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Fundamental Theorem

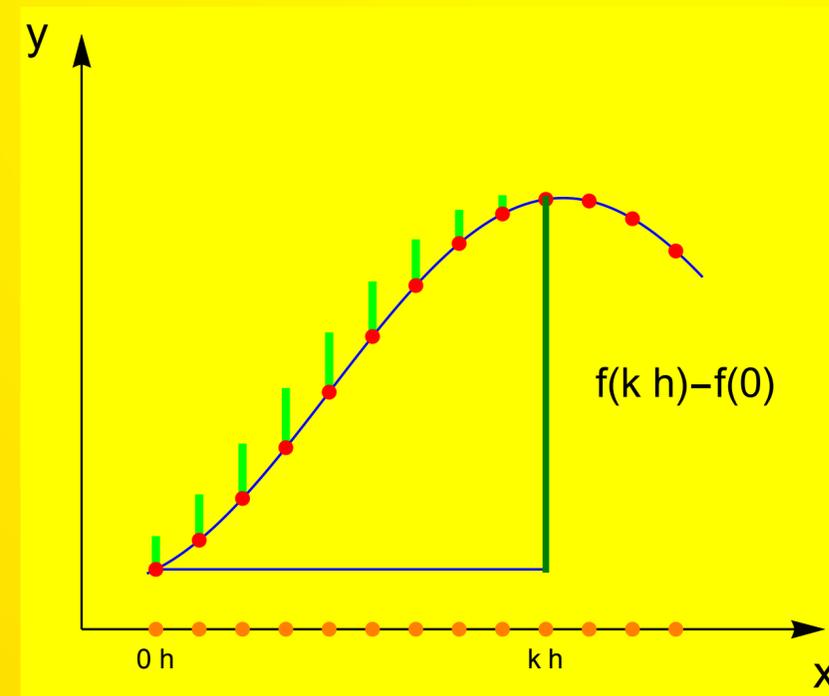
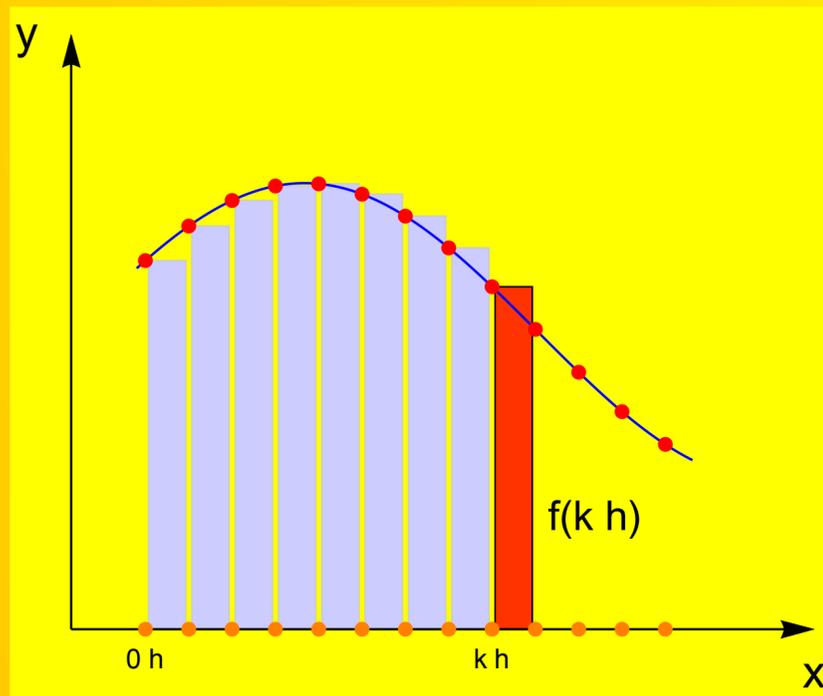
$$Df(x) = [f(x+h) - f(x)]/h$$

