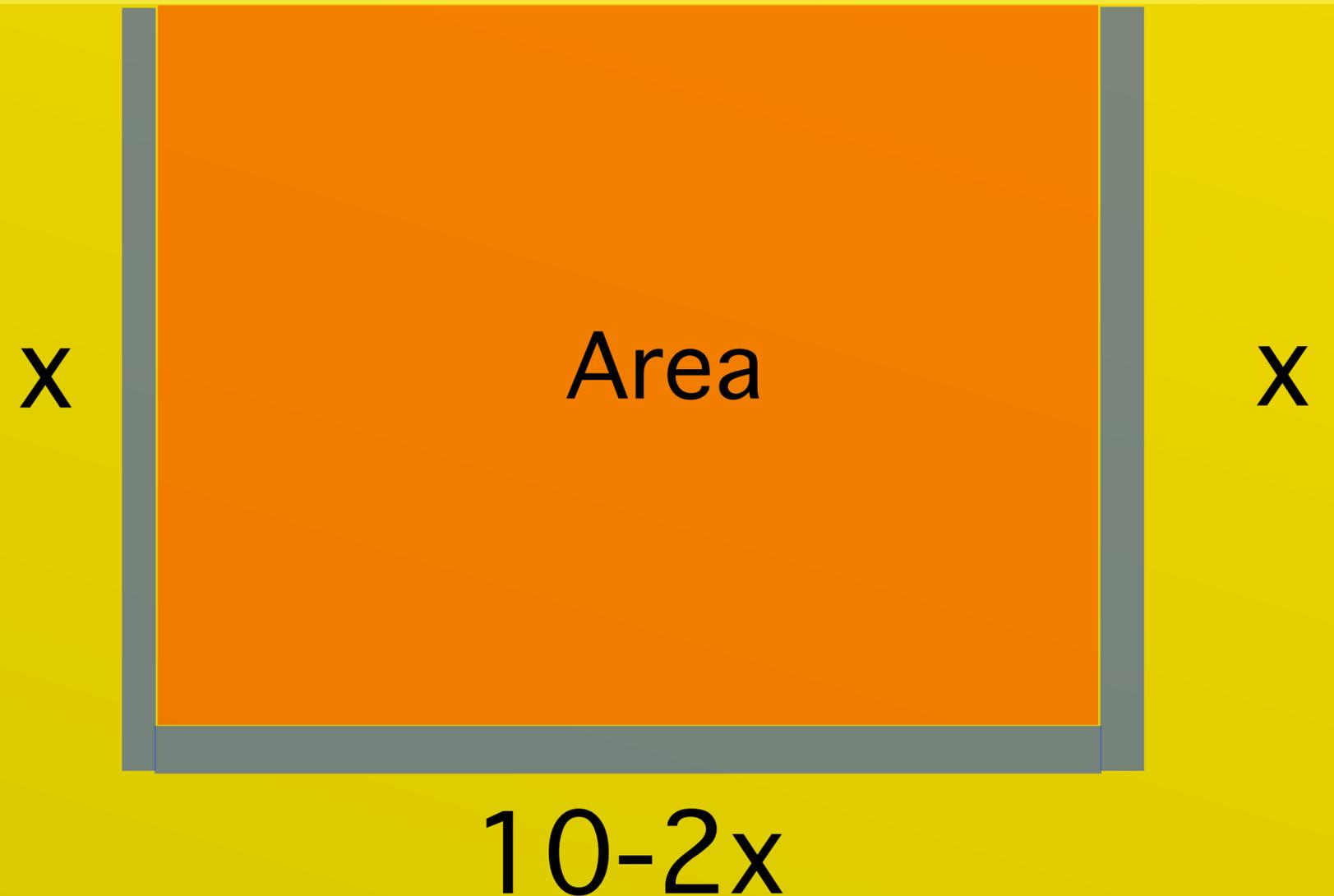
# A) RECTANGLE

What rectangle of total circumference 4 has maximal area?



# *B) CHICKEN COOP*

We have 10 meters  
of chicken wire.  
Maximize the area of  
the field.









