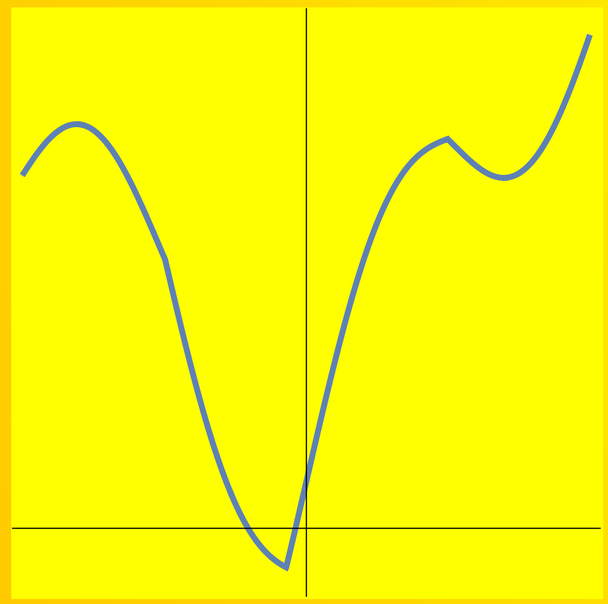
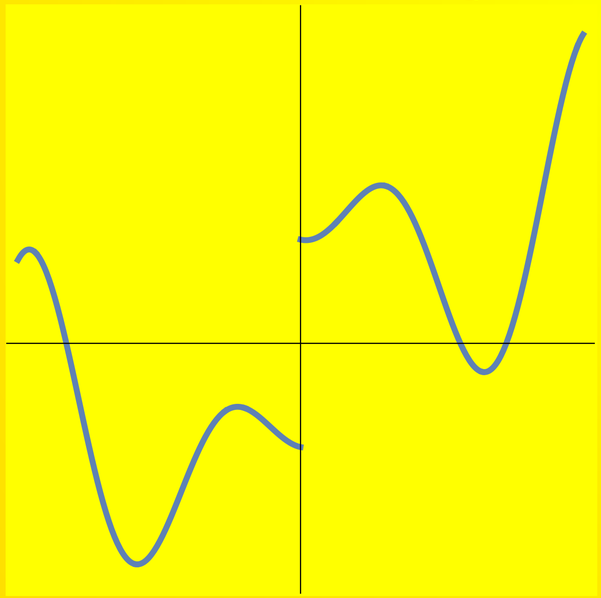
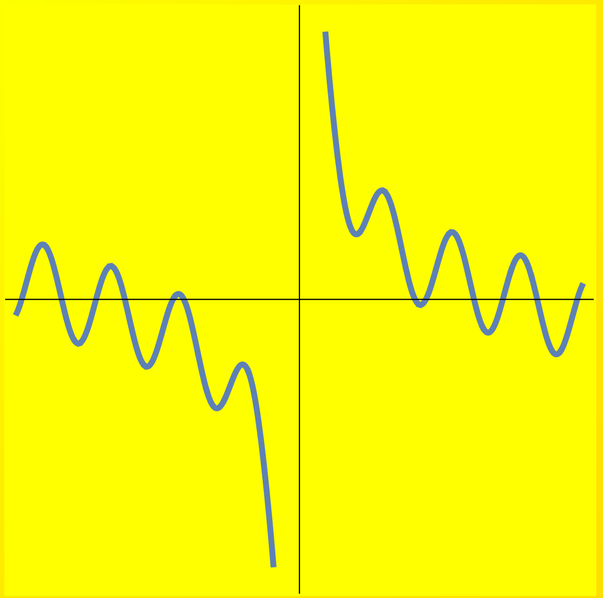
4 Continuity