$$Sf(x) = [f(0) + f(h) + \dots + f((n-1)h)] h$$

$$D S(f)(x) = f(x)$$

$$S D(f)(x) = f(x) - f(0)$$

⇓ later

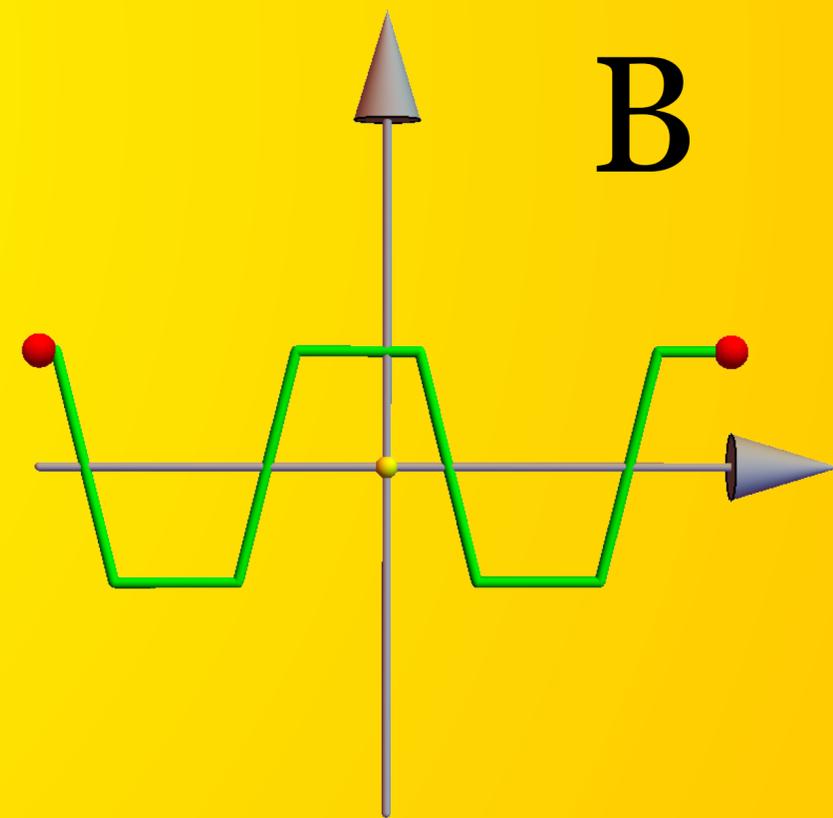
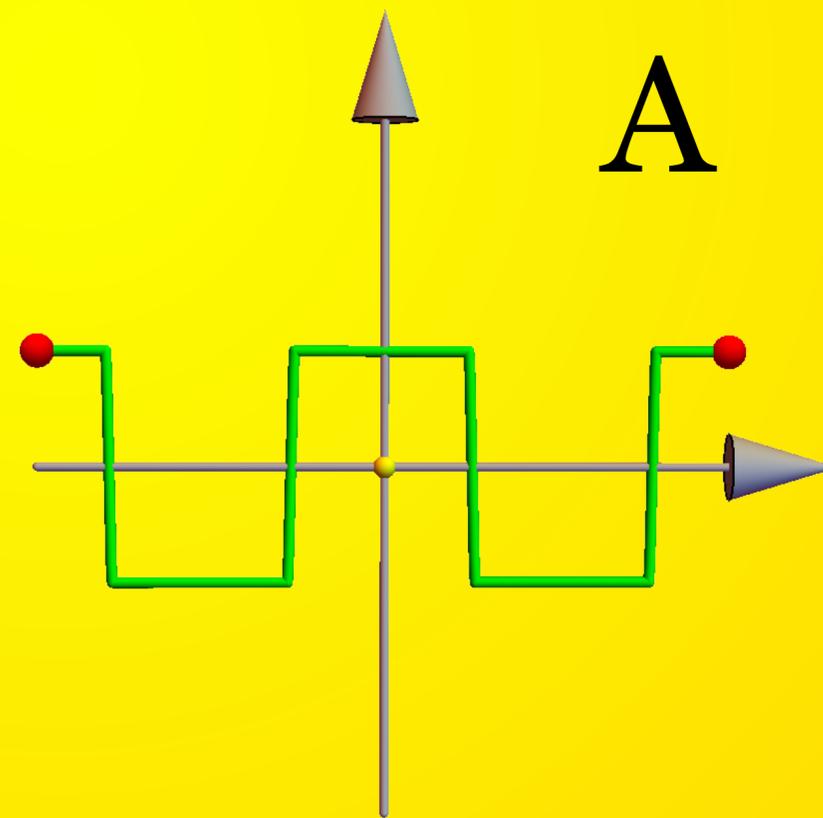
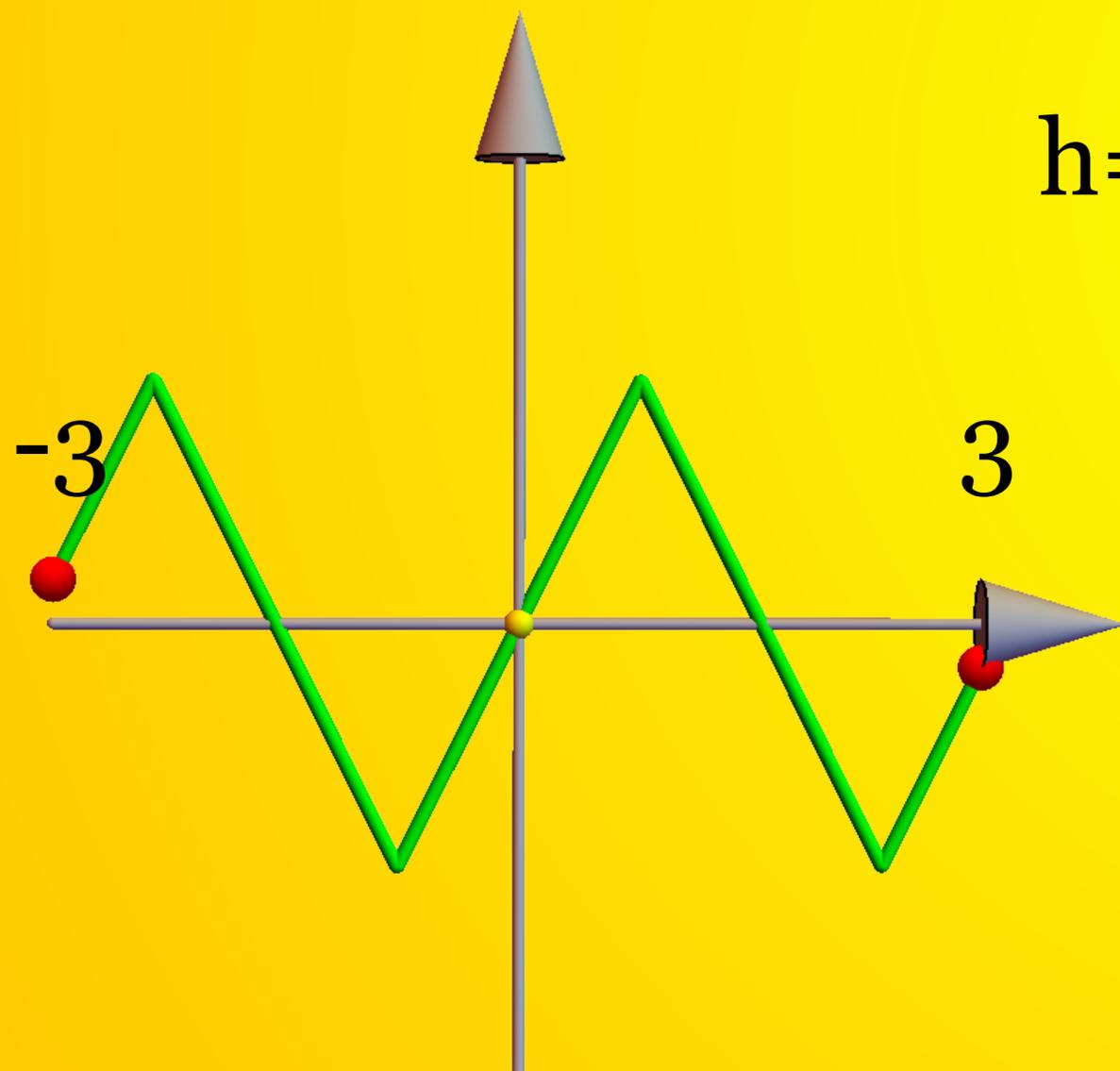


