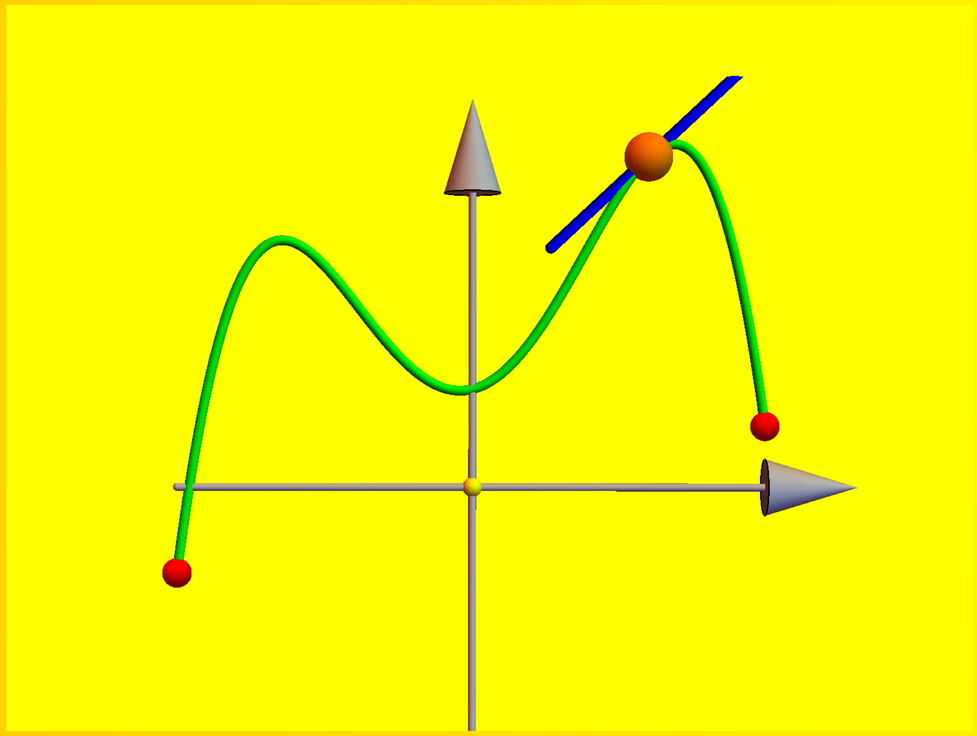
7 Rate of Change



differential quotient

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

rise
over
run

We write df/dx or f '(x)

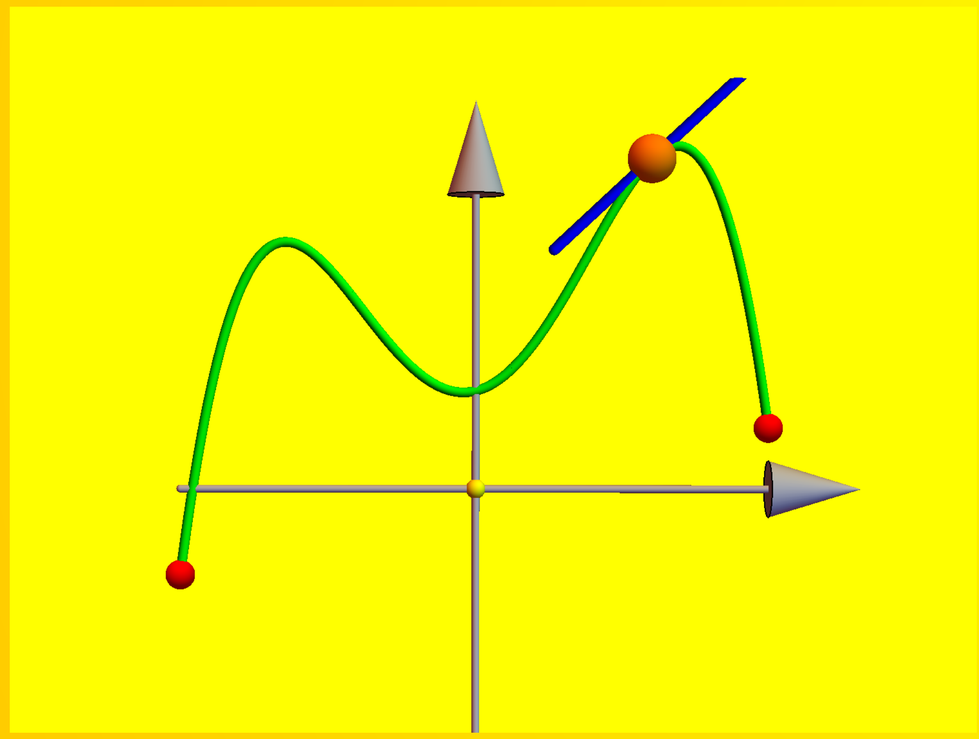
f is differentiable at x if the limit above exists there.