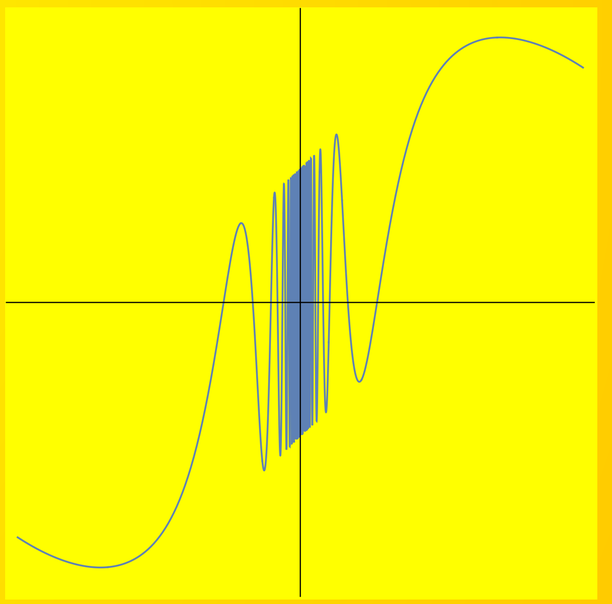
Continuous

Not Continuous









continuous

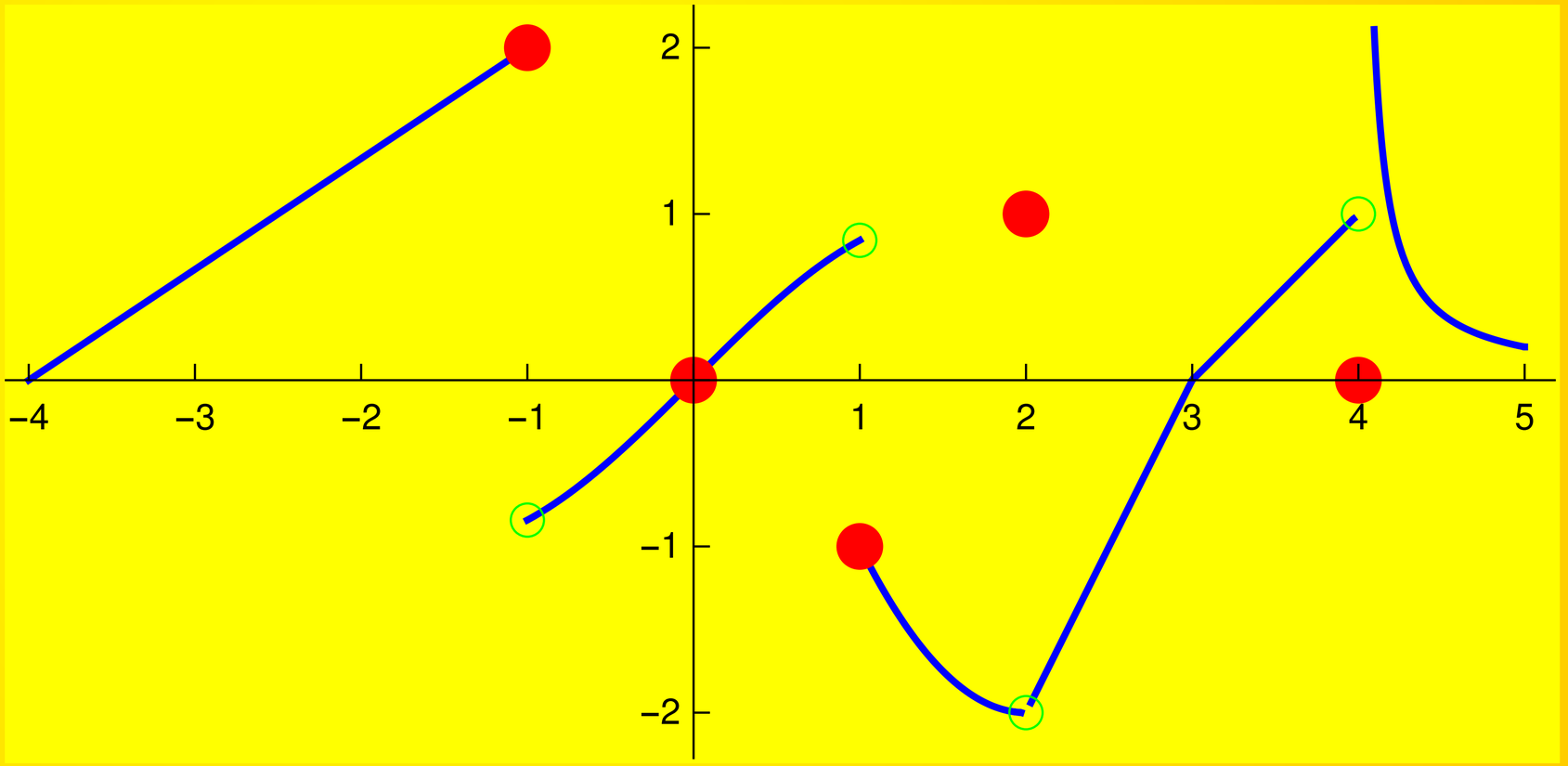
jump

pole

oscillation

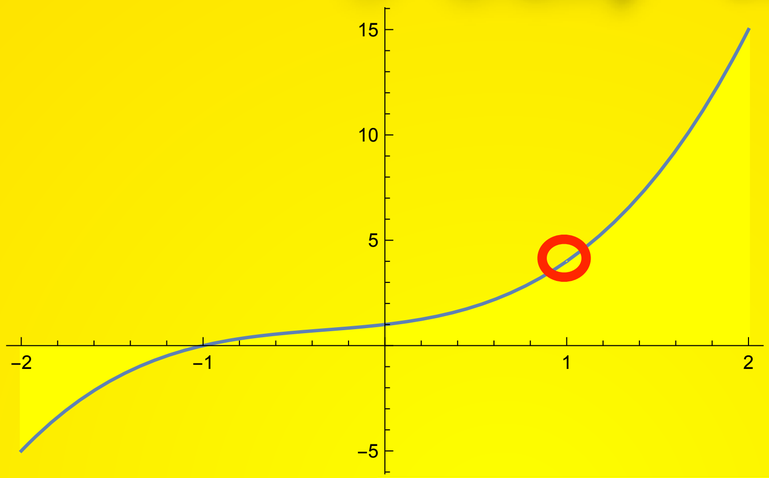
REVIEW

Poll



where continuous
