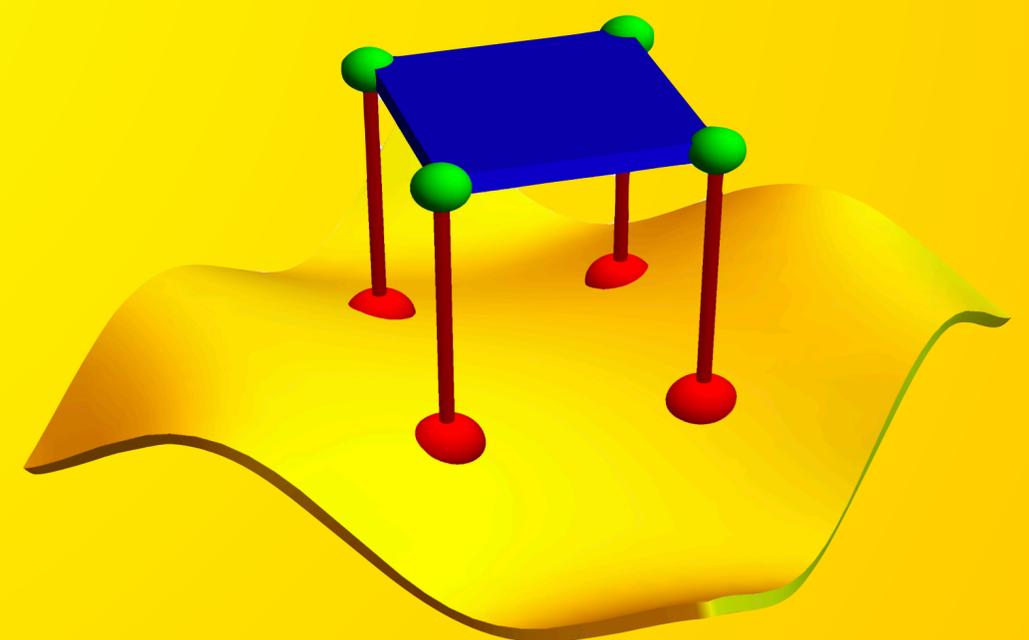
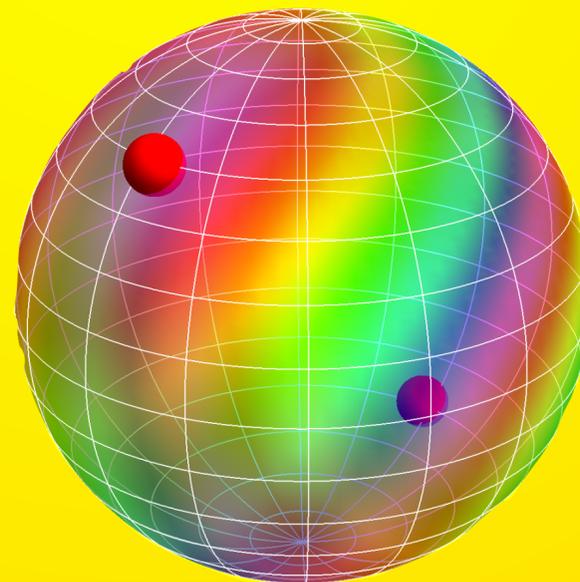
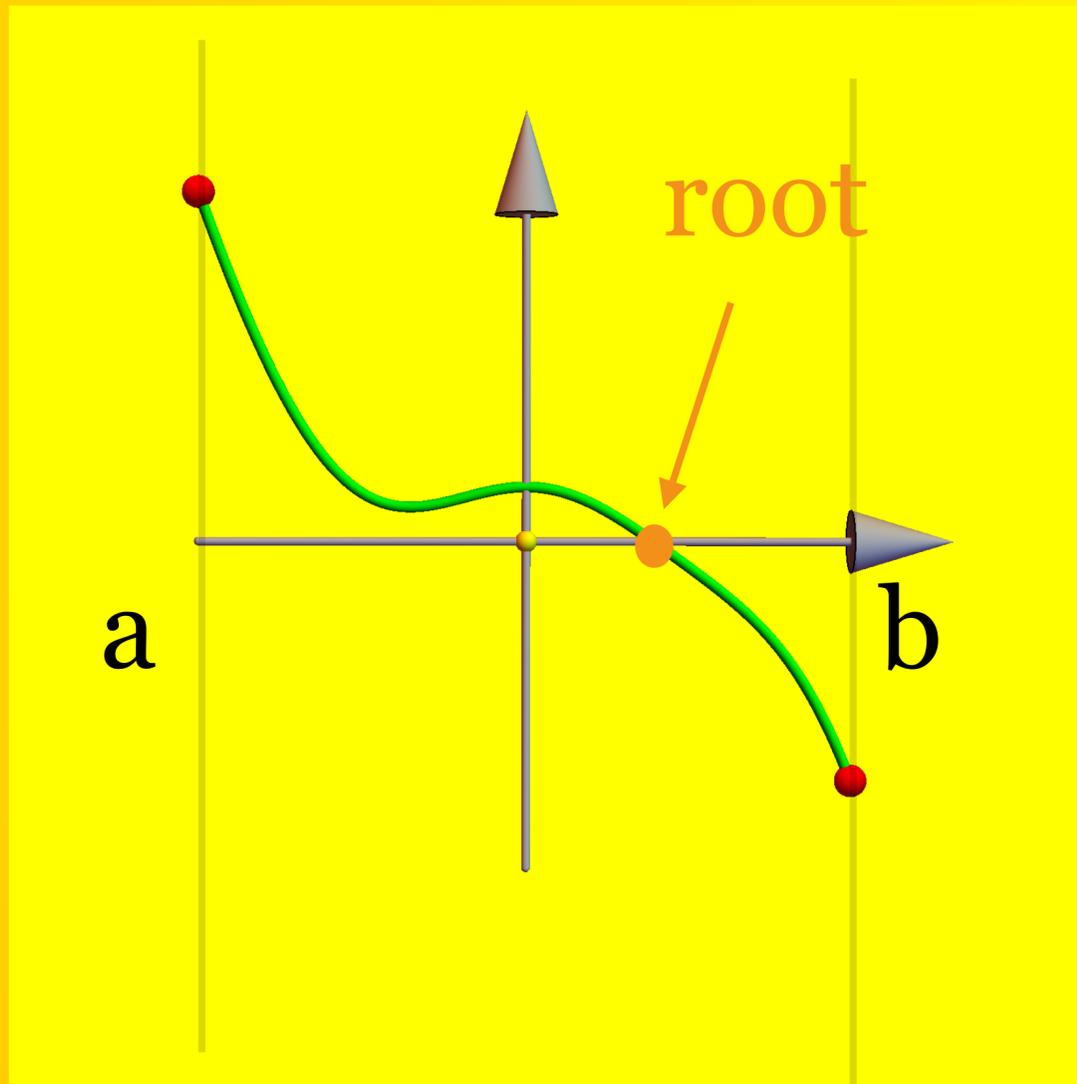


