10 Chain Rule

Chain rule

$$(f(g))' = f'(g)\,g'$$

Example:

$$\frac{d}{dx}\sin(e^x) = \cos(x)e^x$$

Application:

$$\arctan'(x) = \frac{1}{1+x^2}$$

$$x = \tan(\arctan(x))$$

$$1 = \frac{d}{dx}\tan(\arctan(x)) = \sec^2(\arctan(x))\arctan'(x)$$

$$\sec^2(\arctan(x)) = 1 + \tan^2(\arctan(x)) = 1 + x^2$$

PLAN

0. Project Check

1. Poll

2. Chain rule

3.Examples

4. One ring

6. Jam

POLL

What is the derivative of

$$e^{(e^x)}$$

A $e^{(e^x)}$

B $e^x e^x$

C $e^{(e^x)} e^e$

D $e^{e^x} e^x$

CHAIN RULE

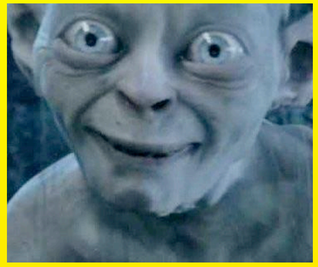
$$(f(g))' = f'(g)g'$$

Proof:

$$\frac{f(g(x+h)) - f(g(x))}{h} = \frac{[f(g(x) + (g(x+h) - g(x))) - f(g(x))]}{[g(x+h) - g(x)]} \cdot \frac{[g(x+h) - g(x)]}{h}$$

VARIABLE-FUNCTION

$$\frac{d}{dx}\sin(x^2) = \cos(x^2)2x$$



For a moment, the function x^2 becomes a variable.

ONE RING

$$f(x) = 1/x$$

Chain rule

$$f(g(x)) = 1/g(x)$$

$$f(g(x))' = -g'(x)/g(x)^2$$

The chain rule implies the reciprocal rule!

ONE RING

One Ring to rule them all
One Ring to find them
One Ring to bring them all,
and in the darkness bind them

ONE RING

Chain rule

$$\frac{d}{dx}(f+g)^2 = 2(f+g)(f'+g')$$

$$= 2(ff' + gg' + gf' + fg')$$

Chain rule

$$\frac{d}{dx}(f^2 + g^2 + 2fg) = 2ff' + 2gg' + 2(fg)'$$

The chain rule implies the product rule!


One ring to bring them all and
in the Darkness bind them.

EXAMPLES

A $\frac{d}{dx}\sin(\log(x))$

B $\frac{d}{dx}e^{-x^2}$

C $\frac{d}{dx}\exp(\log(x))$

D $\frac{d}{dx}\sqrt{x^2+4}$

E $\sin(\cos(x^2))$

WHY CHAIN RULE?

$$\sin(\cos(\exp(\tan(x))))$$

JAM

A $\frac{d}{dx}\sin(\sin(x))$

B $\frac{d}{dx}\sin(\sin(\sin(x)))$

C $\frac{d}{dx}e^{(e^{(e^x)})}$

D $\frac{d}{dx}\sin(\cos(\tan(x)))$

DERIVATIVES OF INVERSE

A $\dfrac{d}{dx}\arctan(x)$

B $\dfrac{d}{dx}\arcsin(x)$

The End