from the right
but not continuous
from the left

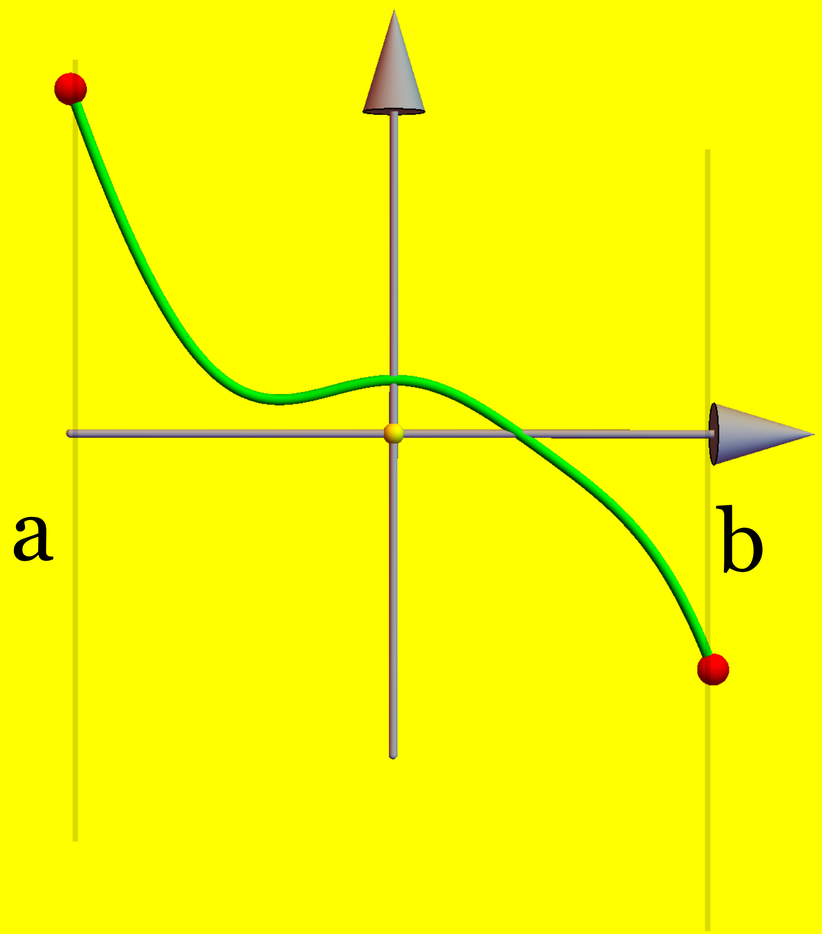
HEALING



$$\lim_{x \to 1} \frac{x^4 - 1}{x - 1}$$

This is a situation where we divide 0/0. The function is broken at x=1.