Poll

What is $\lim_{h \to 0} \dfrac{\sin(\pi + h) - \sin(\pi)}{h}$

A: 1 B: -1 C: 0 D: undefined

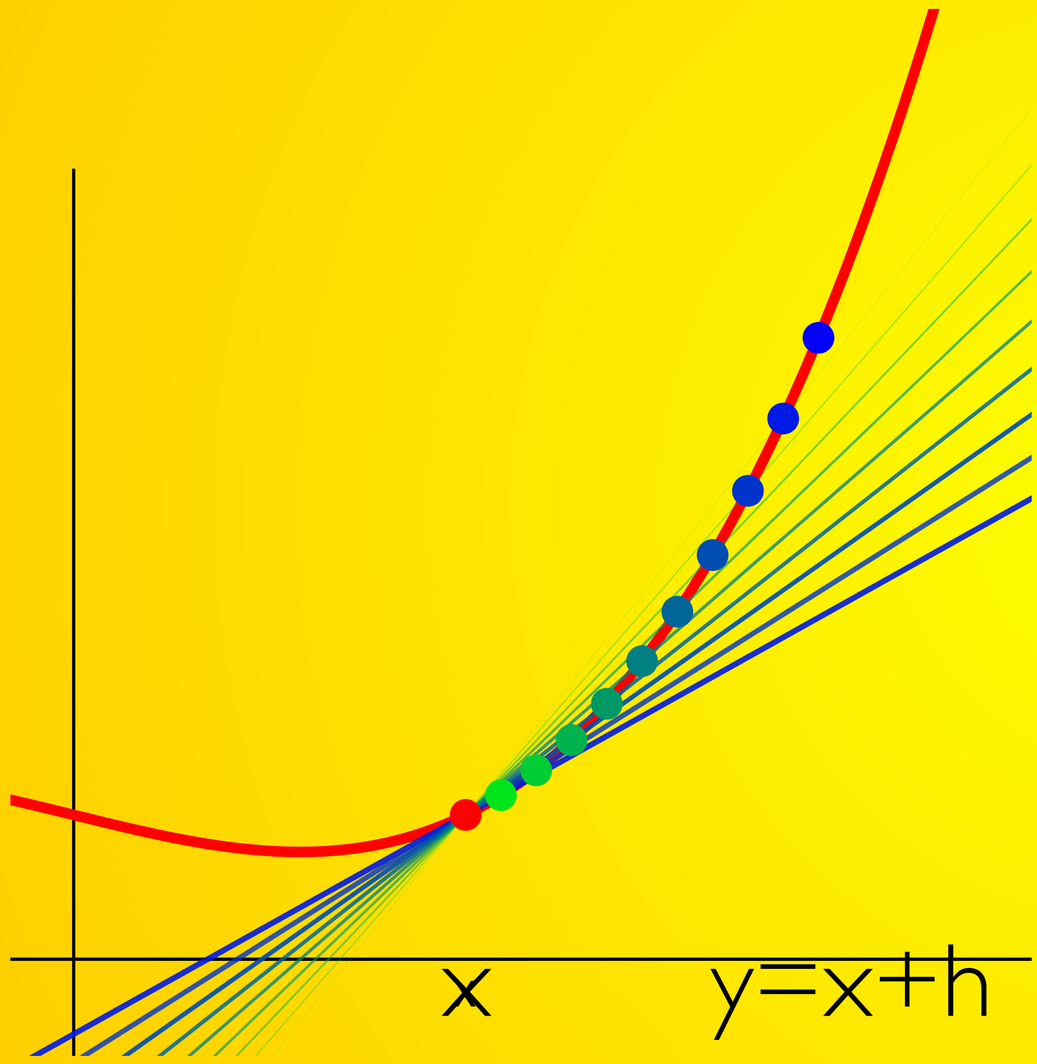
$h \to 0$ limit



$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Indefinite form 0/0.