$$\frac{d}{dx} \int_0^x f(t) dt = f(x)$$

$$\int_0^x f'(t) dt = f(x)$$

Poll

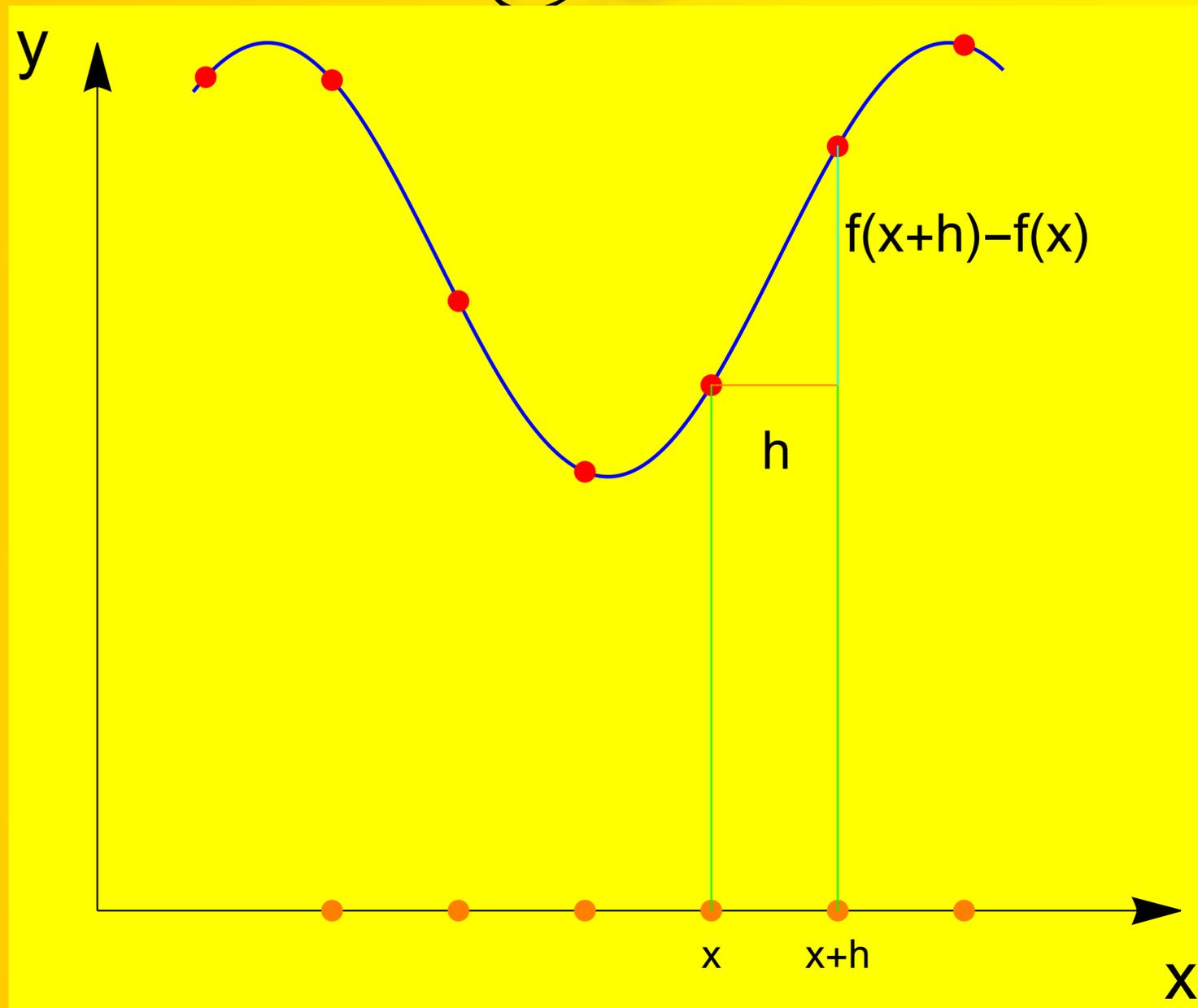
Let $f(x)=\arcsin(\sin(x))$ be the sawtooth graph and $h=0.2$
Which of the graphs is $Df(x) = [f(x+h)-f(x)]/h$?