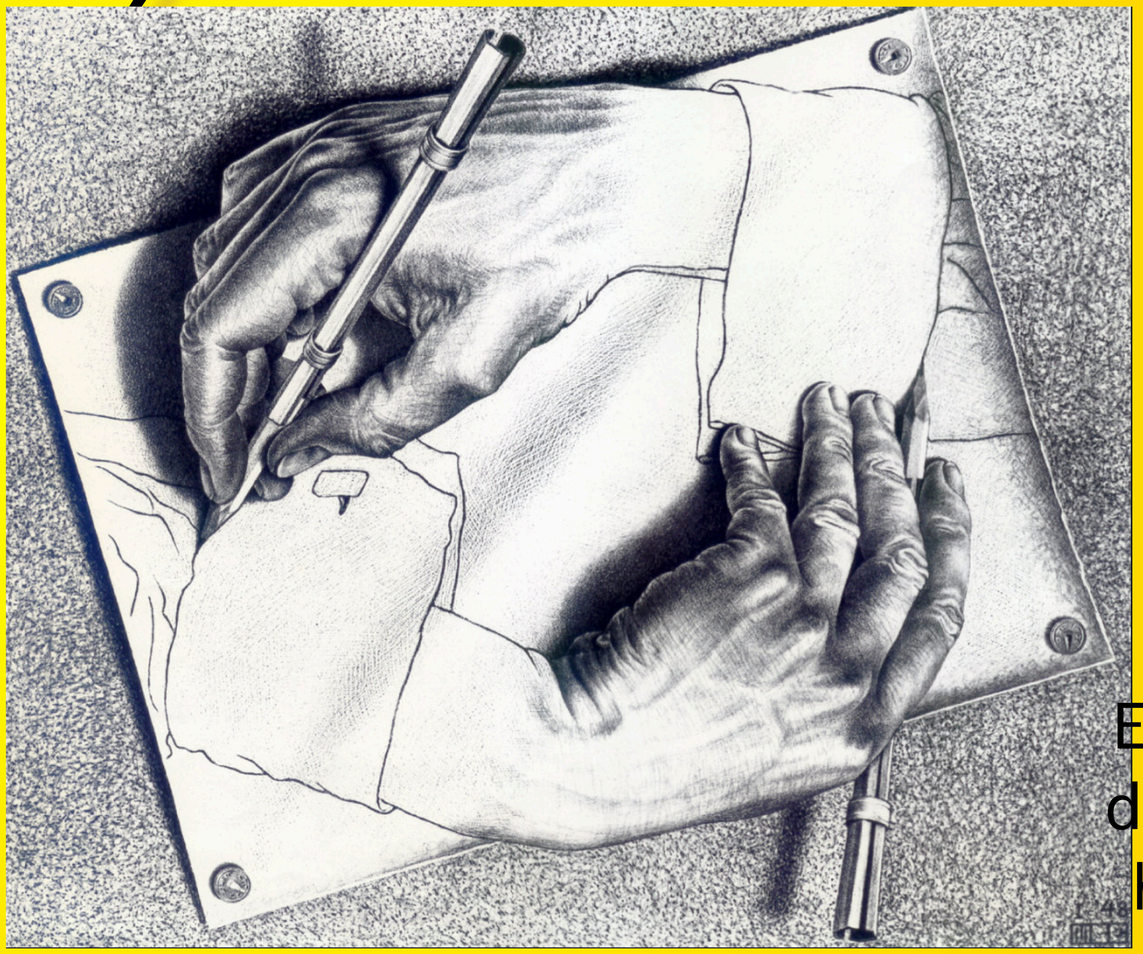
Continuity



A function f is continuous at x=x_0, if there exists a unique value b, such that f(x_0) →b if x → x_0.

