Find roots:

a) $\log|x|$ — 1, −1

b) x^2-25 — 5, −5

c) $\log \sin^2 x$ — $\frac{\pi}{2}$

d) $e^{3x}-1$ — 0

e) $\tan(x^2-1)$ — $x=1$ ✗

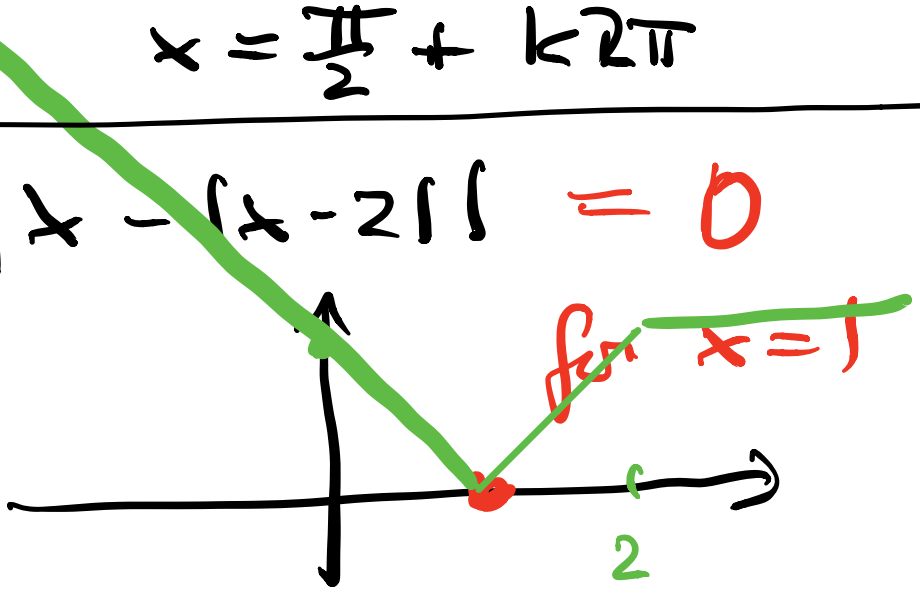
f) $|x-|x-2||$ — 1

c) $\log(\sin^2(x)) = 0$ means

$\sin^2(x) = 1$, which means

$x = \frac{\pi}{2} + k2\pi$

f) $|x-|x-2|| = 0$



continuous.

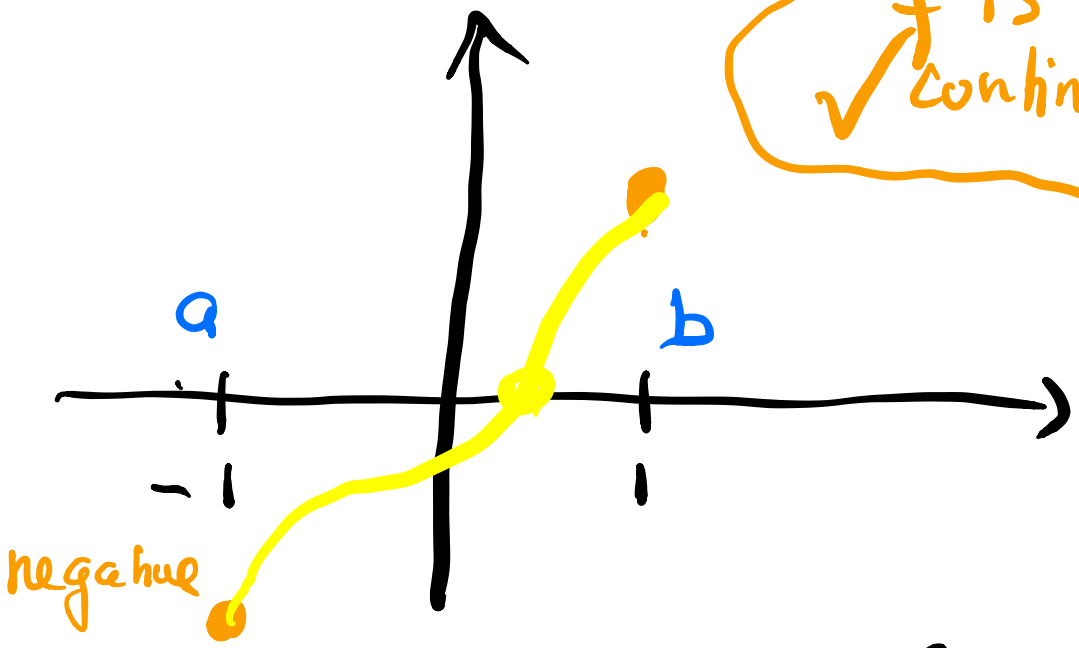
(piecewise linear function).