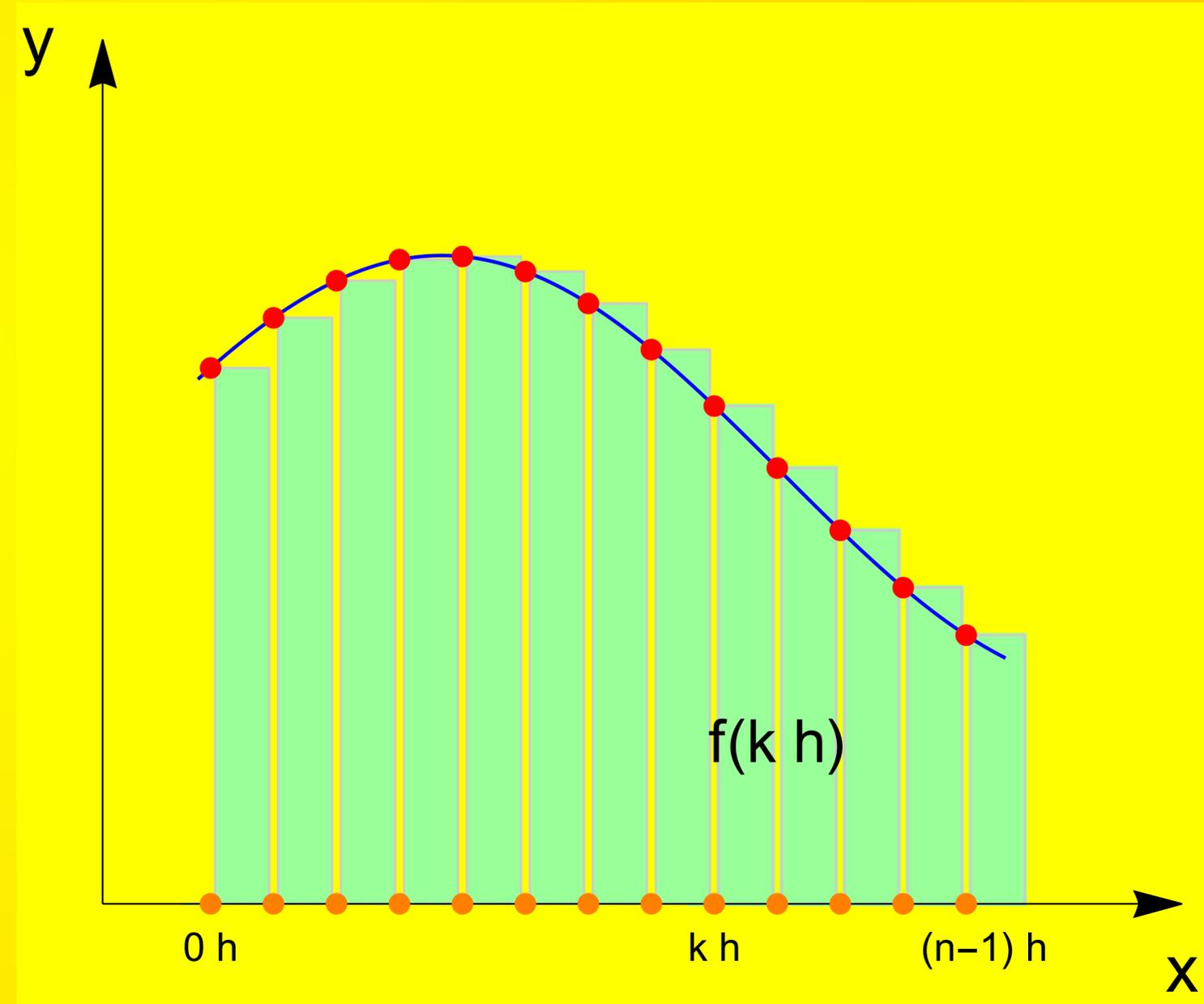


Marvelous Mrs
Maisel, Series 2

Differences and Sums



$$Df(x) = [f(x+h)-f(x)]/h$$



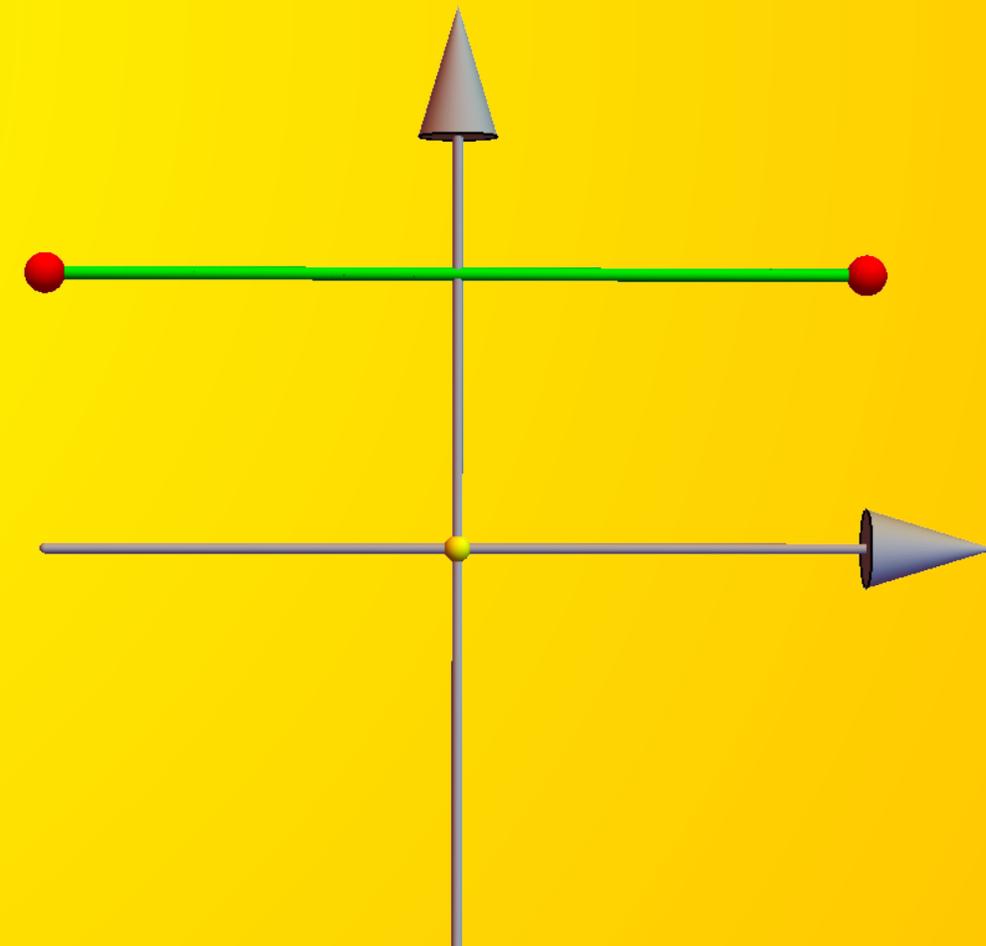
$$Sf(x) = [f(0)+f(h)+\dots+f((n-1)h)] h$$