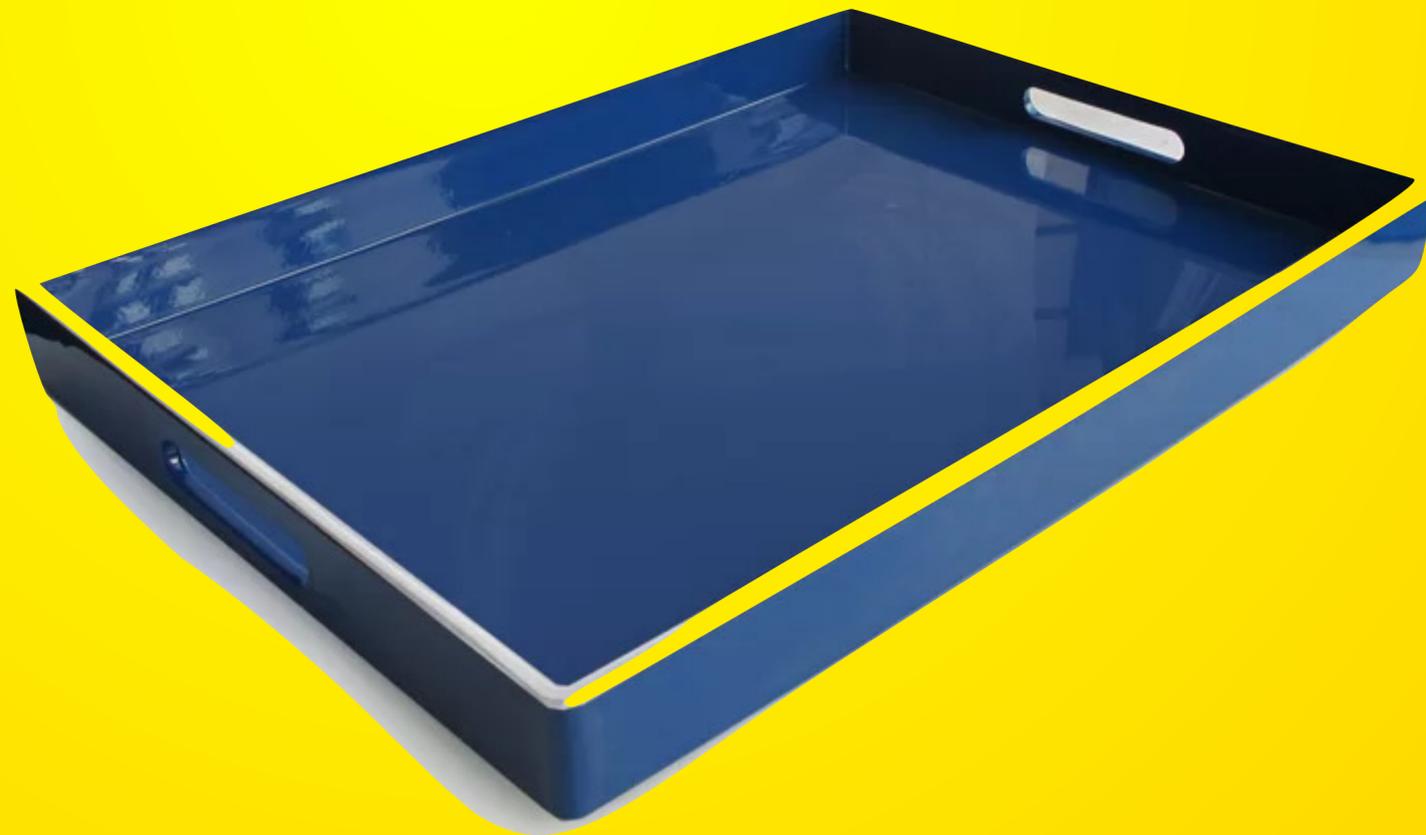






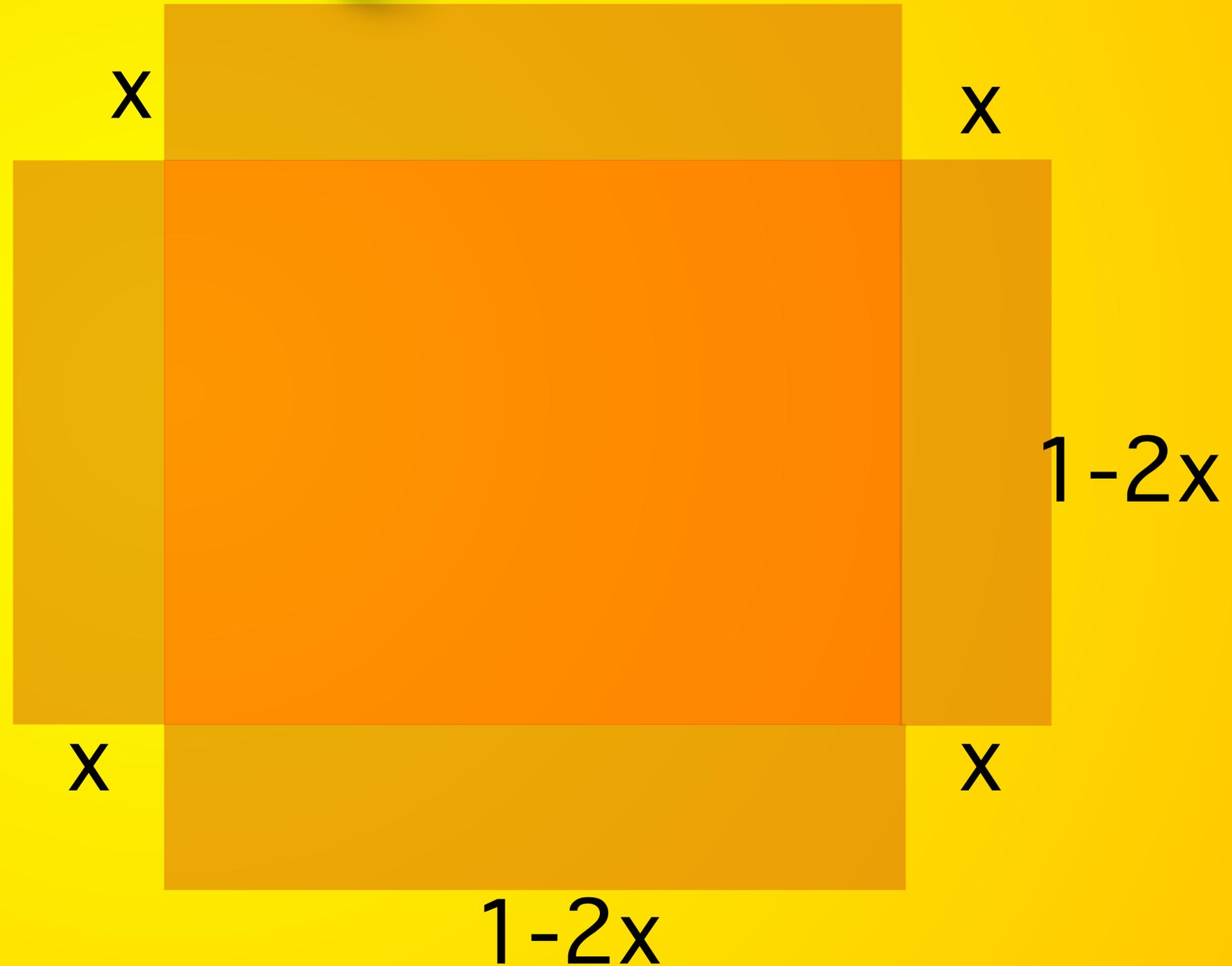
# *C) TRAY*

Find the max and min of the volume of the tray with square base of length  $1-2x$  and height  $x$



# C) TRAY

Find the global max  
and min of  $f(x)$   
volume of the  
tray with square  
base of length  $1-2x$   
and height  $x$



*JAMM*

A

Classify the critical points of  $f(x) = 15x - 5e^x$  using the second derivative test

B

Classify the critical points of  $f(x) = 3x^4 - 4x^3$  using the second derivative test

D

Which rectangle of fixed area  $xy=1$  has minimal circumference  $2x+2y$ ?

$xy=1$

*The End*