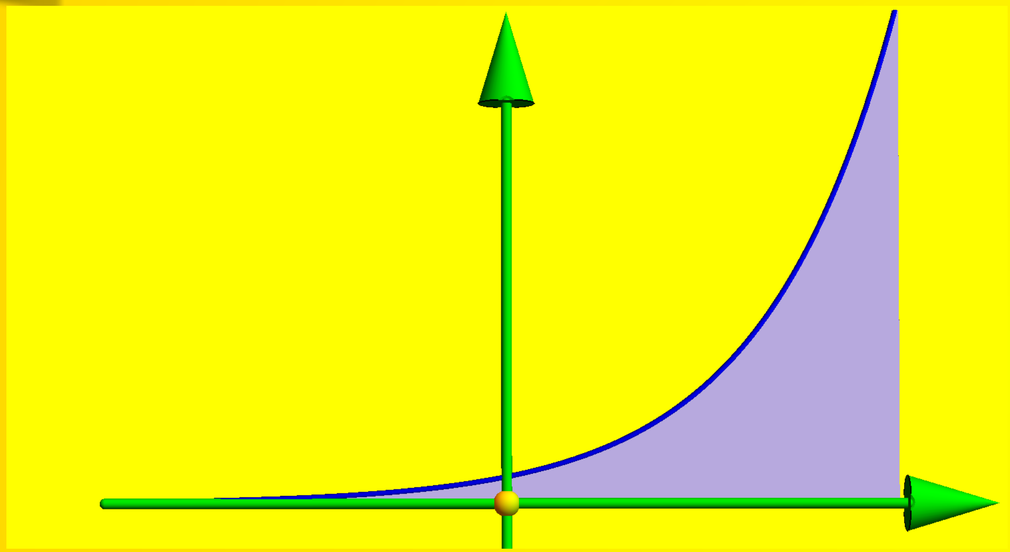
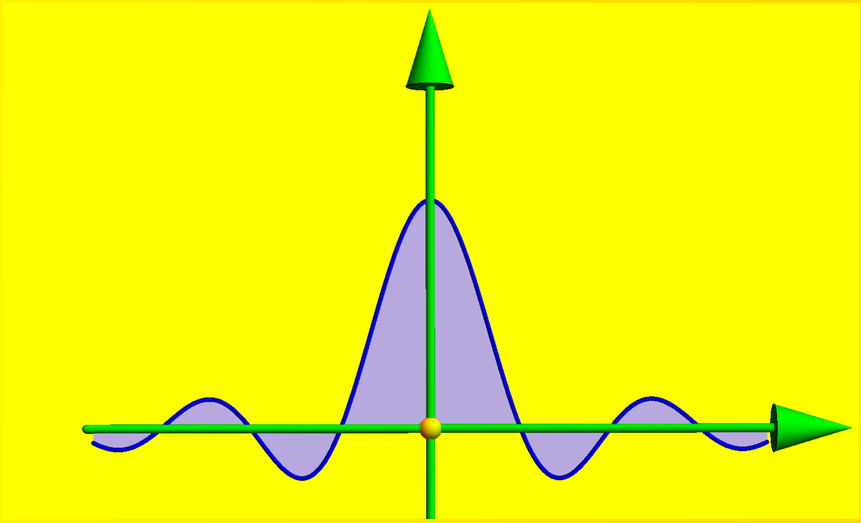
3

Limits





$$\exp(x) = \lim_{h \to 0} (1 + h)^{x/h}$$

$$\operatorname{sinc}(x) = \frac{\sin(x)}{x}$$

PLAN

1. Limits

2. Healing

3. Sin(x)/x

4. Examples

5. Some history

6. Calling on groups

LIMITS

A function has a **limit** at x=a if there exists a value b such that f(x) converges to b when x approaches a.