It is continuous in [a,b] if it is continuous at every point in [a,b].

Informal



Escher, drawing hands

One can draw the entire graph of f without lifting the pen

GOOD NEWS

Polynomials, trig polynomials, exponentials are continuous.

Sums, products and compositions of continuous functions are continuous.

Example: The following function is continuous eveywhere

$$f(x) = 5\sin(3x) + x^5 + 17x^3 + 7e^x$$

BAD NEWS

Logarithms, tangent functions are not continuous everywhere.

Division by a continuous function is not necessarily continuous.

Example: Where is the following function not continuous?

$$f(x) = \log|x| + 1/(3-x) + \tan(x)$$

Which functions are continuous?

EXAMPLES

$$\frac{1}{4-x^2}$$

$$\frac{1}{e^x+\sin(x)+2}$$

$$\frac{1}{\log(x)}$$

$$\sqrt{|x|}$$

$$\log|e^x+1+\sin(x)|$$

$$1/\log(|x|+2)$$

PREDICTION

Continuity can mean also to be able to
predict things.

Examples:

you grow continuously → There was a time in your life when you were exactly 1 meter high!

you run continuously → If you make a run and cross the Charles river once, there has to be a time, when you cross it again.

APROPOS CHARLES

Oliver crossing the Charles in 2002

IMPORTANCE

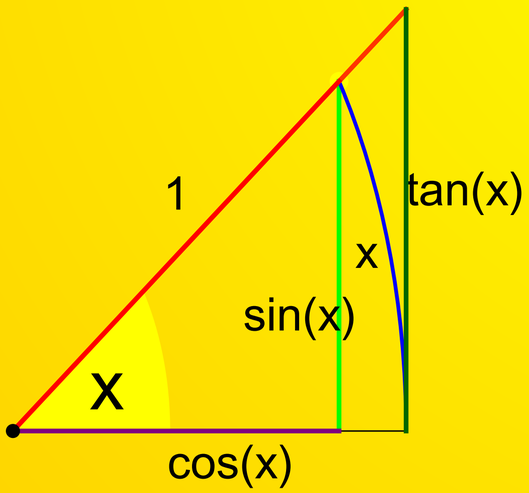
Continuity means **stability**. A small change in x produces a small change in f(x).