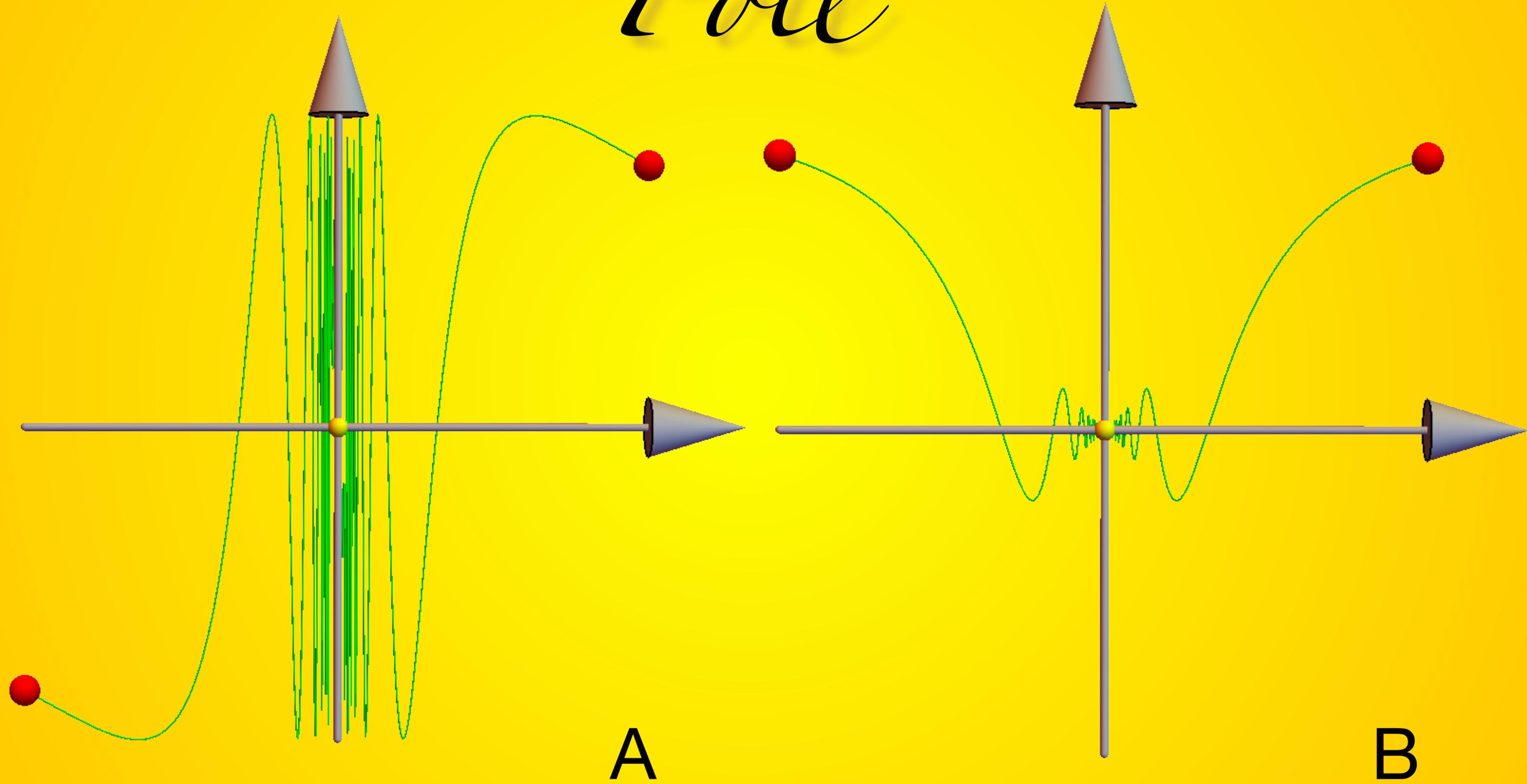
5

Intermediate Value

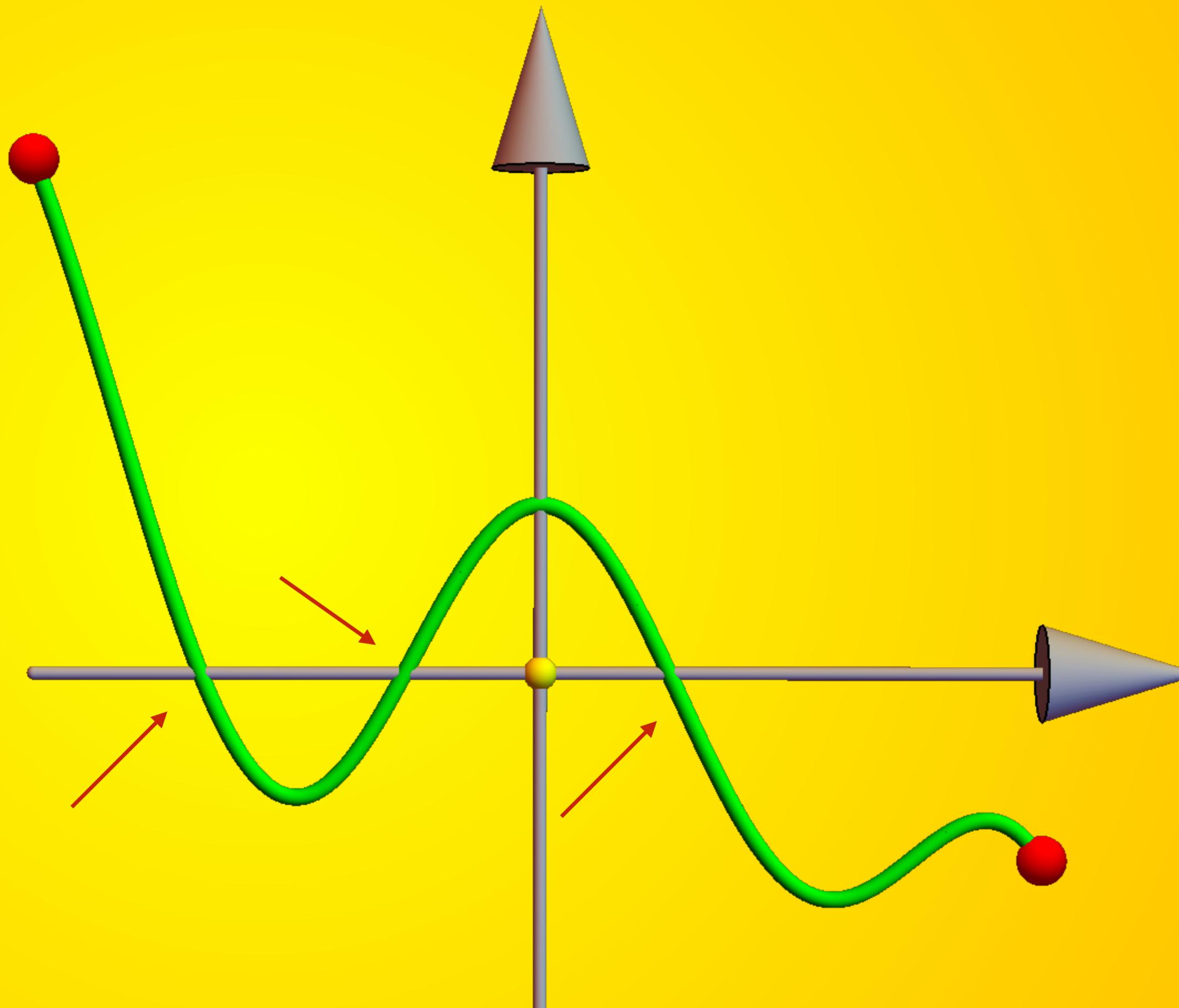
If f is continuous and
if $f(a) \cdot f(b) < 0$, then
there is a root in (a, b) .