Tangent

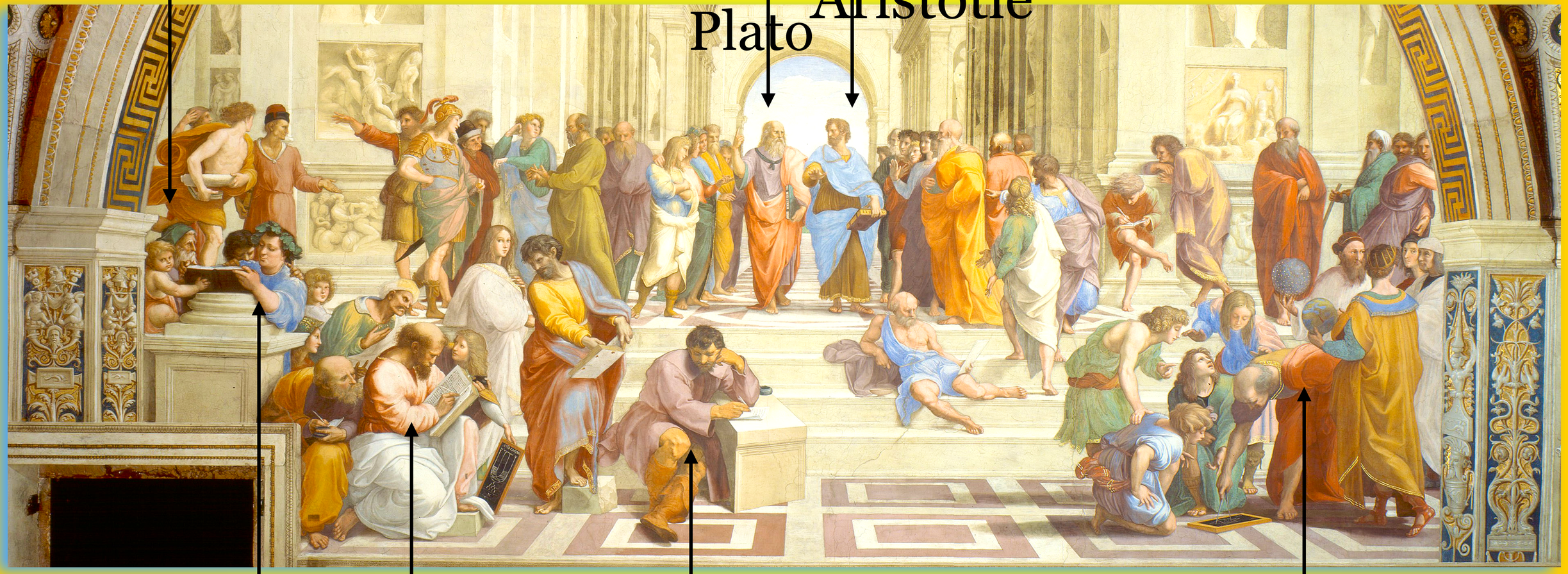


$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

secant lines

become tangent line

Zeno


Zeno
Plato
Aristotle
Epicur
Pythagoras
Archimedes
Euclid

Zeno

490 – c. 430 BC

Fresco in the Library of El Escorial, Madrid.