$$\lim_{x \to a} f(x) = b$$

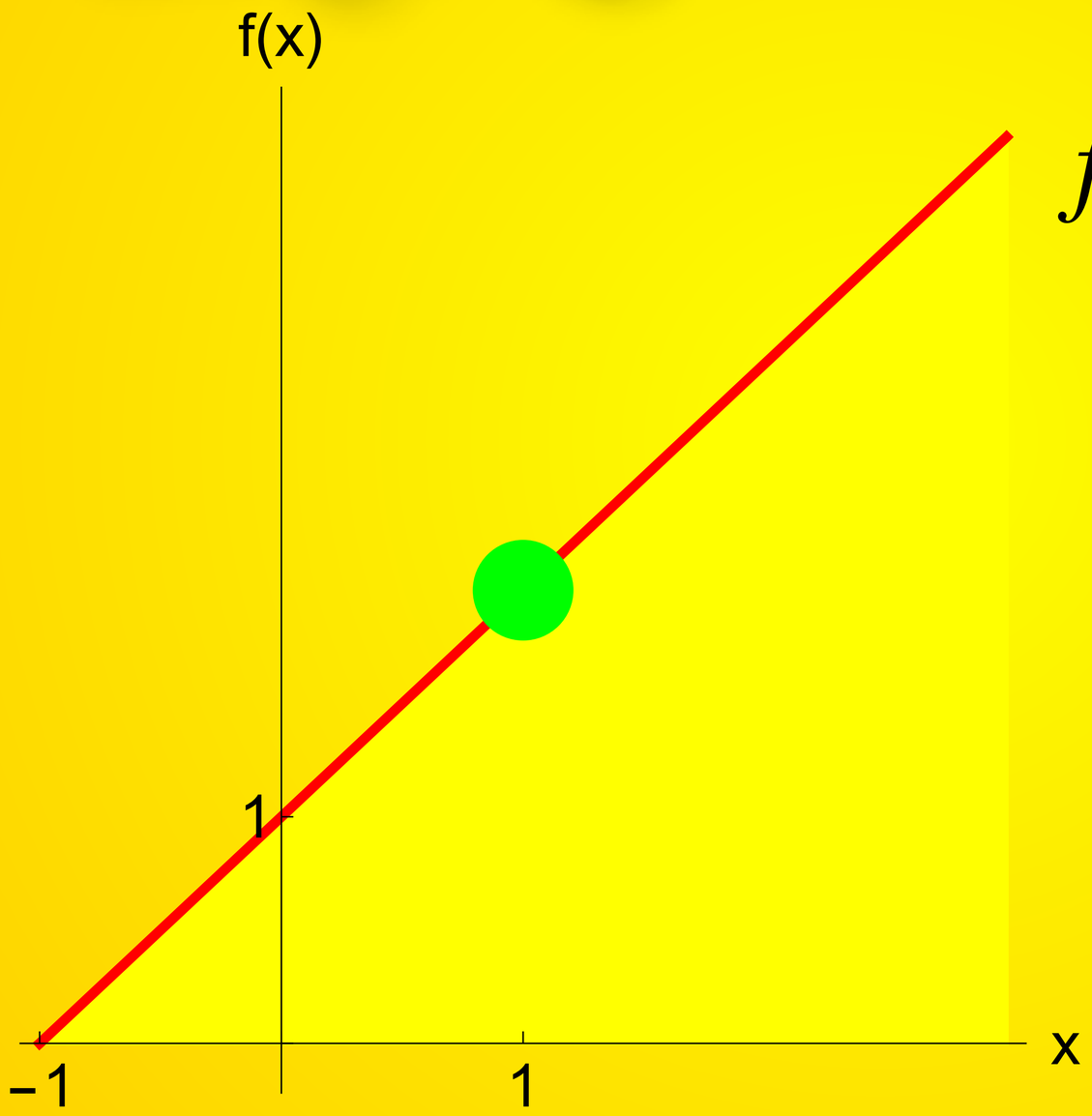
The limit needs to be **finite**, not like with f(x)=1/x at a=0

$$\lim_{x \to a^+} f(x) = b \qquad \lim_{x \to a^-} f(x) = b$$

The limit has to be the same from **both sides**, not like f(x)=x/|x|.

We need convergence to a **unique** value, not like in f(x)=sin(1/x).

BROKEN FUNCTIONS



$$f(x) = \frac{x^2 - 1}{x - 1}$$

f is not defined at x=1 but we can assign a value to x=1.