Poll



Roots



Find the Roots:

$$\log |x|$$

$$x^2 - 25$$

$$\log(\sin^2(x))$$

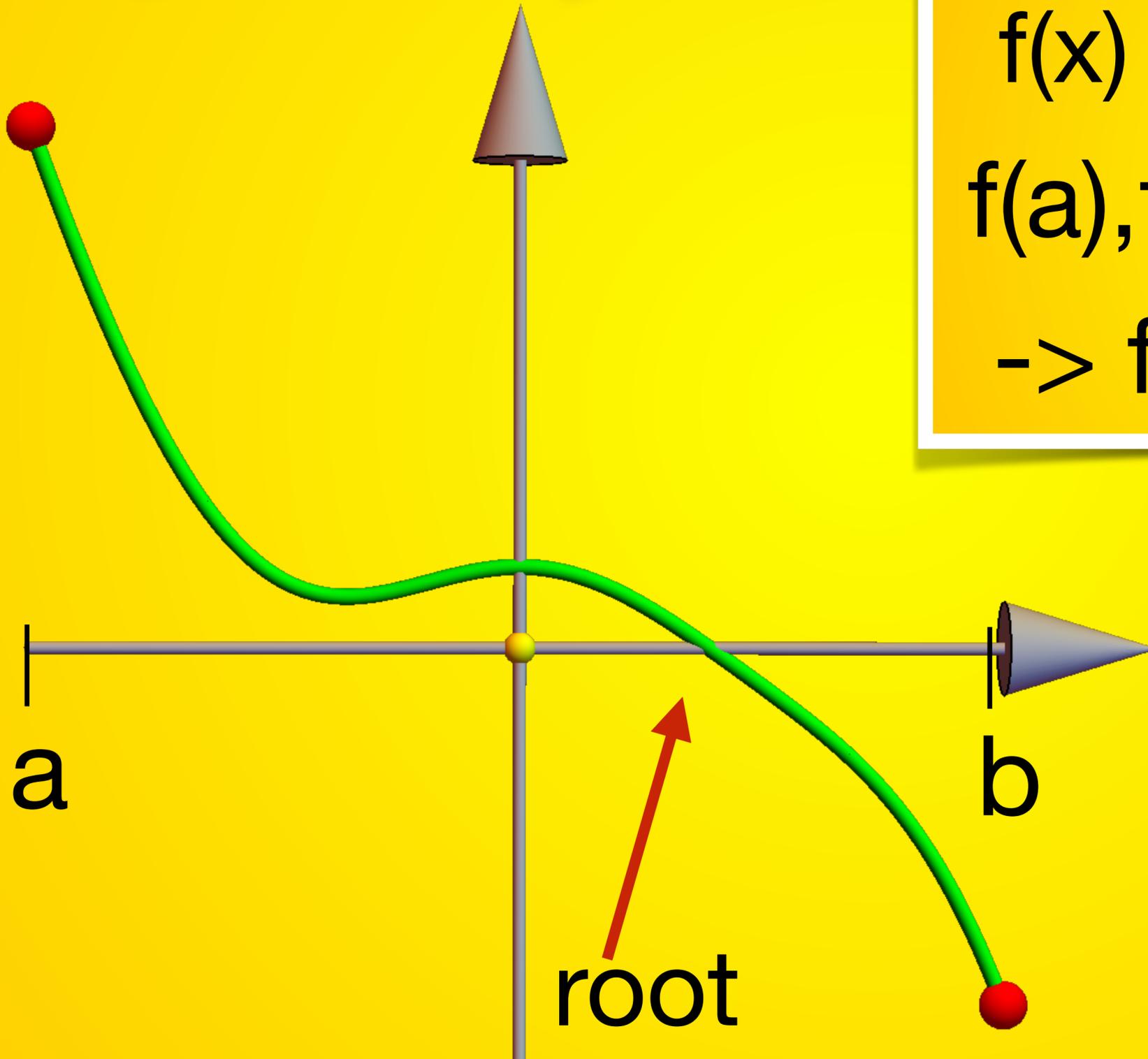
$$\exp(3x) - 1$$

$$\tan(x^2 - 1)$$

$$|x - |x - 2||$$

Theorem

$f(x)$ continuous on $[a,b]$
 $f(a), f(b)$ different signs
 $\rightarrow f$ has a root in $[a,b]$



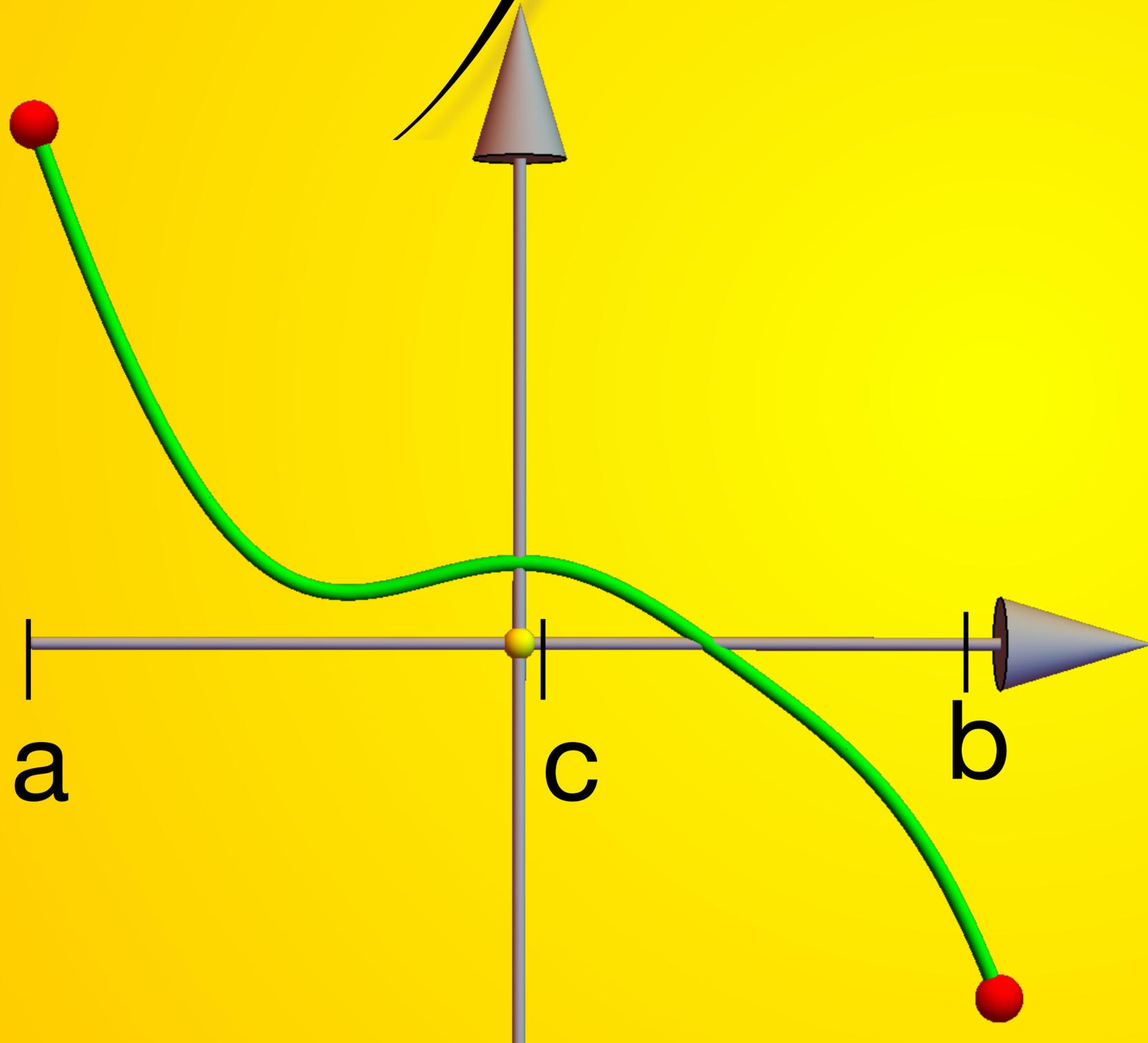
Why

Proof: to get to the other side, you have to cross the road!