Arrow Paradox

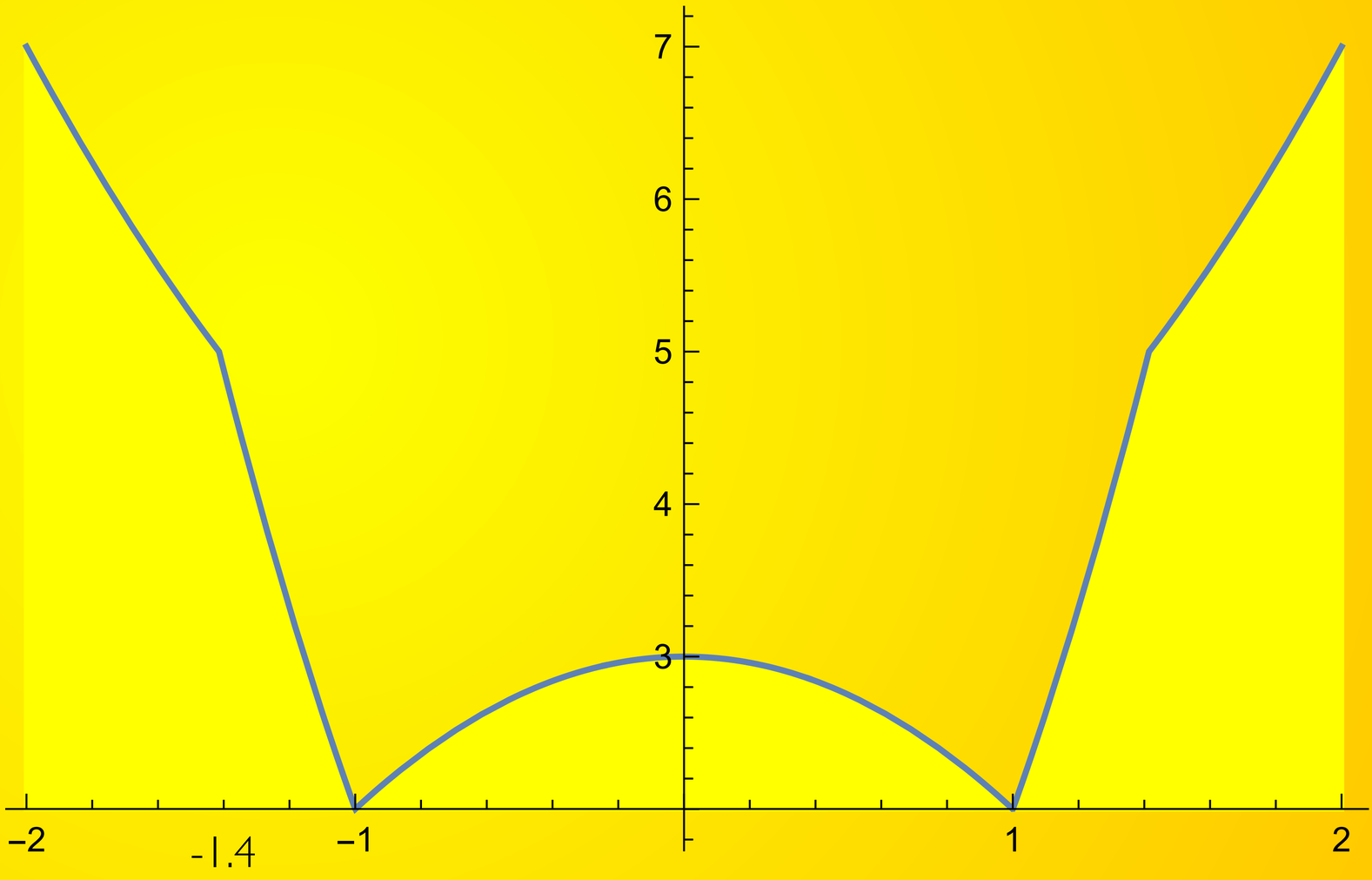
If everything when it occupies an equal space is at rest at that instant of time, and if that which is in locomotion is always occupying such a space at any moment, the flying arrow is therefore motionless at that instant of time and at the next instant of time but if both instants of time are taken as the same instant or continuous instant of time then it is in motion.


Marvelous Mrs Maisel

DIFFERENTIABLE

Continuous and differentiable

Where is f differentiable?
Where is it continuous?
Where is f' continuous?



Power Rule

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{(x+h)^n - x^n}{h} = \frac{x^n + nx^{n-1}h + \ldots + h^n - x^n}{h}$$

