Unit 6

a) $f(x) = x^2$

$$\boxed{Df = \frac{f(x+h) - f(x)}{h}}$$

$$= \frac{(x+h)^2 - x^2}{h} = \frac{\cancel{x^2} + 2\cancel{h}x + h^2 - \cancel{x^2}}{\cancel{h}}$$

$$= 2x + h$$

$2x$ in limit $h \to 0$

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b) $f(x) = x(x-h) = [x]^2$

$f(x+h) - f(x)$

$$\overset{(*)}{=} \frac{(x+h) \cdot x - x(x-h)}{h}$$

$$= \frac{x^2 + hx - x^2 + xh}{h}$$

$$= 2x$$

$$= 2[x]$$

$(*)$ $f(x) = x(x-h)$

$f(x+h) = (x+h)(x+h-h)$

Formula:

$$\boxed{\frac{d}{dx}[x]^n = n\,[x]^{n-1}}$$

$$[x]^n = \underbrace{x(x-h)\dots(x-(n-1)h)}_{n \text{ terms}}$$

leave away brackets:

$$\boxed{\frac{d}{dx}x^n = n\,x^{n-1}}$$

$$D[x]^n = n\,[x]^{n-1}$$

In the HW, you will check!

$$\boxed{D e^{ax} = a e^{ax}}$$

$$e^{ax} = (1+ah)^{x/h}$$

compound interest formula.

$n=4$

Homework

$$[x]^2 = x(x-h)$$

c)

$$f(x) = x(x-h)(x-2h)$$

$$f(x+h) = (x+h)\cdot x(x-h)$$

$$\frac{f(x+h)-f(x)}{h}$$

$$= \frac{x(x-h)\underbrace{[\cancel{x}+h-(\cancel{x}-2h)]}_{3h}}{h}$$

$$= x(x-h)\cdot 3$$

$$D[x]^3 = 3[x]^2$$

(d)

$$\sqrt{x} = x^{1/2}$$

$$\frac{d}{dx} x^n = n x^{n-1}$$

also holds for $n = \frac{1}{2}$

$$\boxed{\frac{d}{dx} x^{1/2} = \frac{1}{2\sqrt{x}}}$$

$$D\sqrt{x} = \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

How do we simpliphiy.

$$\frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h(\sqrt{x+h} + \sqrt{x})}$$

we get a simple expression

$$\frac{\cancel{x}+\cancel{h}-\cancel{x}}{\cancel{h}(\sqrt{x+h}+\sqrt{x})} = \frac{1}{\sqrt{x+h}+\sqrt{x}}$$

$$h \to 0 : \quad \frac{1}{\sqrt{x}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

Trick used several times is a method.

Gauss! complex numbers:

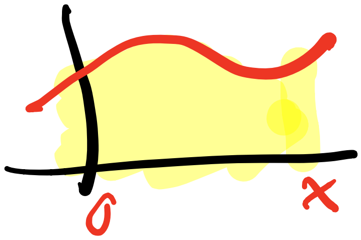
Jan:

$$\boxed{\frac{d}{dx} e^{ax} = a e^{ax}}$$

$$\boxed{\int e^{ax} dx = \frac{e^{ax}}{a} - \frac{1}{a}}$$

x

We want $\int_0^x f(t)\,dt$



$= Sf(x)$

to be zero if $x = 0$

$$\int \cos ax \, dx = \frac{\sin ax}{a}$$

$$\int \sin ax \, dx = -\frac{\cos ax}{a} + \frac{1}{a}$$

$$= \frac{1 - \cos ax}{a}$$

We want the integral to be 0 for $x = 0$.
sum

$$\boxed{Sf(x) \approx h\left[f(0) + f(h) + \dots f((n-1)h)\right]}$$

$x = nh$

$x = 0 \quad Sf(0) = 0$

$$\frac{d}{dx} \log(1 - x^2)$$

$$= \frac{d}{dx} \log((1-x)(1+x))$$

$$= \frac{d}{dx} \log(1-x) + \frac{d}{dx} \log(1+x)$$

$$\frac{d}{dx} \log(a+x) = \frac{1}{a+x}$$