Proof



Divide and conquer
Define $c = (a+b)/2$

If $f(c) f(b) < 0$, take
new interval $[c, b]$

Otherwise, take
interval $[a, c]$.

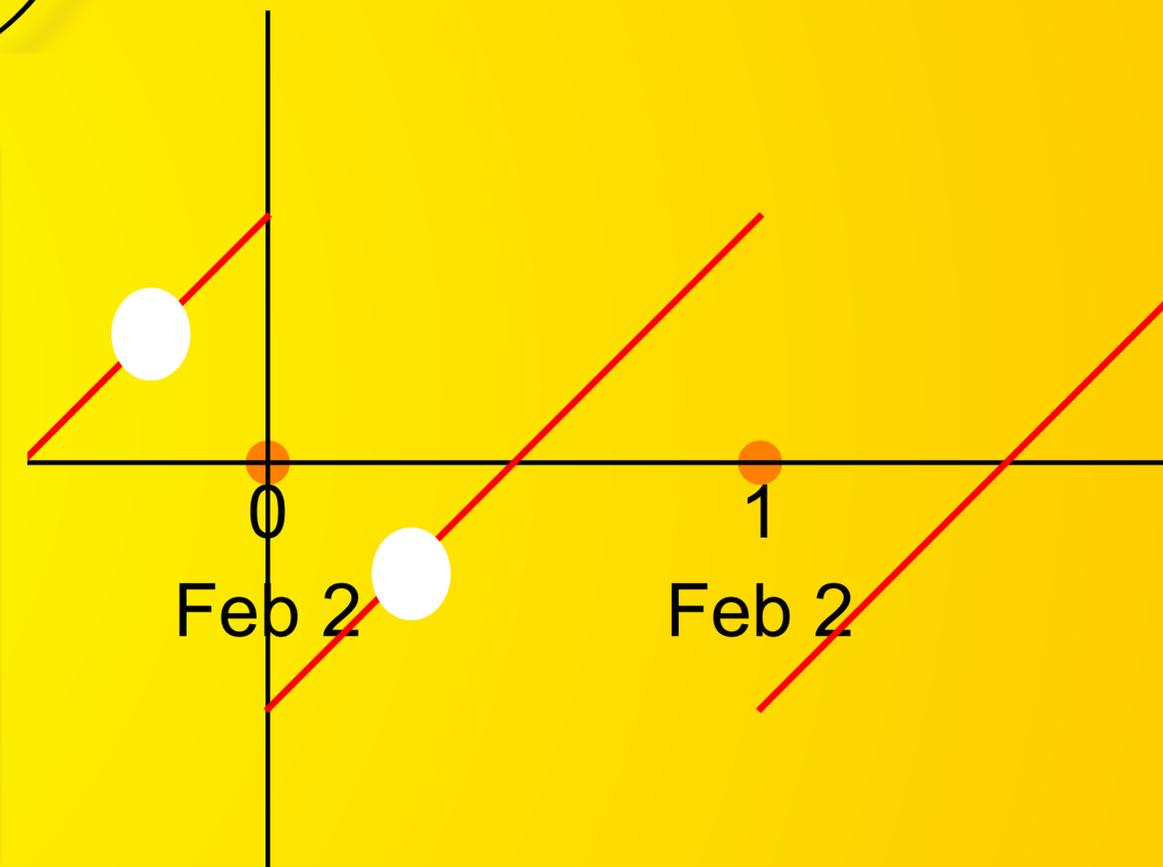
Continue to narrow
down.

Ground Hog

The function

$$f(x) = x - \text{floor}(x)$$

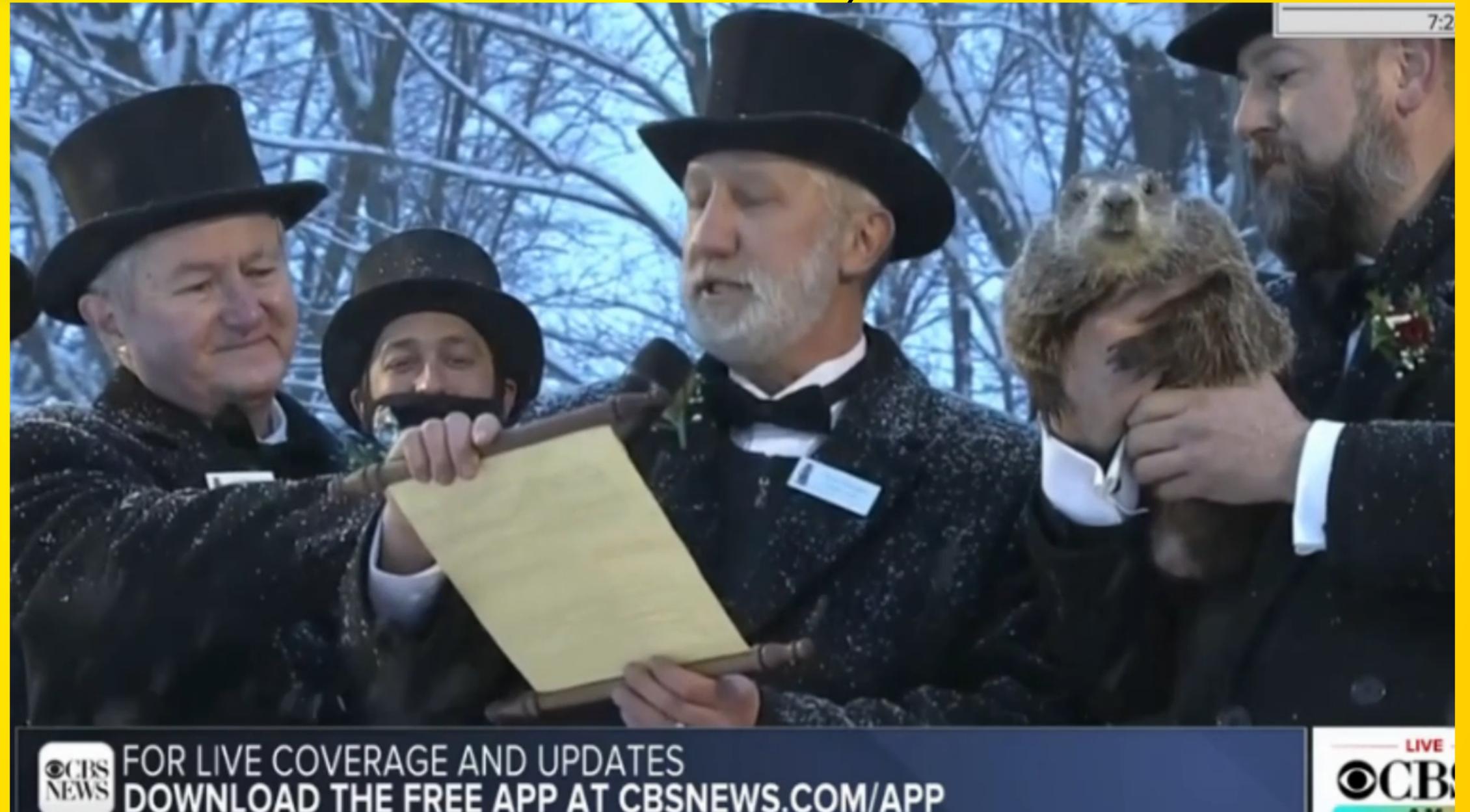
is the ground hog function.