$$\frac{d}{dx}x^n = nx^{n-1}$$ *Powers Rule*

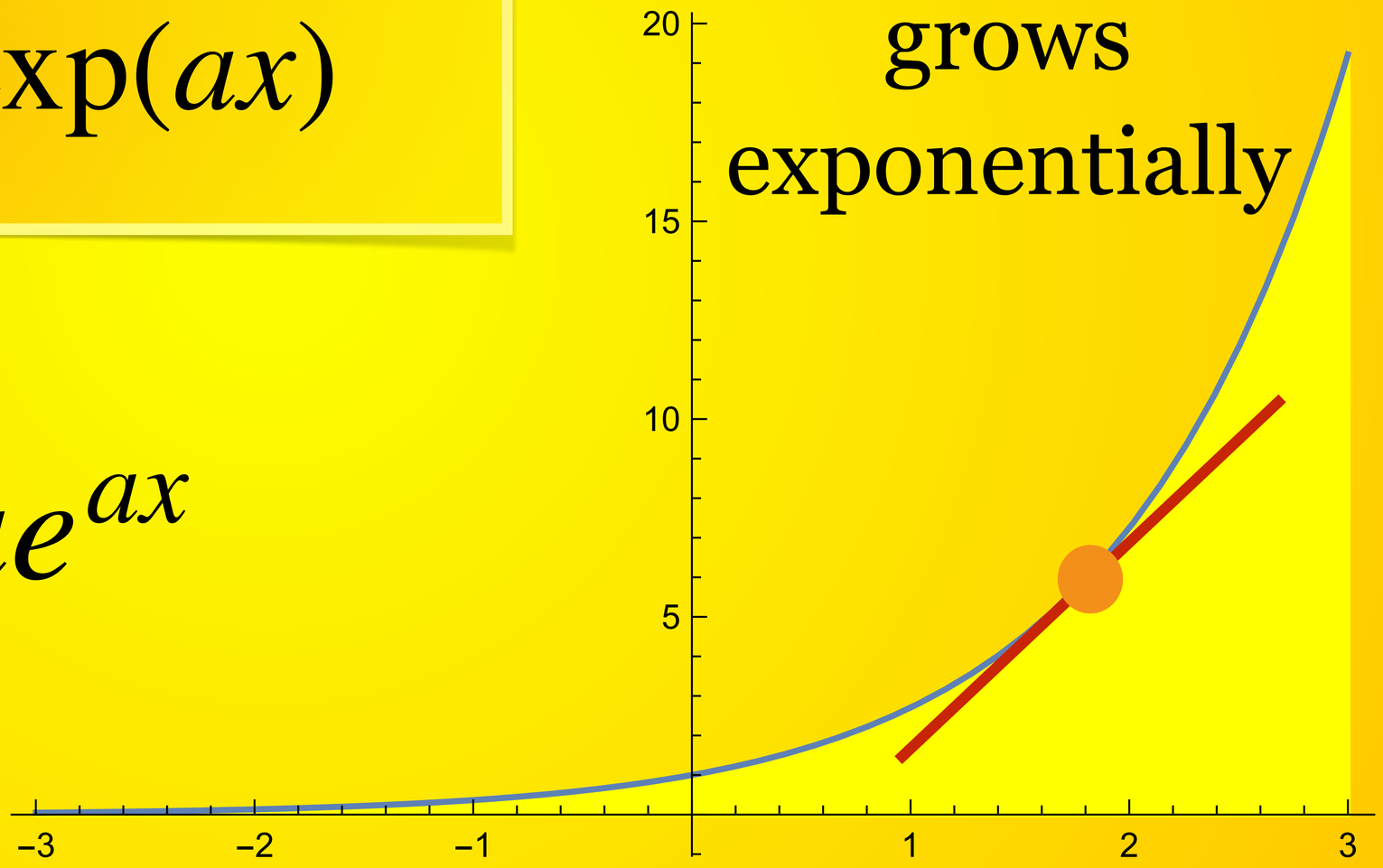
POWERS RULE

EXPONENTIAL FUNCTION

$$f(x) = \exp(ax)$$

$$\frac{d}{dx} e^{ax} = a e^{ax}$$

the slope grows exponentially




WHY

EULER NUMBER

$$f(x) = \exp(x)$$

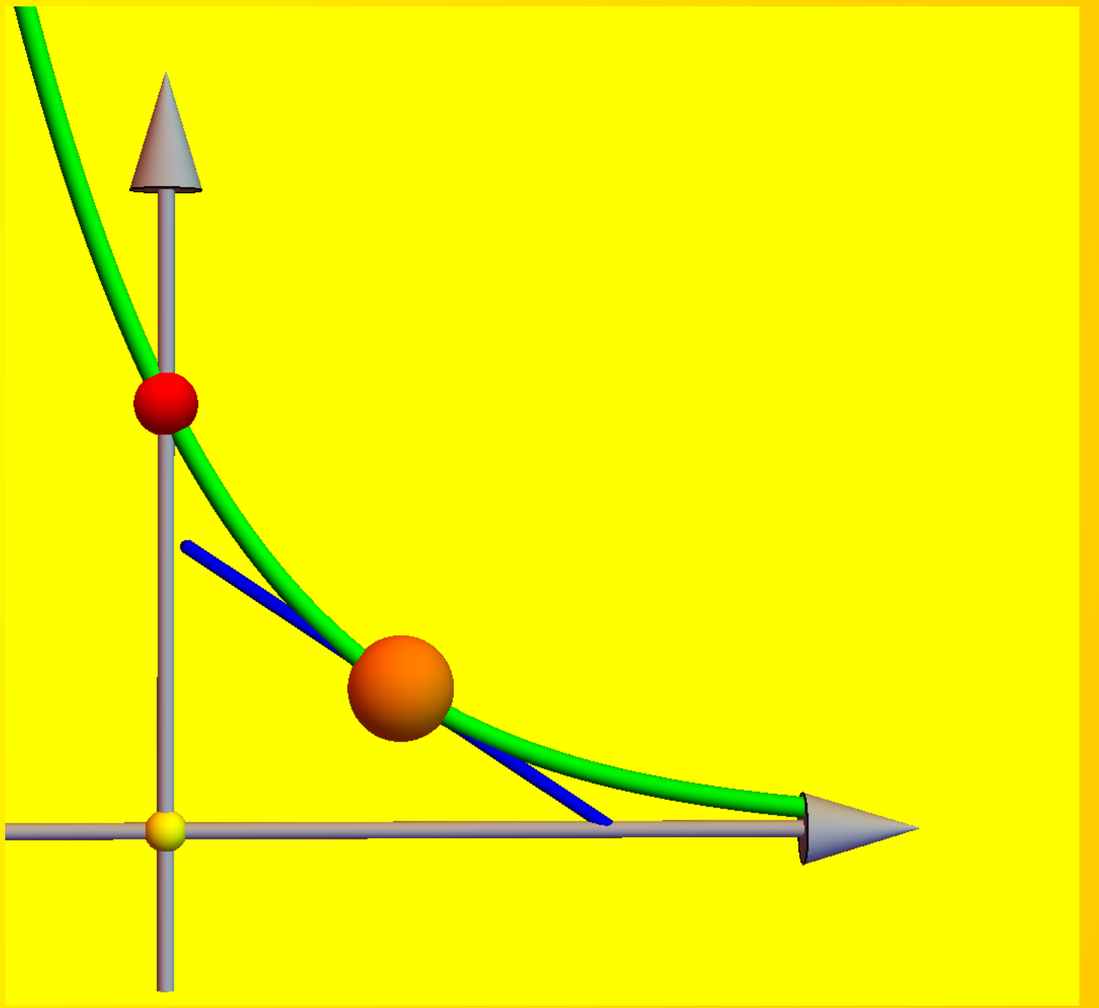
$$(1 + \frac{1}{1000})^{1000} = \frac{1001^{1000}}{1000^{1000}}$$

```
N[E]

2.71828

10 001 ^ 10 000 / 10 000 ^ 10 000

27 181 459 268 252 248 640 376 646 749 131 465 36
   730 527 814 778 395 201 398 550 252 332 514 27
   566 558 206 314 343 048 315 259 135 263 509 00
   303 668 822 824 462 791 135 530 003 628 766 46
   259 954 301 399 733 108 900 225 169 844 041 75
   881 462 502 271 091 693 041 486 593 950 434 71
   371 858 592 516 953 304 552 040 316 434 171 68
   961 040 780 048 700 209 032 684 748 287 421 12
   501 034 207 235 344 284 020 120 789 837 778 23
   065 124 410 505 303 449 906 955 143 451 190 83
   079 812 214 852 953 846 100 533 365 529 668 86
```

DECAY

$$f(x) = \exp(-ax)$$

$$\frac{d}{dx} e^{-ax} = (-a)e^{-ax}$$



WHY EXPONENTIAL?

TRIG FUNCTIONS

$$\frac{d}{dx}\sin(ax) = a\cos(ax)$$

$$\frac{d}{dx}\cos(ax) = -a\sin(ax)$$

WHY?

$$\exp(iax) = \cos(ax) + i\sin(ax)$$

$$ia\exp(iax) = ia\cos(ax) - a\sin(ax)$$

POLYNOMIALS

By linearity, we can
now differentiate
any polynomial

$$(3x^4 - 2x^2 + 5x)' = 12x^2 - 4x + 5$$

There is an error in this computation. Can you spot it?