Constant Function

If $f(x)=c$ is constant, then $Df(x) = 0$

$$Df(x) = [f(x+h)-f(x)]/h$$

This rule will remain valid also
in the limit $h \rightarrow 0$

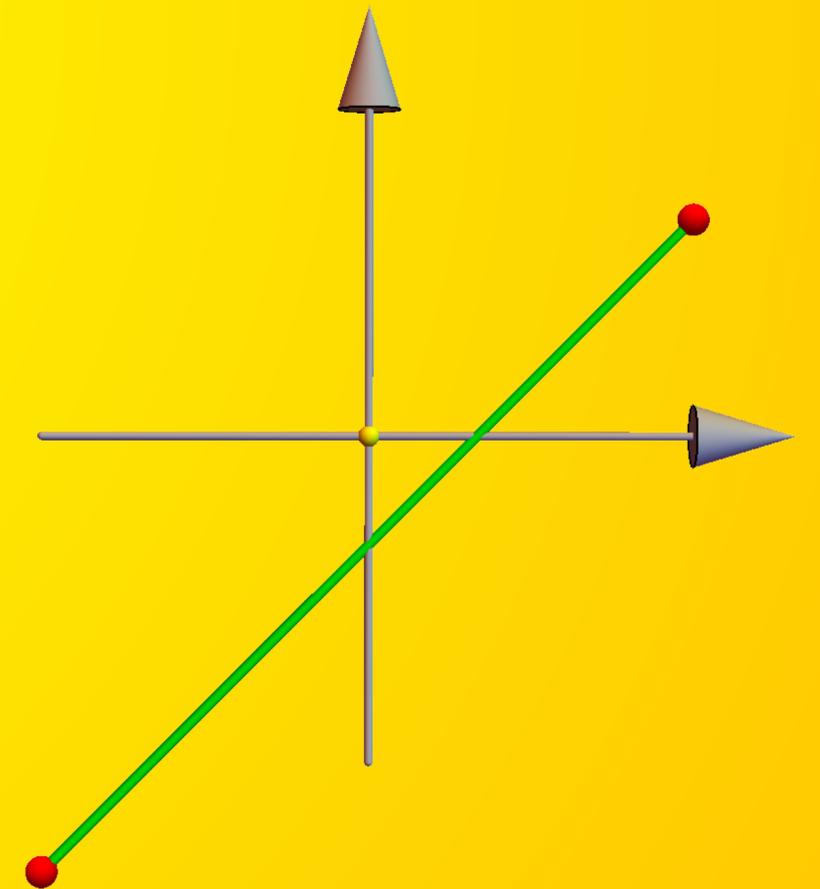


Linear Function

If $f(x)=mx+b$, then $Df(x) = m$

$$Df(x) = [f(x+h)-f(x)]/h$$

This rule will remain valid also
in the limit $h \rightarrow 0$

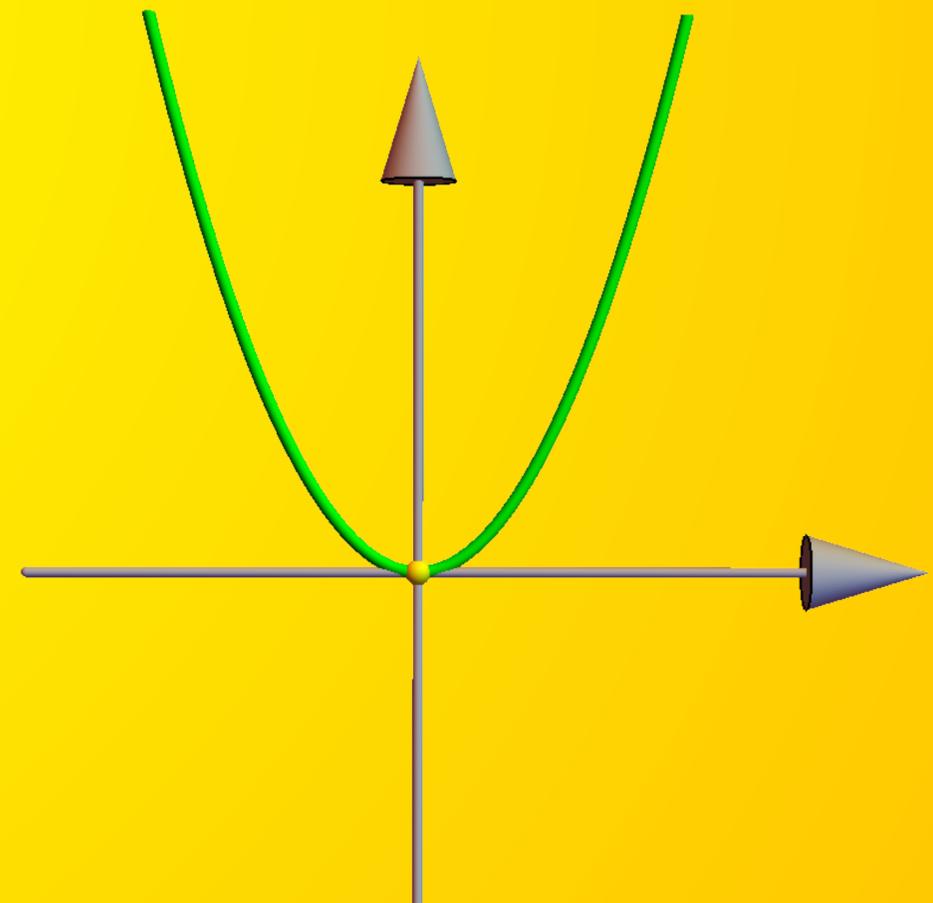


Quadratic Function

If $f(x)=x^2$, then $Df(x) = 2x+h$

$$Df(x) = [f(x+h)-f(x)]/h$$

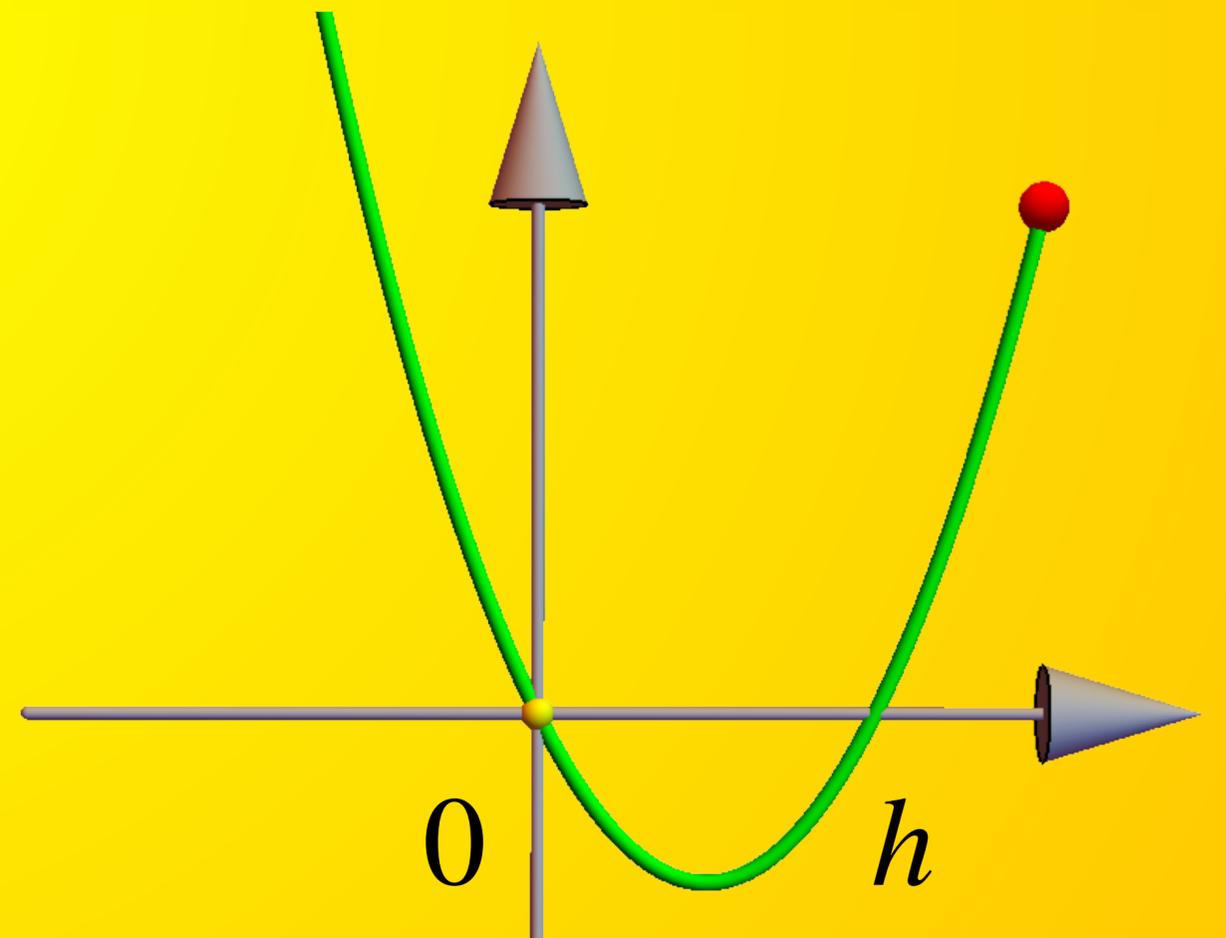
This rule will remain valid also
in the limit $h \rightarrow 0$