$f(-0.5)$ and $f(0.5)$ have different signs. Why is there no root?

Ground Hog

yesterday



Solving equations

Prove that

$$f(x) = \sin(x) + x^3 - |x|$$

have a root.

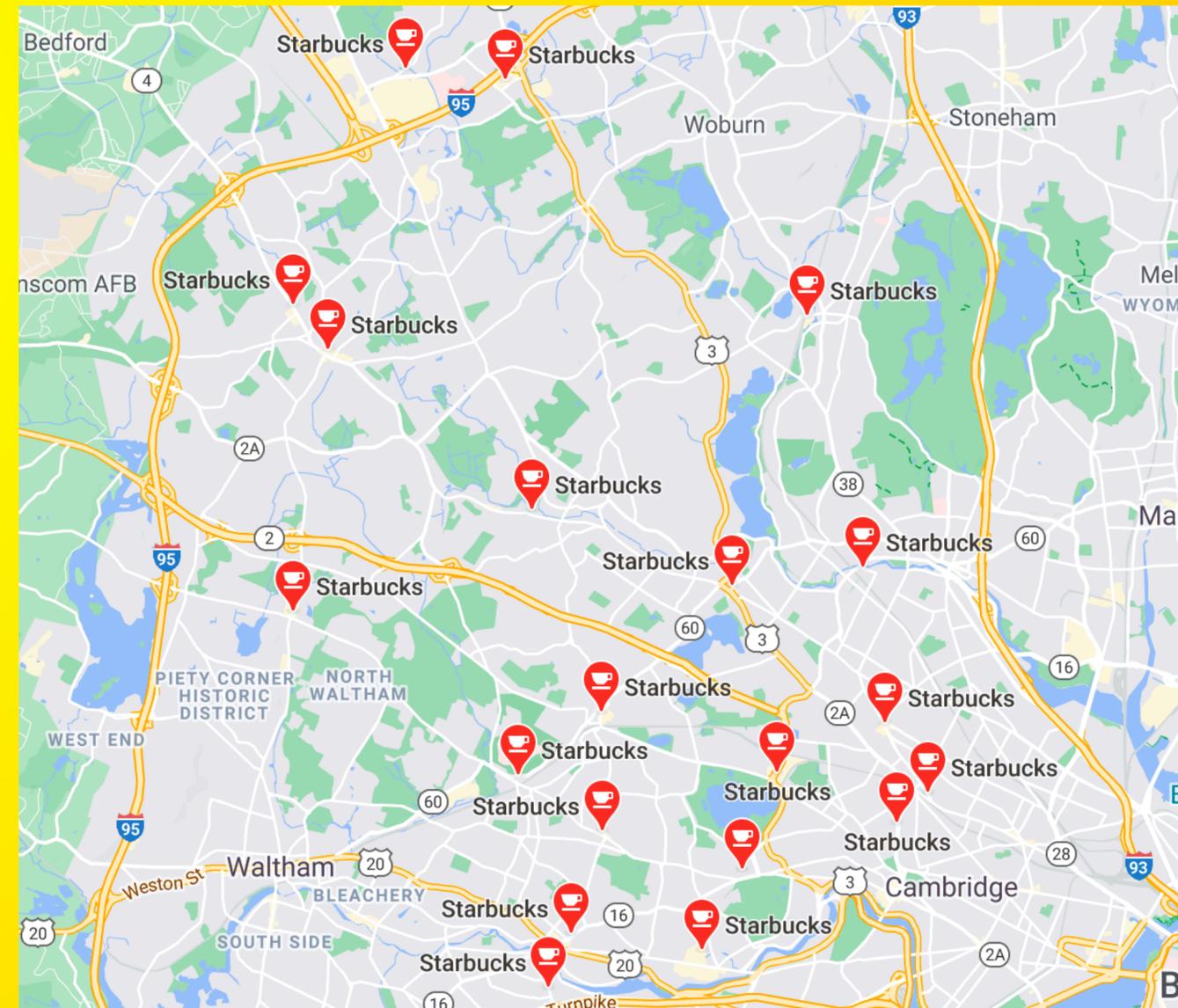
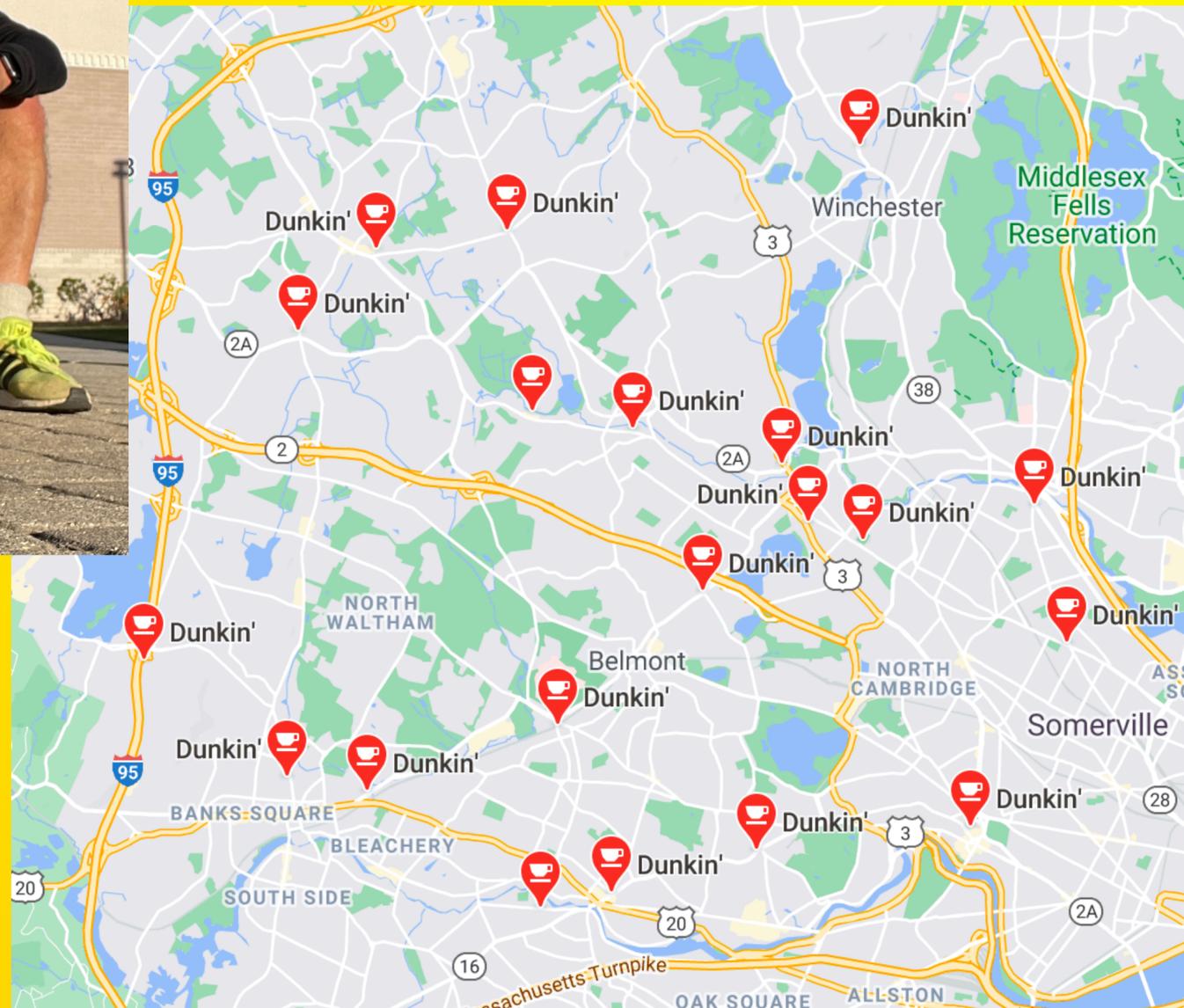
Archilles and Tortoise