Example

verify that f has a root:

$f(x) = \sin x + x^3 - |x|$

✓ f is continuous

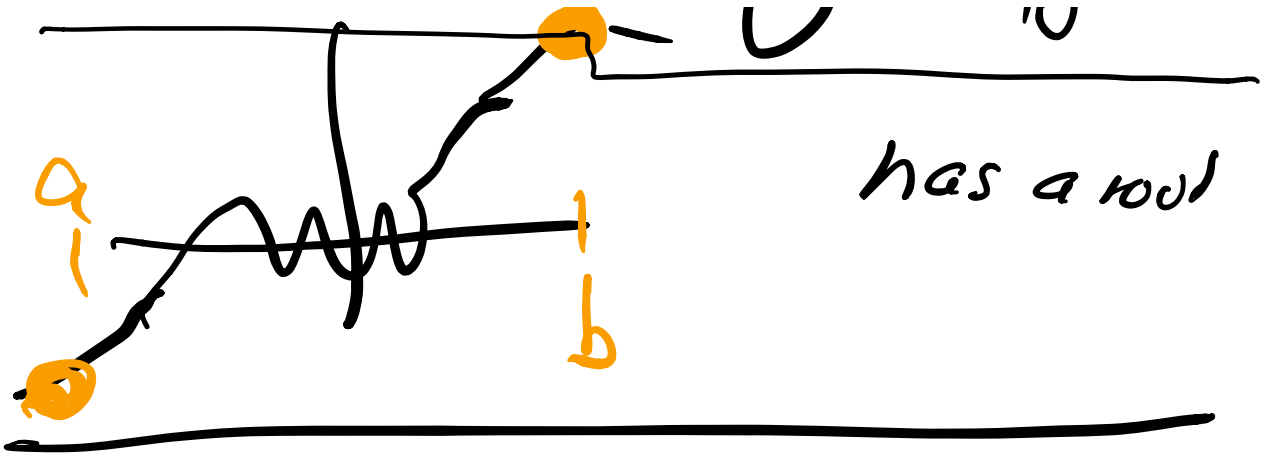


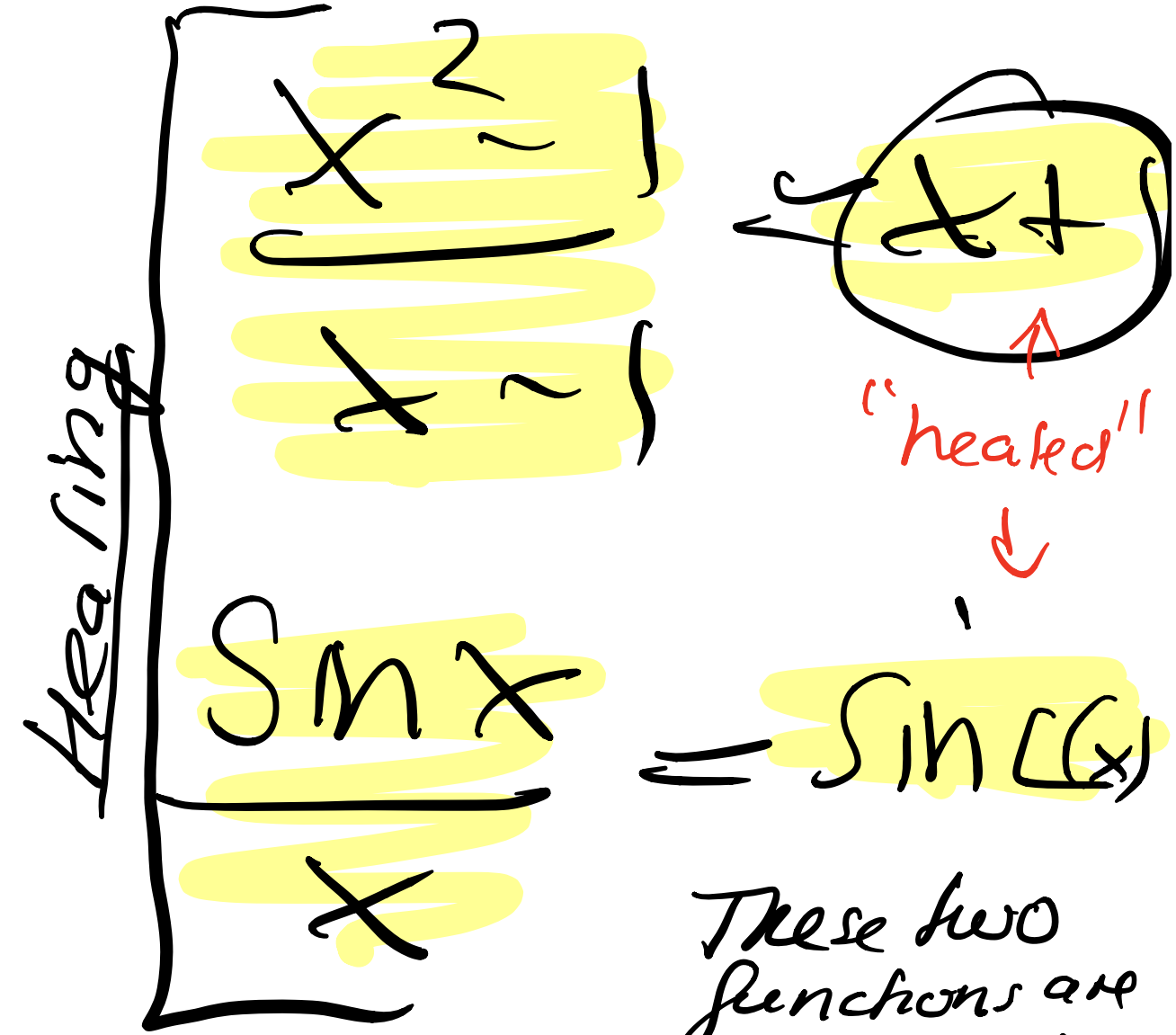
$a = -1000000\pi$ $\quad \sin a = 0$

$b = 1000000\pi$ $\quad \sin b = 0$

Ex:

$$x^{77} - x^5 + 1000$$


has a root
a
b


$x^2 - 1$
$x + 1$
"healed"
$\frac{\sin x}{x}$
$\sin c(x)$
These two functions are considered continuous everywhere!