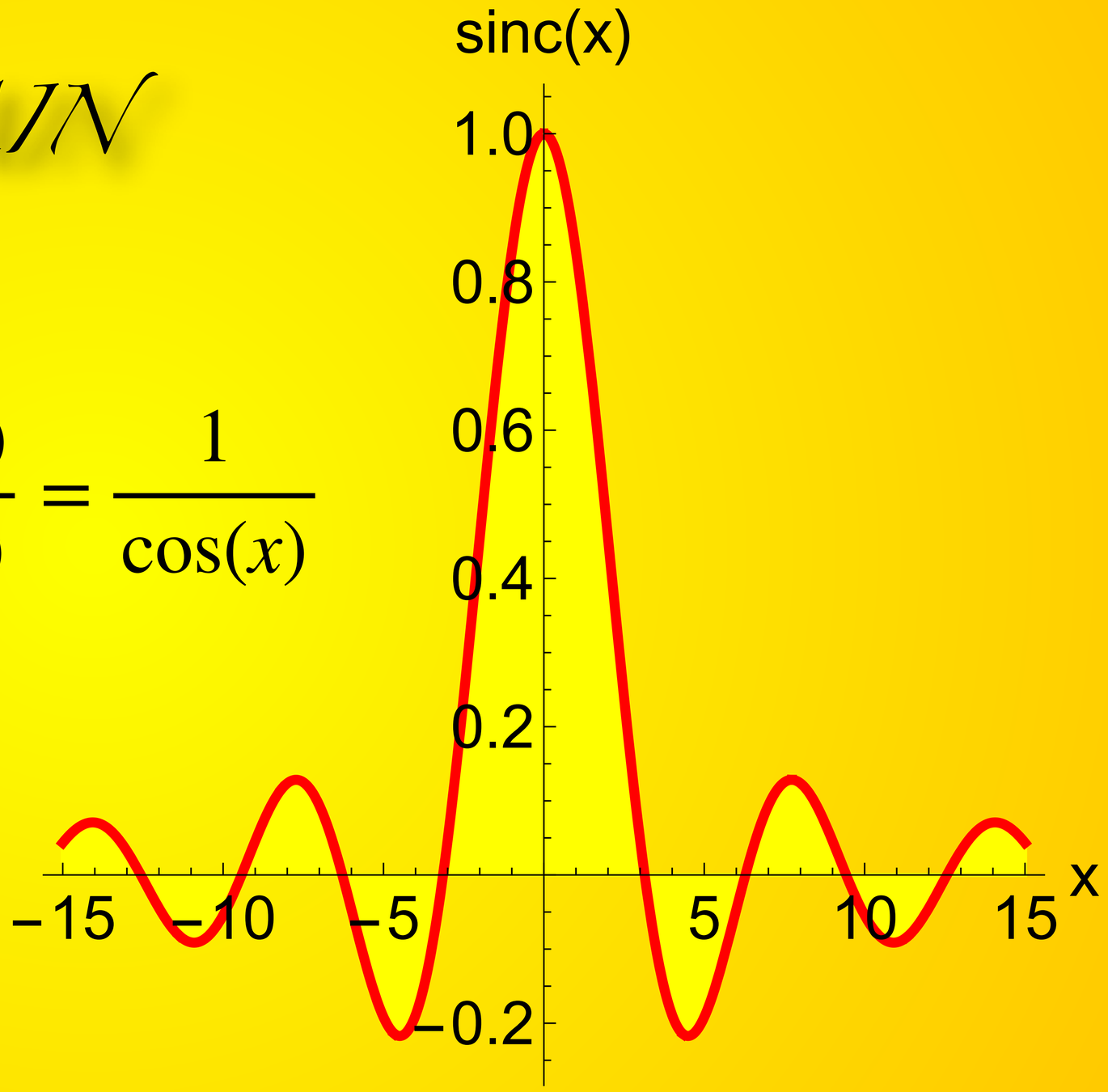
Discontinuity can mean **unpredictability**. It can mean catastophes, fast changes.

With continuity of the derivatives one can even do more and **control** things

SIN(X)/X AGAIN

$$1 = \frac{\sin x}{\sin x} \leq \frac{x}{\sin x} \leq \frac{\tan(x)}{\sin(x)} = \frac{1}{\cos(x)}$$