2020
summer

These days...

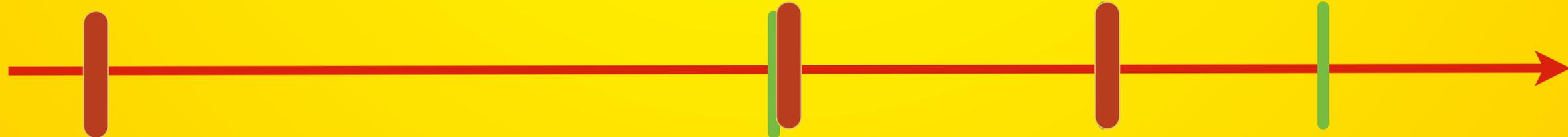
Running forth and back from
Dunkin' Doughnuts ← → to Starbucks



2021
winter

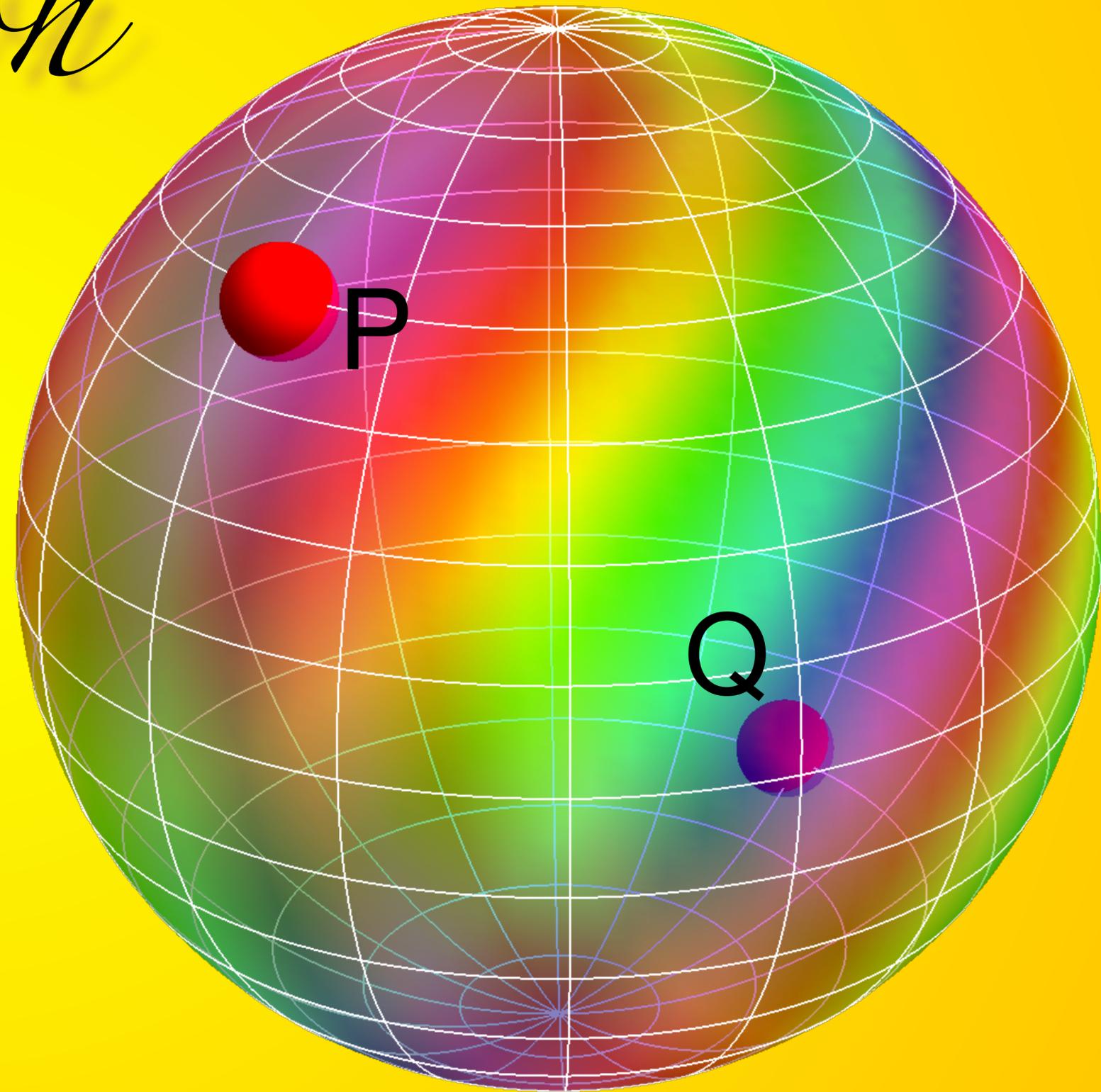


Archilles and Tortoise



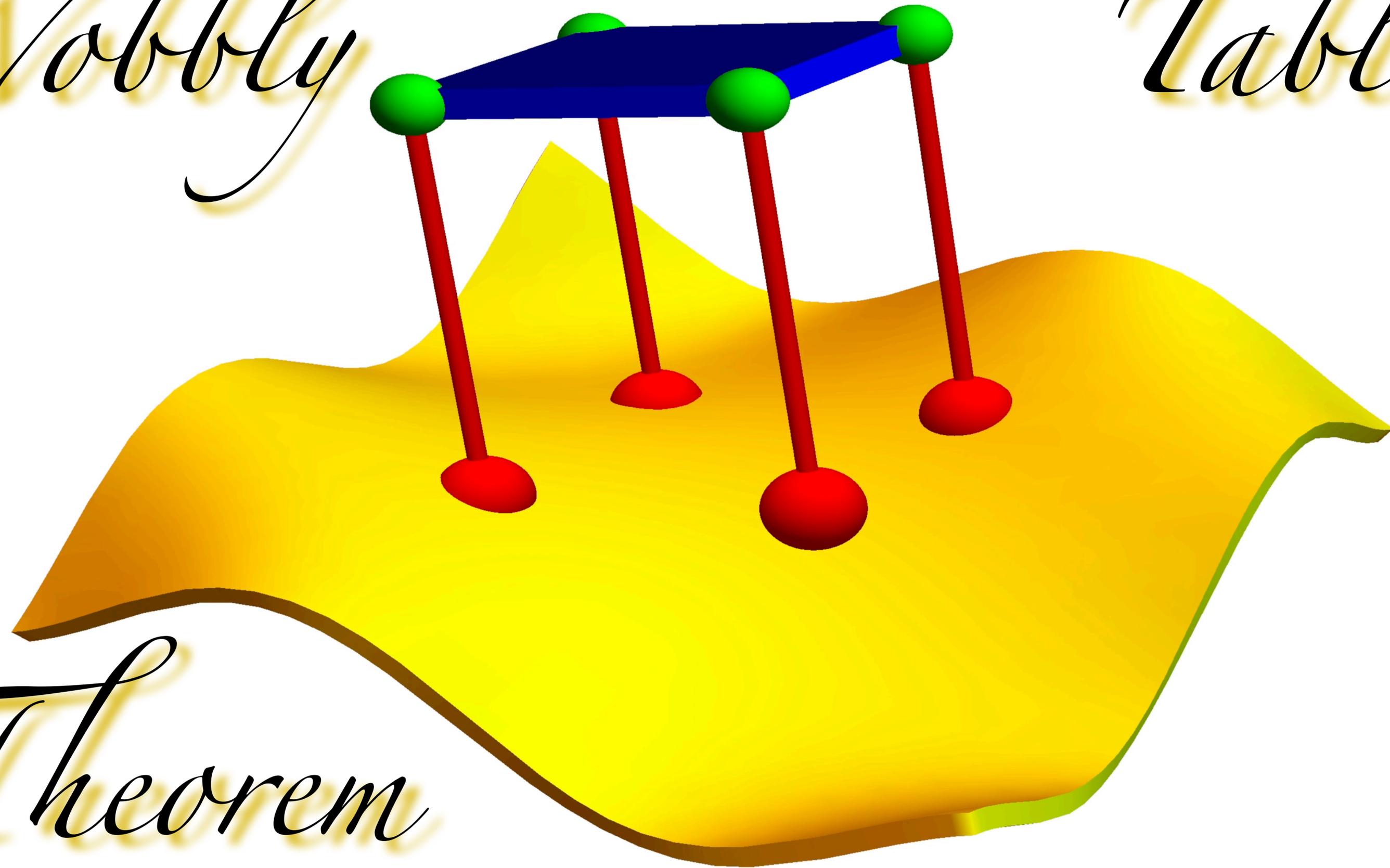
Application

There is a point on earth where the temperature and pressure are the same as on the antipode