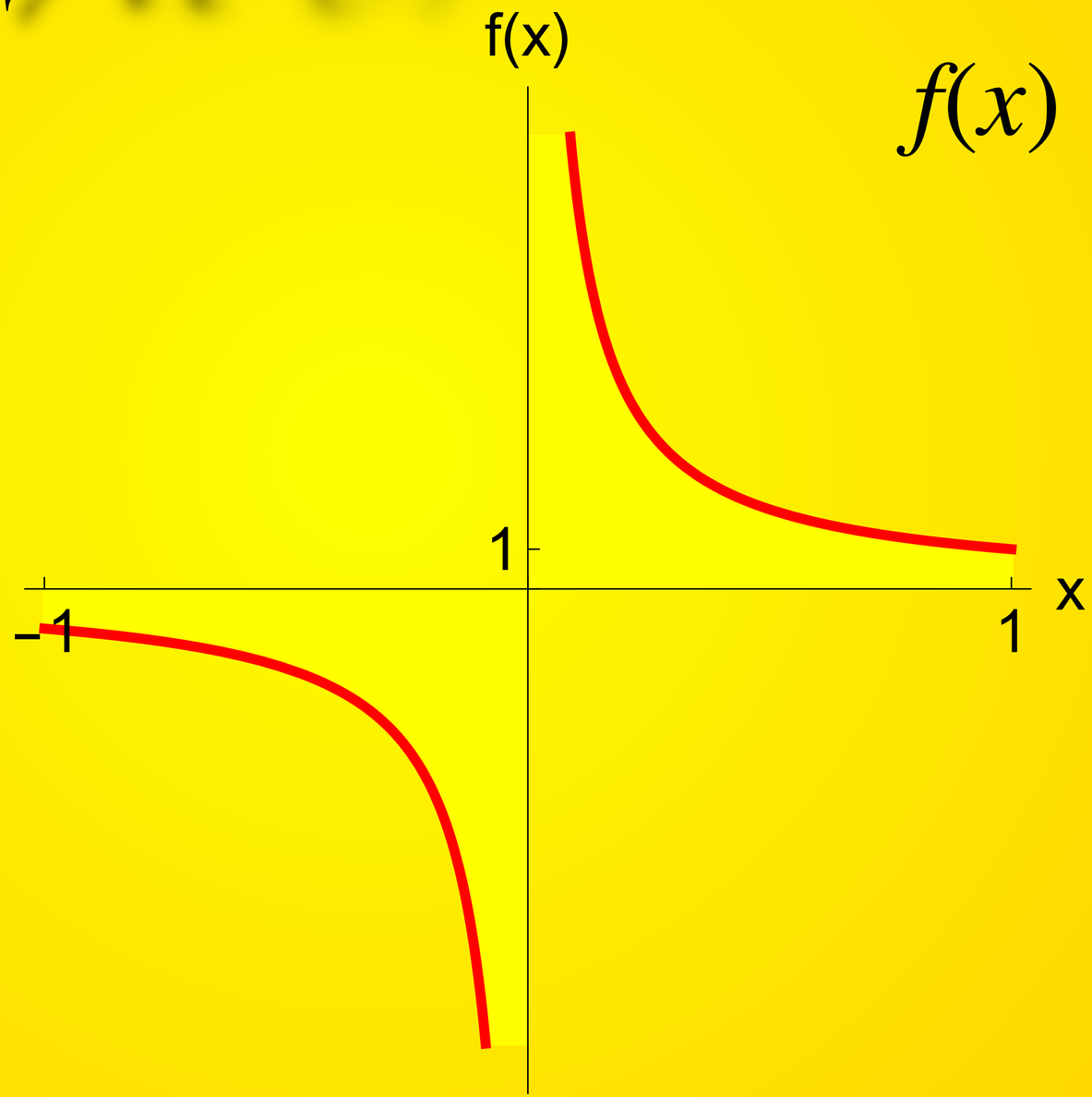
POLL

What is:

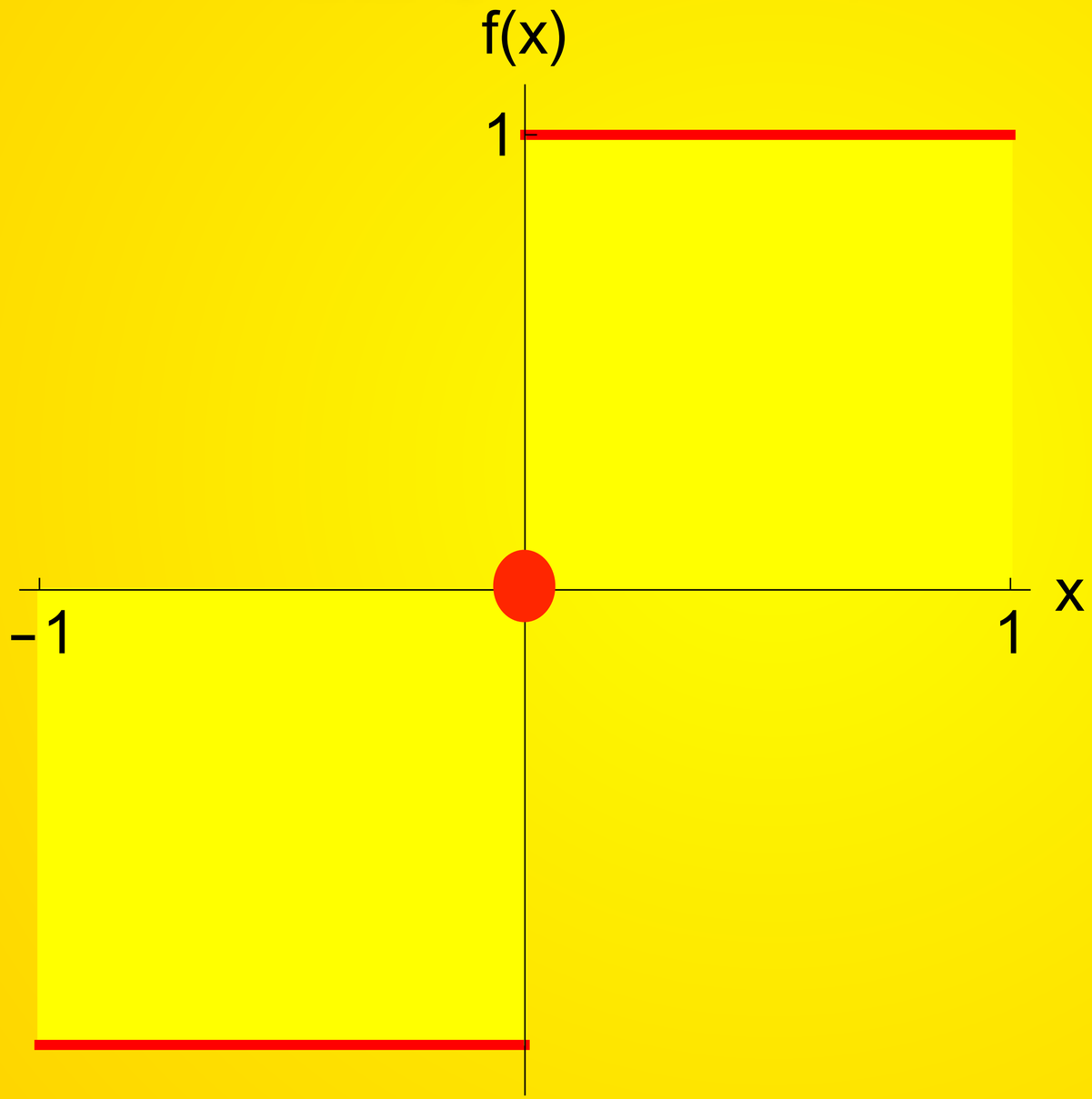
$$\lim_{x \to 2} \frac{(x^2 - 5x + 6)x}{(x - 2)}$$