Question

What is

$$\frac{d}{dx}\frac{1}{x}$$

$\log(x)$

$\frac{1}{x}$

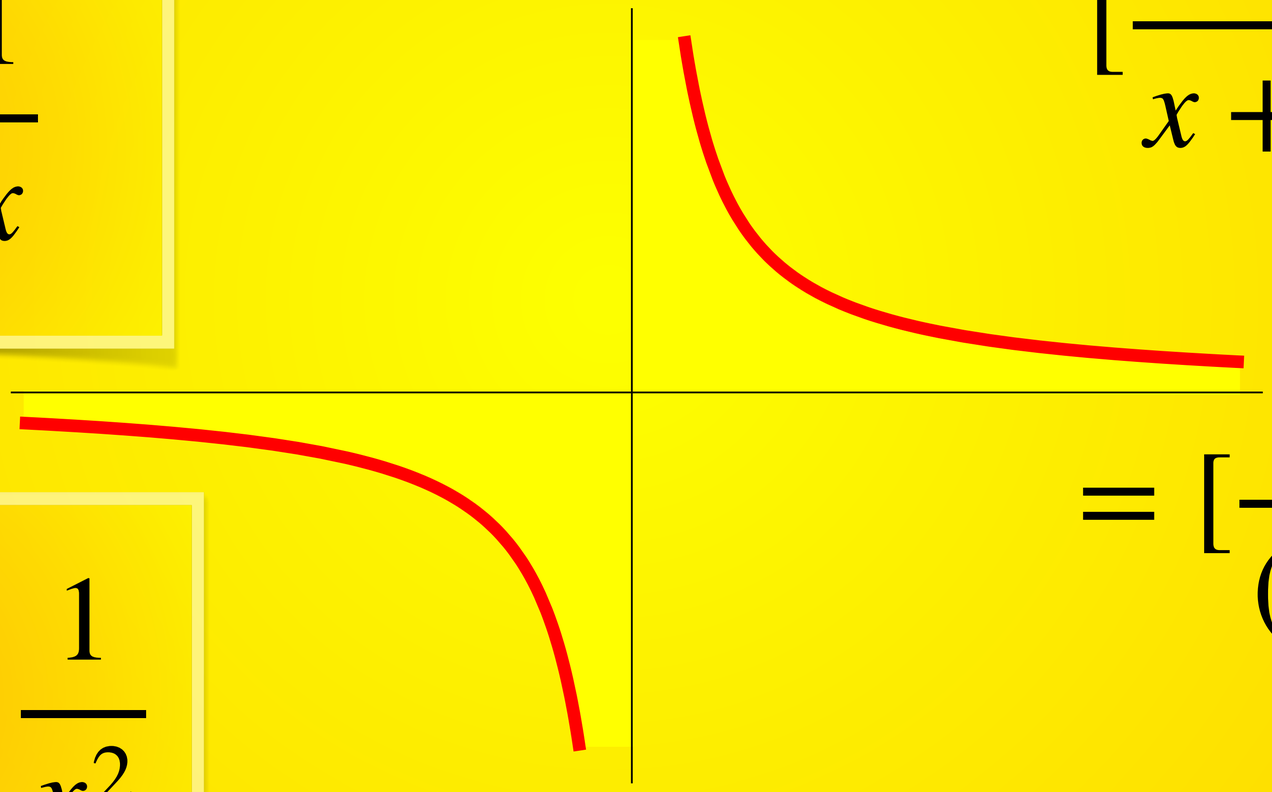
$\frac{1}{x^2}$

$\frac{-1}{x^2}$

$$F(x)=1/x$$

$$f(x)=\frac{1}{x}$$

$$f'(x)=-\frac{1}{x^2}$$



$h \to 0$

$$\frac{f(x+h)-f(x)}{h}=$$

$$\left[\frac{1}{x+h}-\frac{1}{x}\right]\frac{1}{h}$$

$$=\left[\frac{-h}{(x+h)x}\right]\frac{1}{h}$$

$$=\frac{-1}{(x+h)x}$$

INCREASING DECREASING