Wobbly

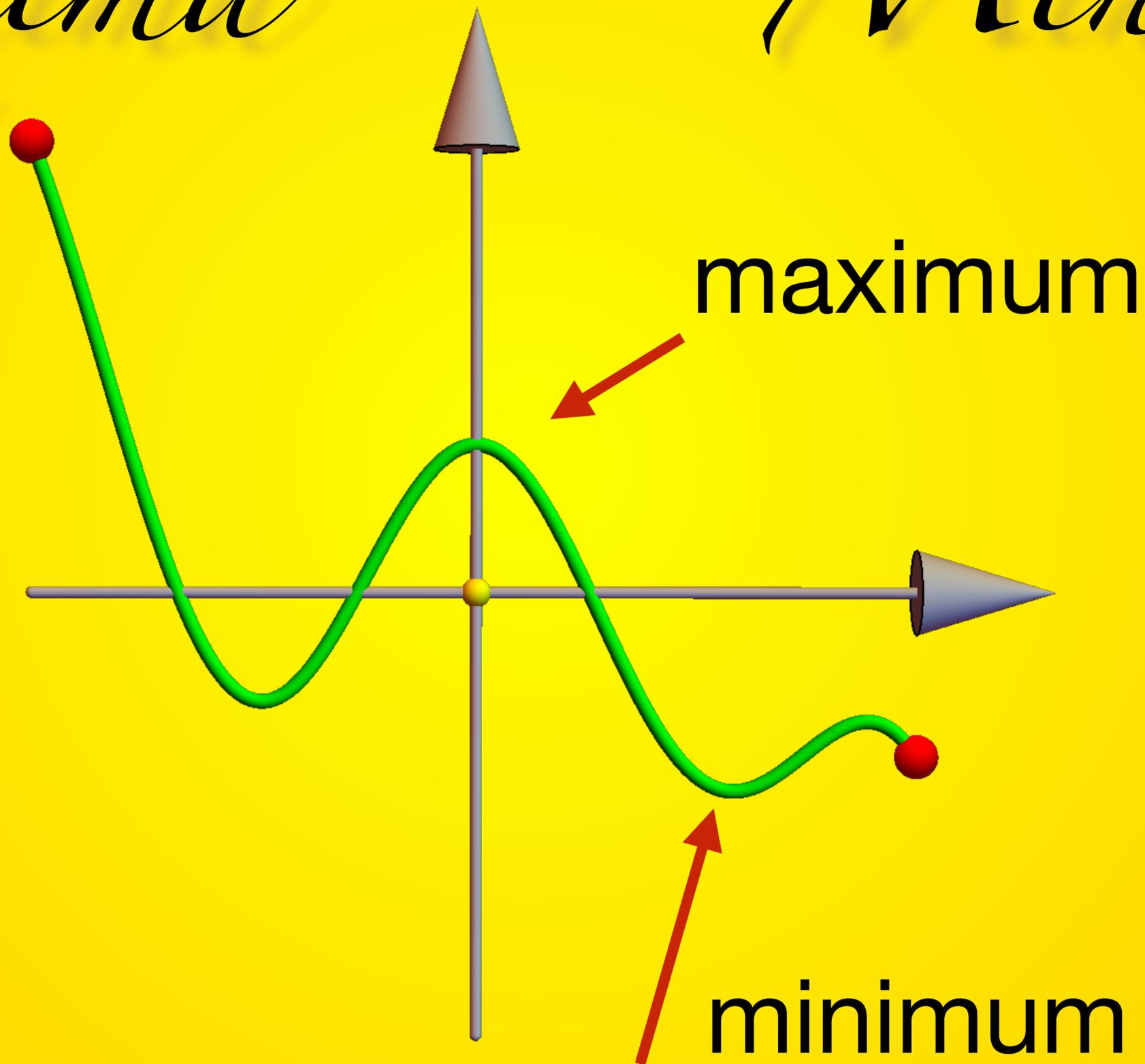
Table



Theorem

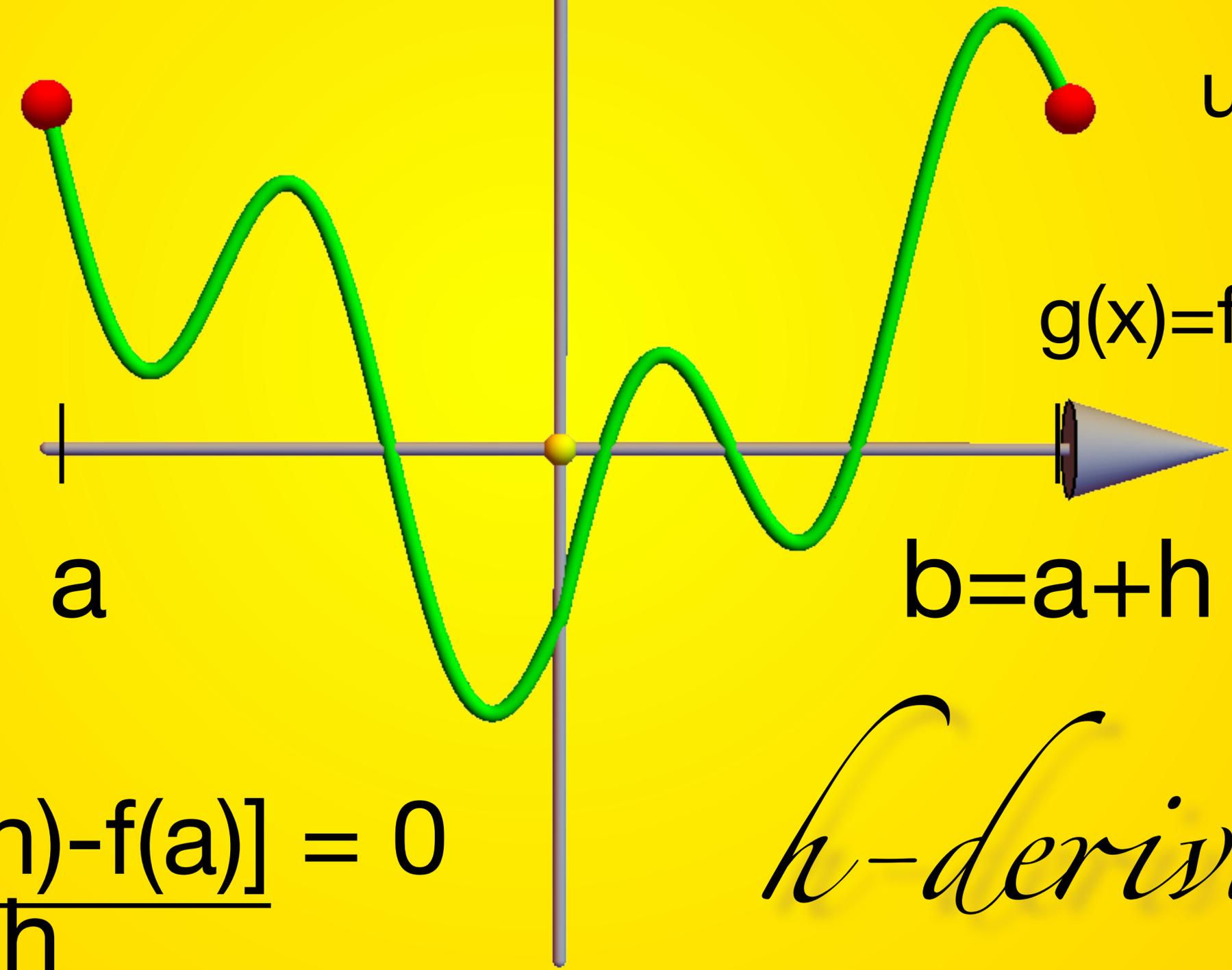
Maxima

Minima



Extremal Values

$Df(a)=0$
gives
max or min
in interior



use IVT
for
 $g(x)=f(x+h/2)-f(x)$

$$Df = \frac{f(a+h) - f(a)}{h} = 0$$

h-derivative

Growth



was there a time when they had the same height?

See Homework

NETFLIX



BETH & BENNY