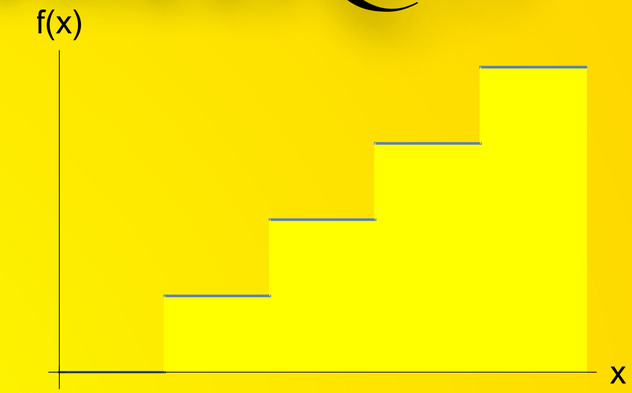


CONTINUITY FAILURES

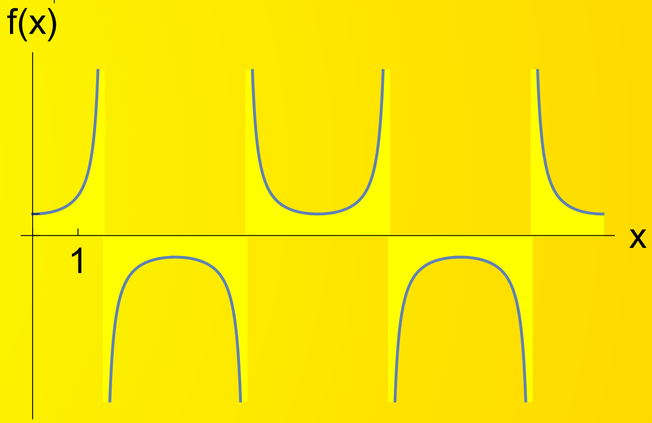
Jumps

Infinity

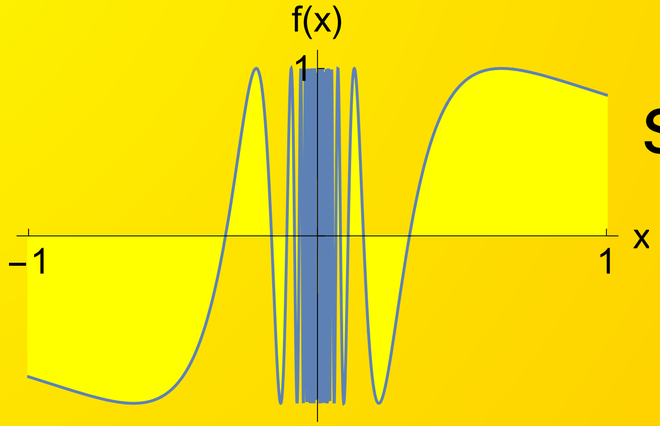
Oscillation



floor(x)



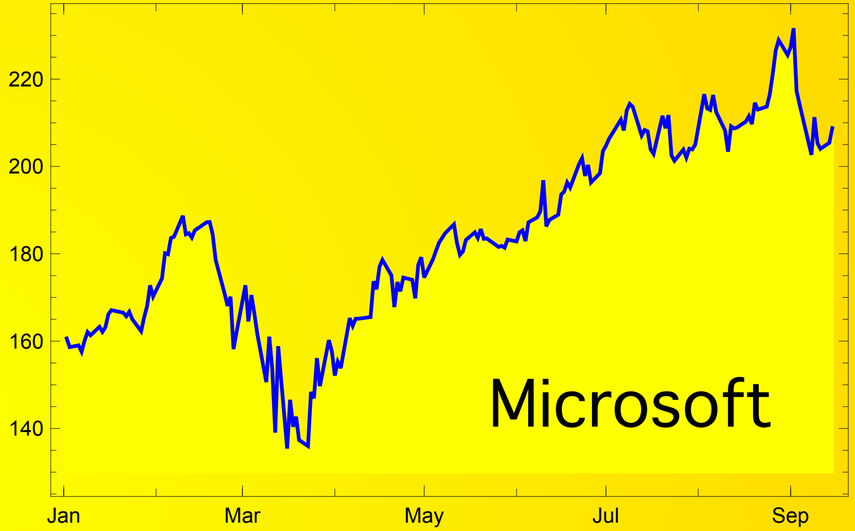
sec(x)



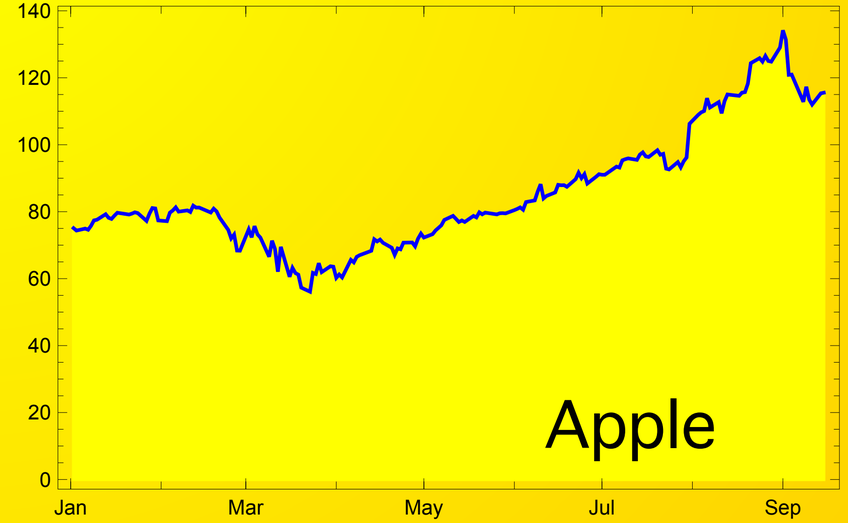
sin(1/x)