Modified

If $f(x) = [x]^2 = x(x - h)$, then $Df(x) = 2x$

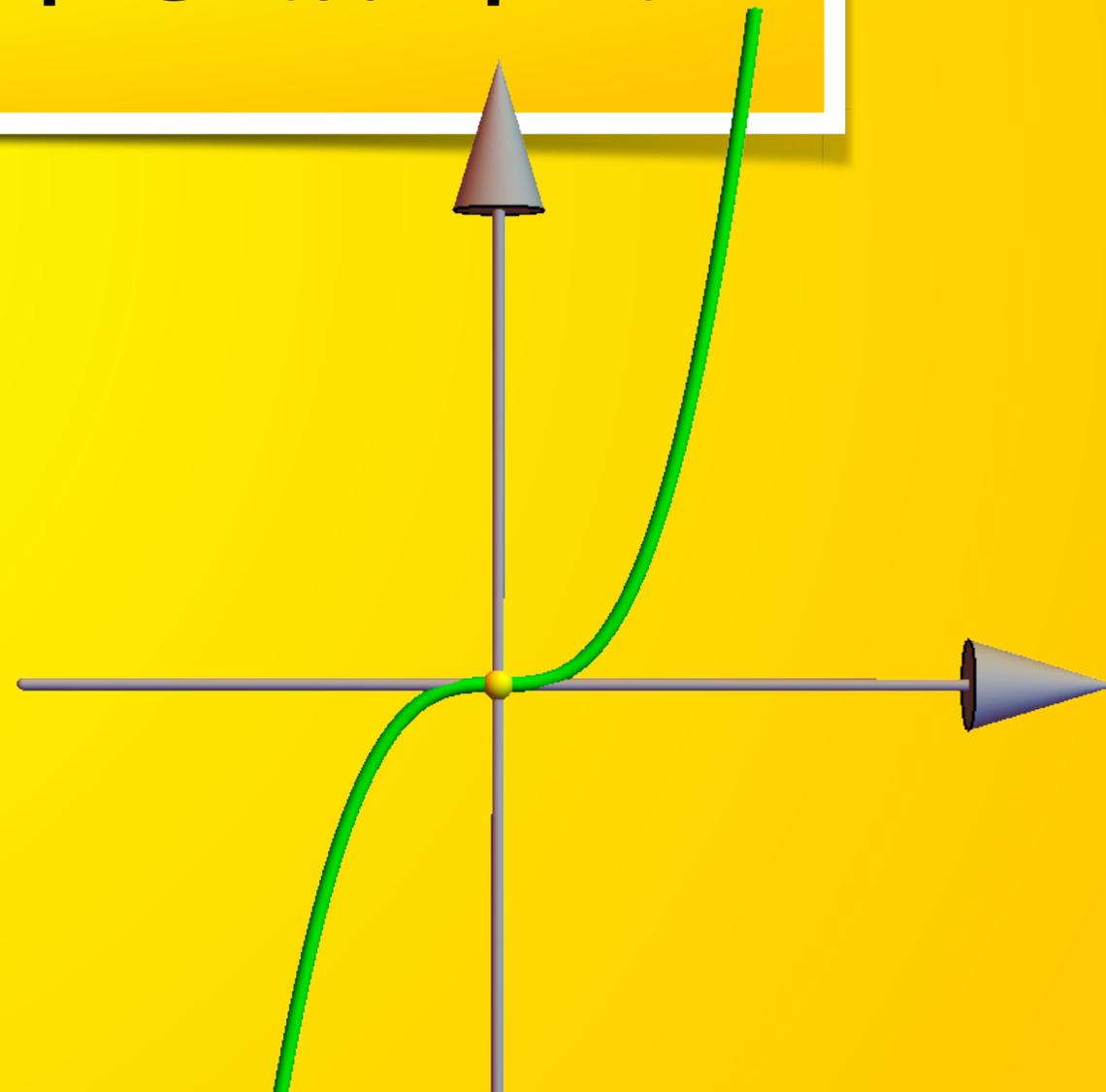
$$\begin{aligned} Df(x) &= [f(x+h) - f(x)]/h \\ &= [(x+h)x - x(x-h)]/h \end{aligned}$$