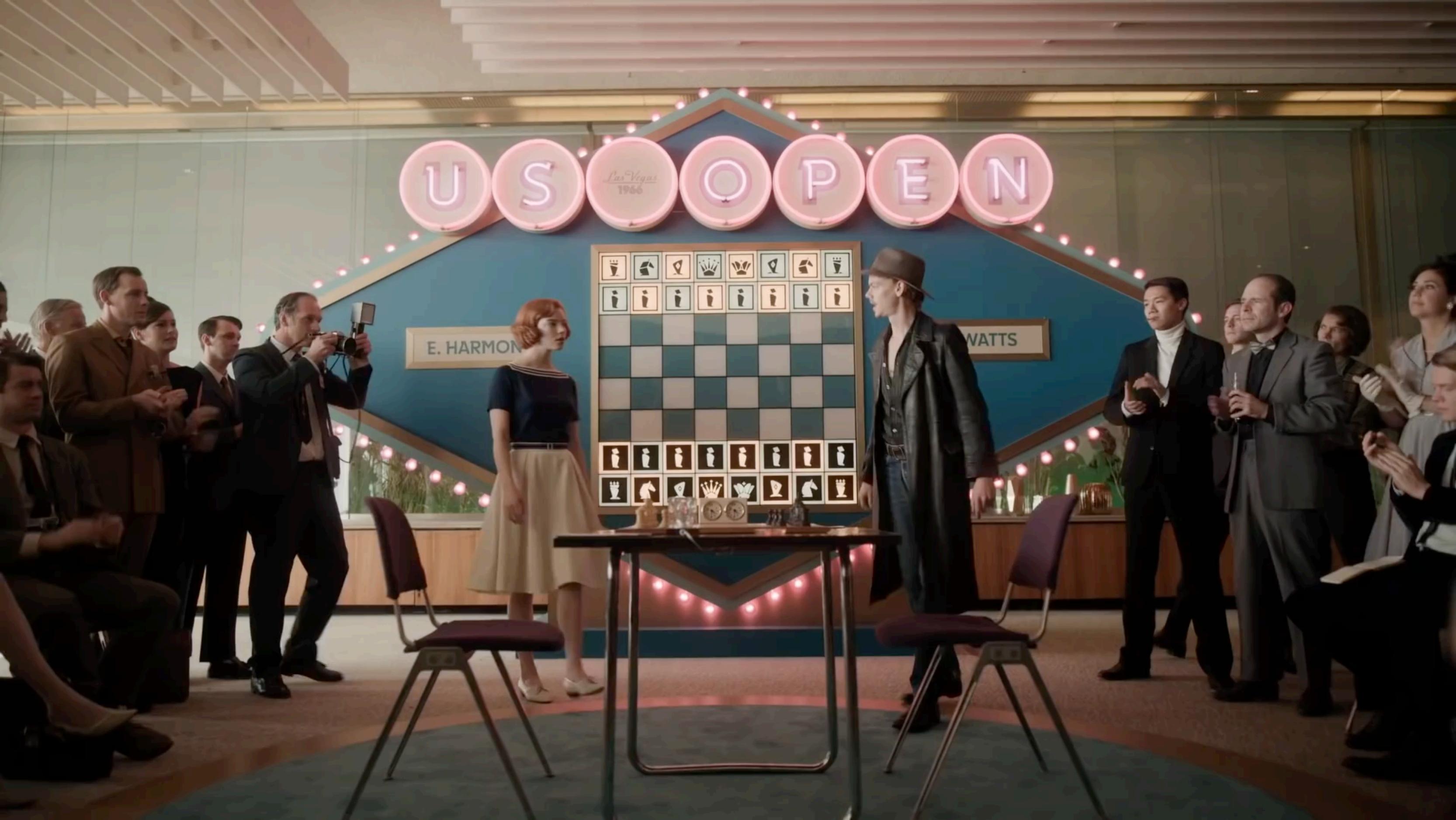
U S O P P E N

Las Vegas
1966

E. HARMON

WATTS

♠	♣	♠	♣	♠	♣	♠	♣	♠	♣
♟	♟	♟	♟	♟	♟	♟	♟	♟	♟
♟	♟	♟	♟	♟	♟	♟	♟	♟	♟
♠	♣	♠	♣	♠	♣	♠	♣	♠	♣



JAM

Does the following equation have a solution?

$$x^{11} - \sin(x) = x^5 + |x| + \cos(x)$$

Argue why there was a time in your life when your height was 1000 times the length of your teeth.

The End