FINANCE


3500
3000
2500
2000
1500
1000
500
0
Jan
Mar
May
Jul
Sep
Amazon


220
200
180
160
140
Jan
Mar
May
Jul
Sep
Microsoft


1700
1600
1500
1400
1300
1200
1100
1000
Jan
Mar
May
Jul
Sep
Google


140
120
100
80
60
40
20
0
Jan
Mar
May
Jul
Sep
Apple

Market Summary > GameStop Corp.

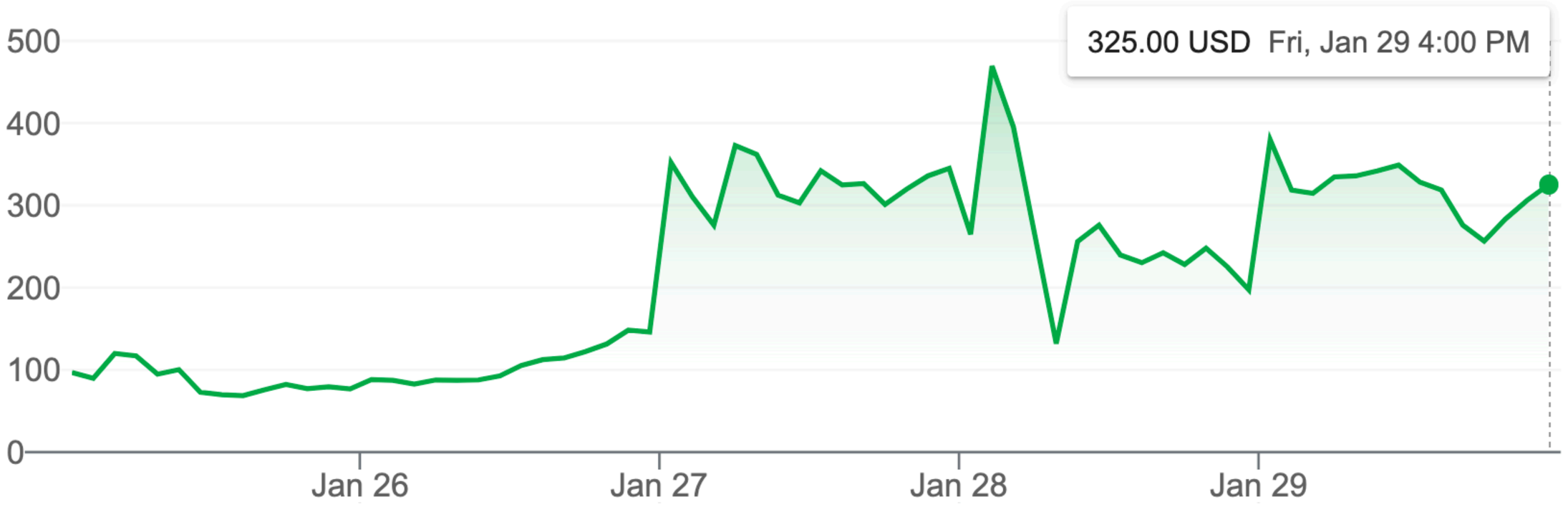
NYSE: GME

325.00 USD +131.40 (67.87%) ↑

Closed: Jan 29, 7:59 PM EST · Disclaimer

After hours 312.01 −12.99 (4.00%)

1 day | 5 days | 1 month | 6 months | YTD | 1 year | 5 years | Max



Which functions are continuous?

CHALLENGE

$\sin(\arcsin(x))$

$\arcsin(\sin(x))$

$\arctan(\tan(x))$

$\tan(\arctan(x))$

$\log|\exp(x)|$

$\exp(\log|x|)$

Which functions are continuous?

JAM

$1/\sin(x)$

$$\frac{1}{5+\sin(x)}$$

$$\frac{1}{x^2-5x+6}$$

$$\frac{1}{x^2+100+x}$$

$\log|x+1|$

$\log(|x|+1)$

End