Cubic Function

$$\text{If } f(x)=x^3, \text{ then } Df(x) = 3x^2 + 3hx + h^2$$

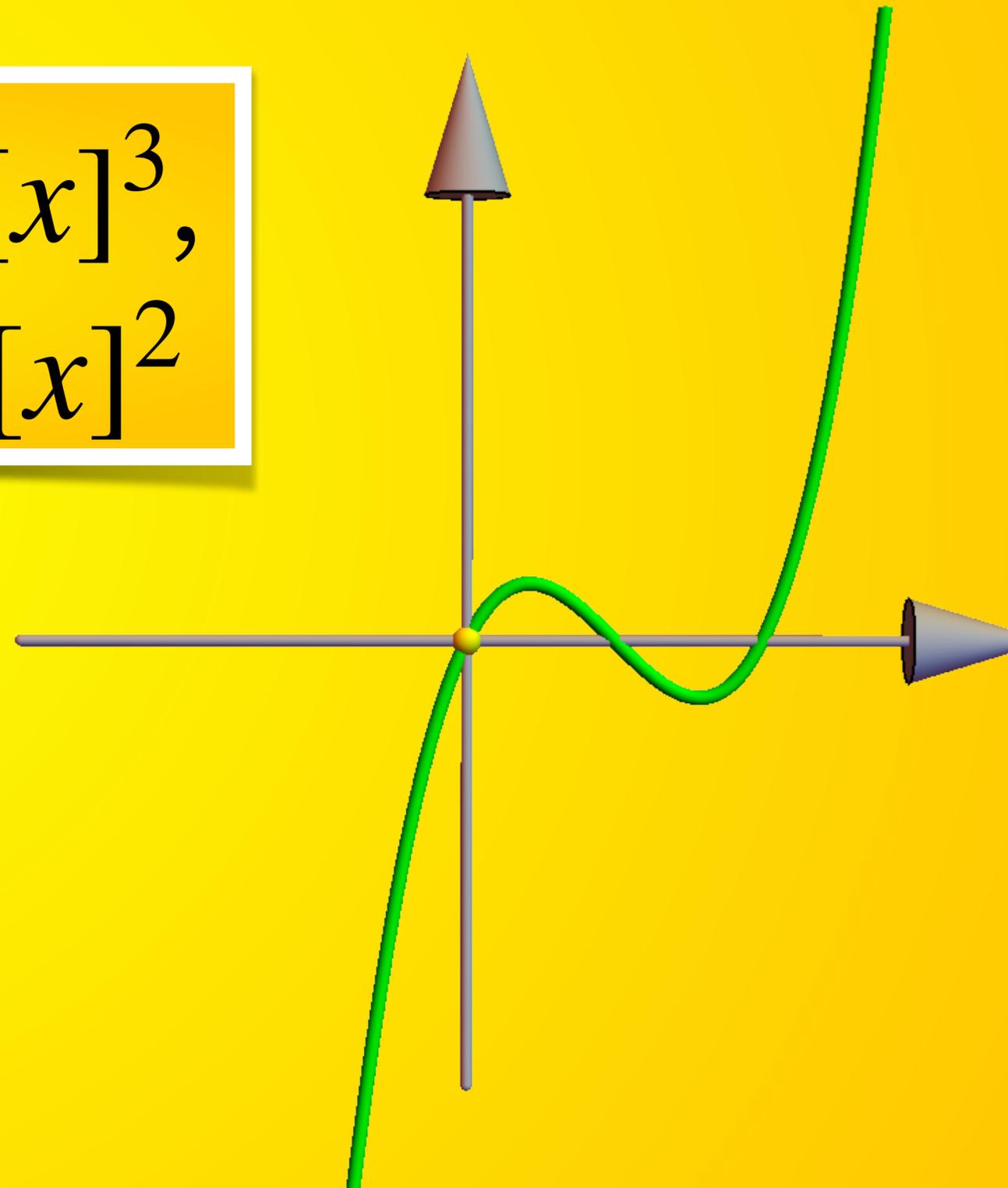
$$Df(x) = [f(x+h)-f(x)]/h$$



Modified

If $f(x) = x(x - h)(x - 2h) = [x]^3$,
then $Df(x) = 3x(x - h) = 3[x]^2$

$$Df(x) = [f(x+h) - f(x)]/h$$

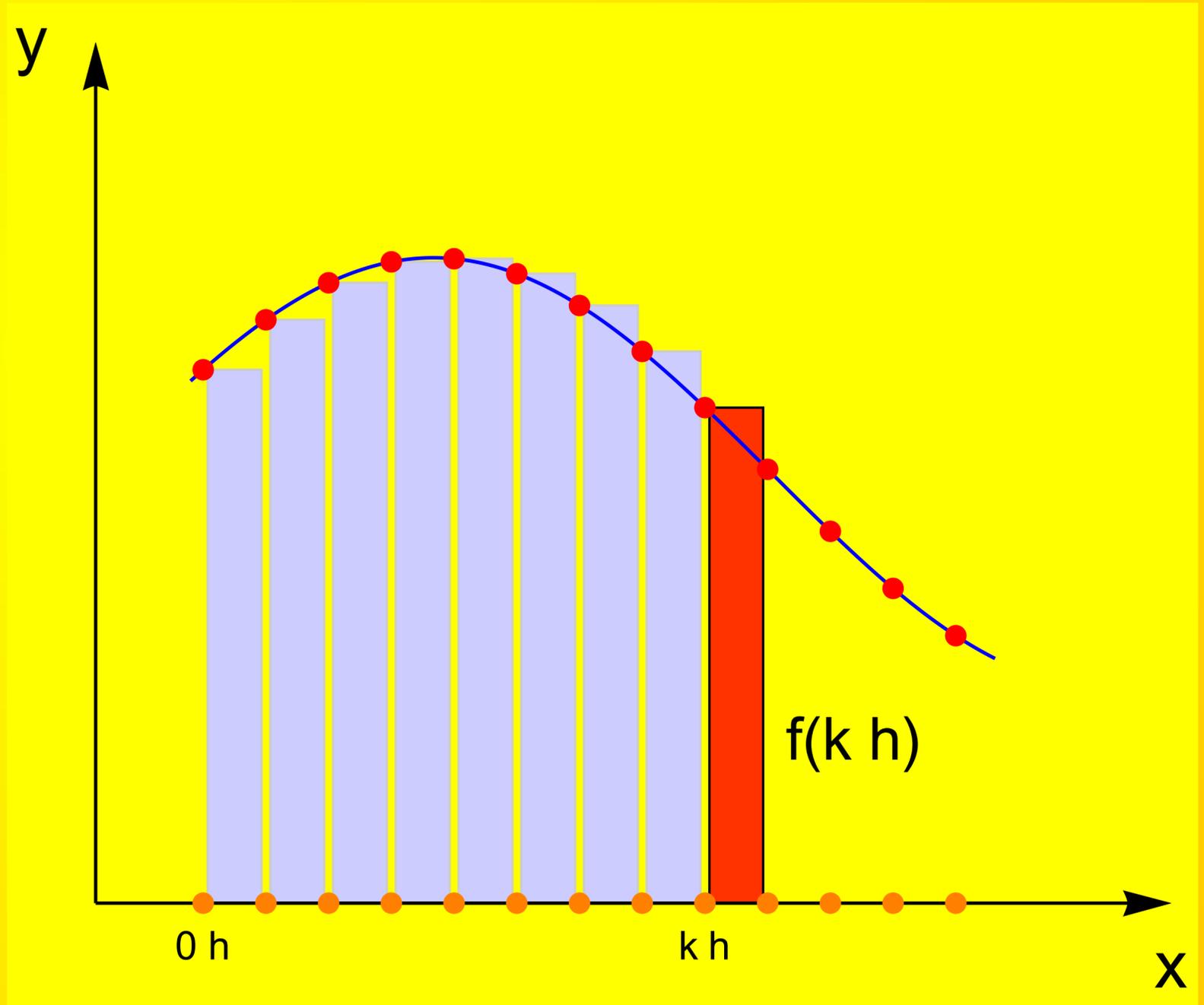


Fundamental Theorem 1

$$D S(f)(x) = f(x)$$



$$\frac{d}{dx} \int_0^x f(t) dt = f(x)$$

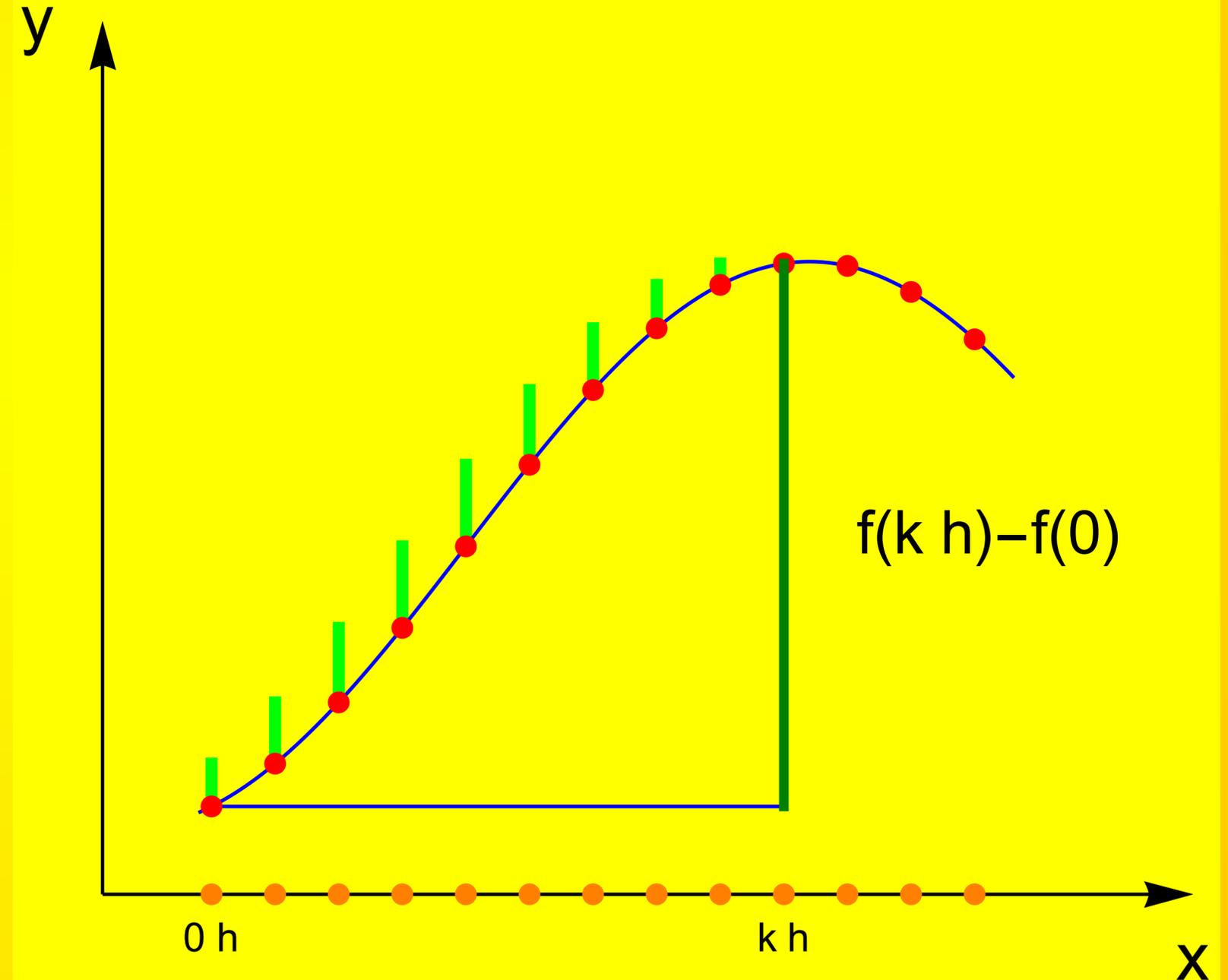


Fundamental Theorem II

$$SD(f)(x) = f(x) - f(0)$$



$$\int_0^x f'(t) dt = f(x)$$



Derivatives and Integrals

$$\diamond D f(x) = f(x+1) - f(x)$$

$$S f(x) = f(0) + f(1) + f(2) + \dots + f(x-1)$$

$$D x^n = n x^{n-1}$$

$$\begin{aligned} & (x+1)x(x-1)(x-2) \\ & - x(x-1)(x-2)(x-3) \\ & = 4x(x-1)(x-2) \end{aligned}$$

$$D \exp(ax) = a \exp(ax)$$