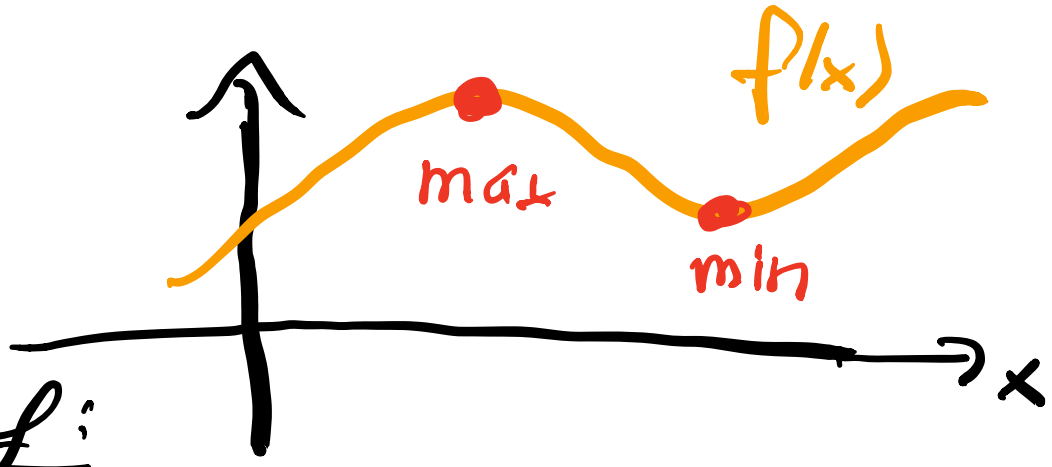
$$\frac{x^{77}-1}{x-1}$$

$$x^{76}+x^{75}+x^{74}$$

$$\vdots$$

Healed.

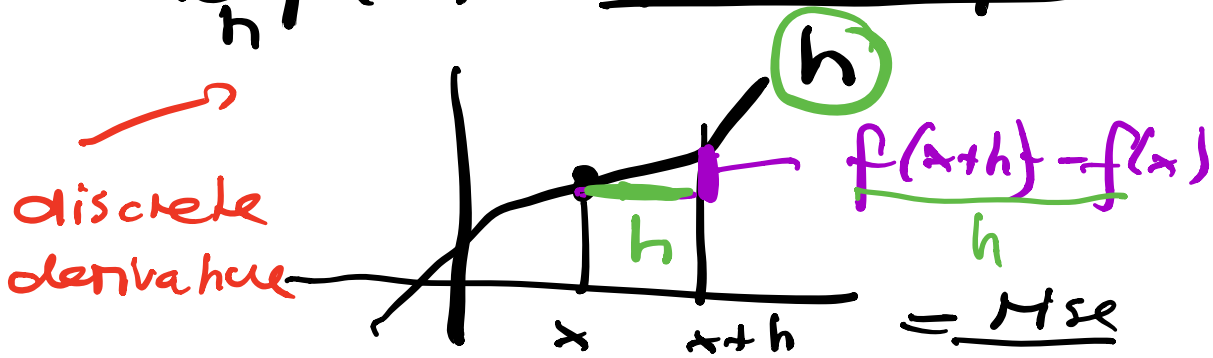
Finding maxima and minima using intermediate value thm:



Def:

$$D_h f(x) = \frac{f(x+h) - f(x)}{h}$$

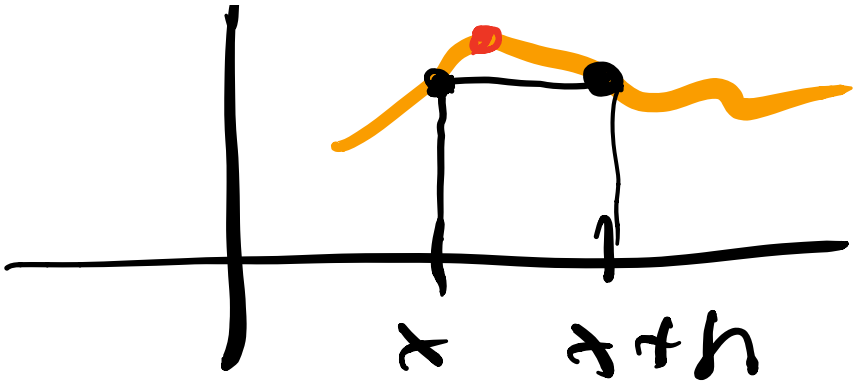
discrete derivative



$= \frac{\text{rise}}{\text{run}}$

Def | h - critical point is a point x, where $D_h f(x) = 0$

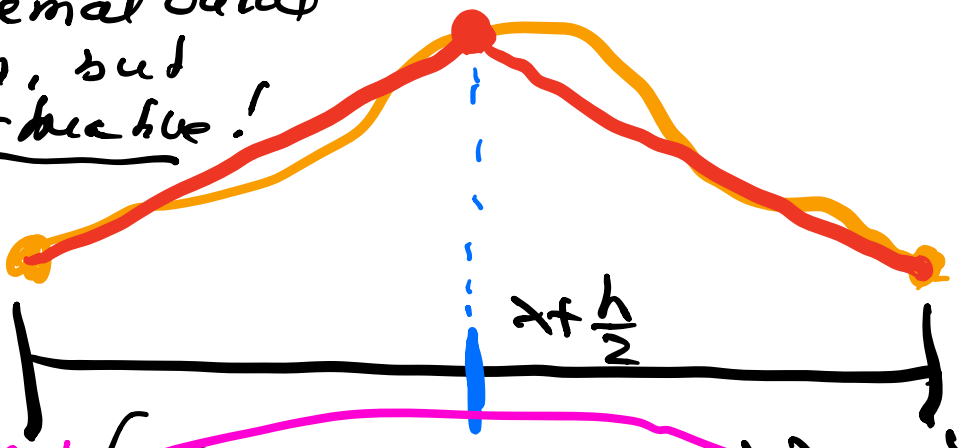
$\uparrow \quad f(x+h) = f(x)$



Theorem: (Bernard Bolzano)

If a is a h-critical point, then there is a max or min on $[a, a+h]$

Related to extremal value thm, and constructive!

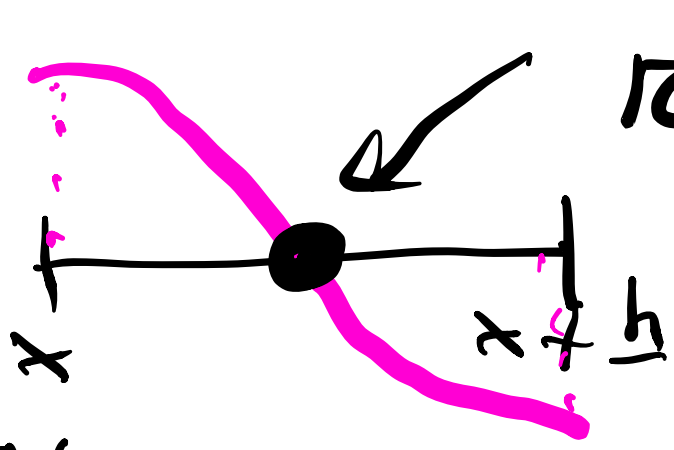


$$g(x)\left(f\left(x+\frac{h}{2}\right)-f(x)\right)$$

$$= -\left(f(x+h)-f\left(x+\frac{h}{2}\right)\right)$$

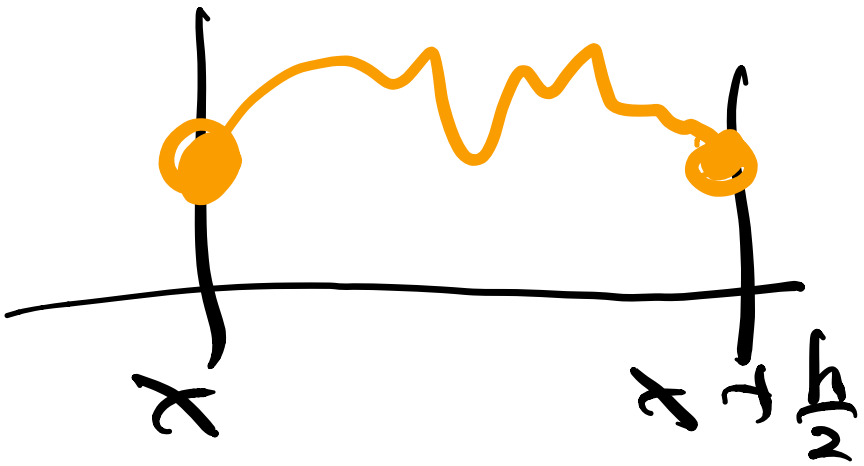
$g(x)$ $g(x+\frac{h}{2})$

$= -g(x+\frac{h}{2})$



Now!

$g(x) \approx g(x+h/2)$



Same situation, but

half of the interval!

Now repeat!

constructive!!!

amazing!

In step n, we are

$$\frac{|b-a|}{2^n}$$

close to a

maximum or minimum.

→ Machine learning in nonsmooth situations