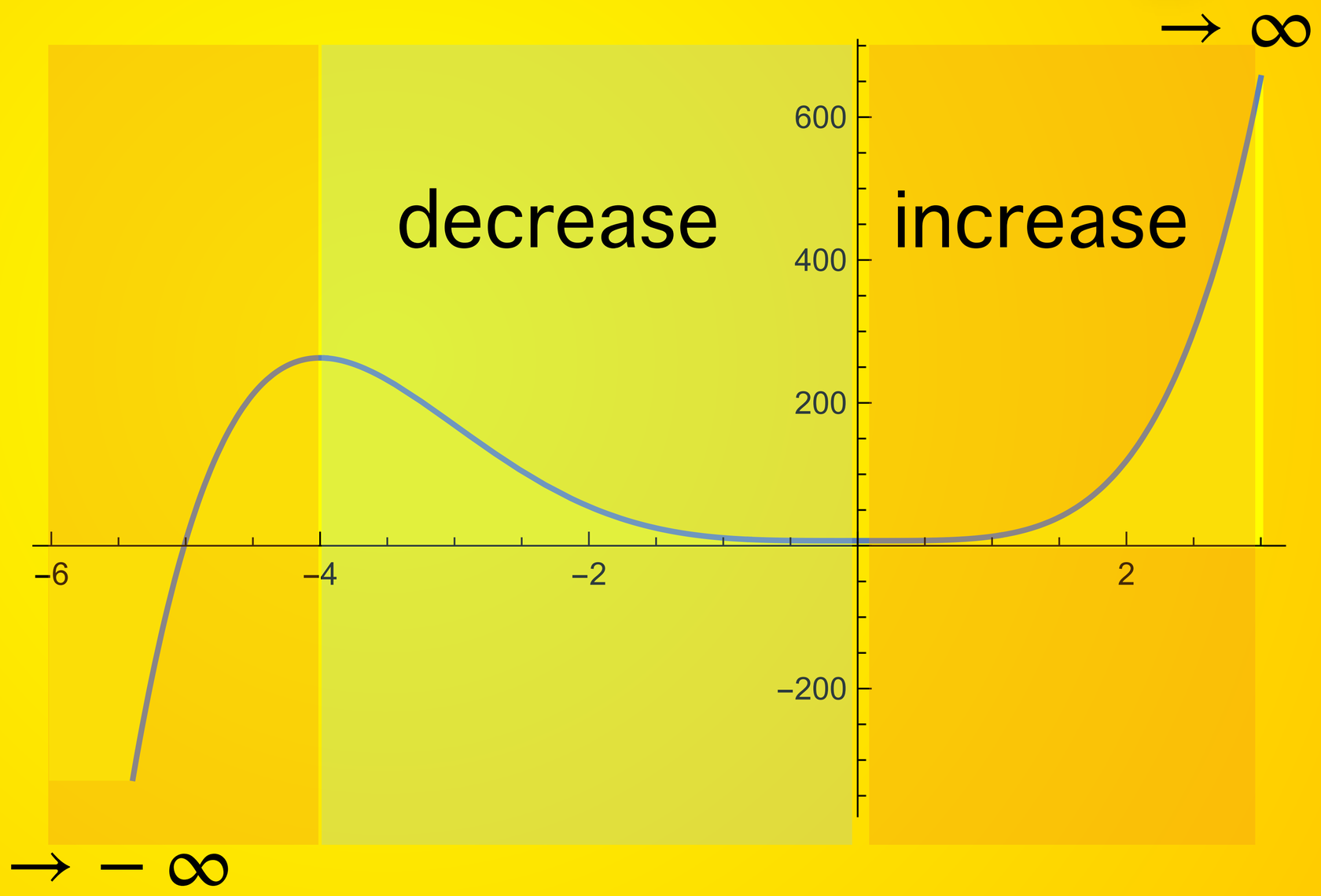
$$x^5 + 5x^4 + 7$$

$$f'(x) > 0$$

increase

$$f'(x) < 0$$

decrease



February 10th Jam

Find the derivatives at x=0

$f(x) = 10^{2+x}$

$f(x) = (x+2)^{10}$

$f(x) = (2+x)^{10}$

$f(x) = 2\cos(10x)$

Jam

Where does the following function have positive slope, where negative slope?

$$f(x) = 12x - 9x^2 + 2x^3$$

Can you draw the graph of f '?

More challenging: Can you draw the graph of f

End