$$\begin{aligned} & (1+a)^{(x+1)} - (1+a)^x \\ & = a(1+a)^x \end{aligned}$$



Exponential

$$D \exp(ax) = a \exp(ax)$$

$$\exp(ax) = (1 + ah)^{x/h}$$

Polynomials

$$D x^n = n x^{n-1}$$

After the modification.

Trig Functions

$$D \exp(ix) = i \exp(ix)$$

Rules

$$\frac{d}{dx} x^n = n x^{n-1}$$

$$\frac{d}{dx} e^{ax} = a e^{ax}$$

$$\frac{d}{dx} \cos(ax) = -a \sin(ax)$$

$$\frac{d}{dx} \log(a + x) = \frac{1}{a + x}$$

$$\frac{d}{dx} \sin(ax) = a \cos(ax)$$

Rules

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

$$\int e^{ax} dx = \frac{e^{ax}}{a}$$

$$\int \cos(ax) dx = \frac{\sin(ax)}{a}$$

$$\frac{d}{dx} \log(a+x) = \frac{1}{a+x}$$

$$\frac{d}{dx} \sin(ax) = a \cos(ax)$$

$$\frac{d}{dx} [5e^{12x} + 9 \sin(7x)]$$

Jam

$$\int [3 \cos(23x) + 6e^{4x}] dx$$

$$\int \left[3 + 6 \sin(8x) + \frac{1}{1+x} \right] dx$$

$$\frac{d}{dx} [\log((1-x^2))]$$

End