LIMIT AT 0?

$$f(x) = \frac{1}{x}$$



LIMIT AT 0?



$$f(x) = \operatorname{sign}(x)$$

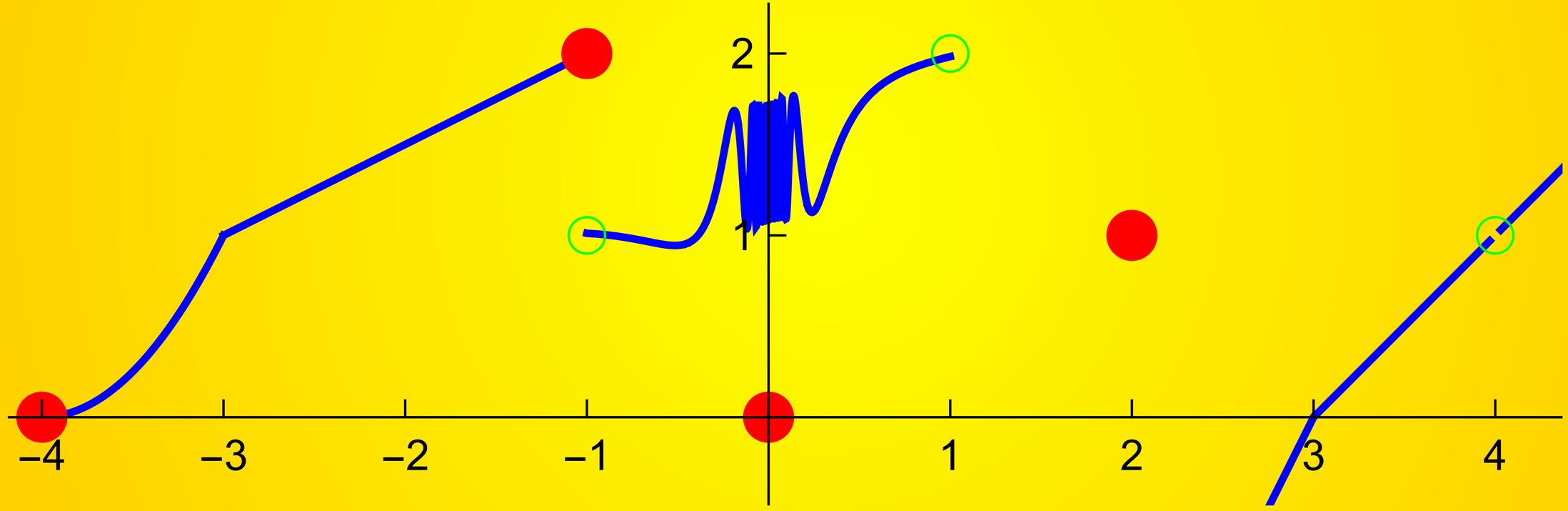
EXAMPLES

a) $$\lim_{x \to -1} \frac{x^4 - 1}{x + 1}$$

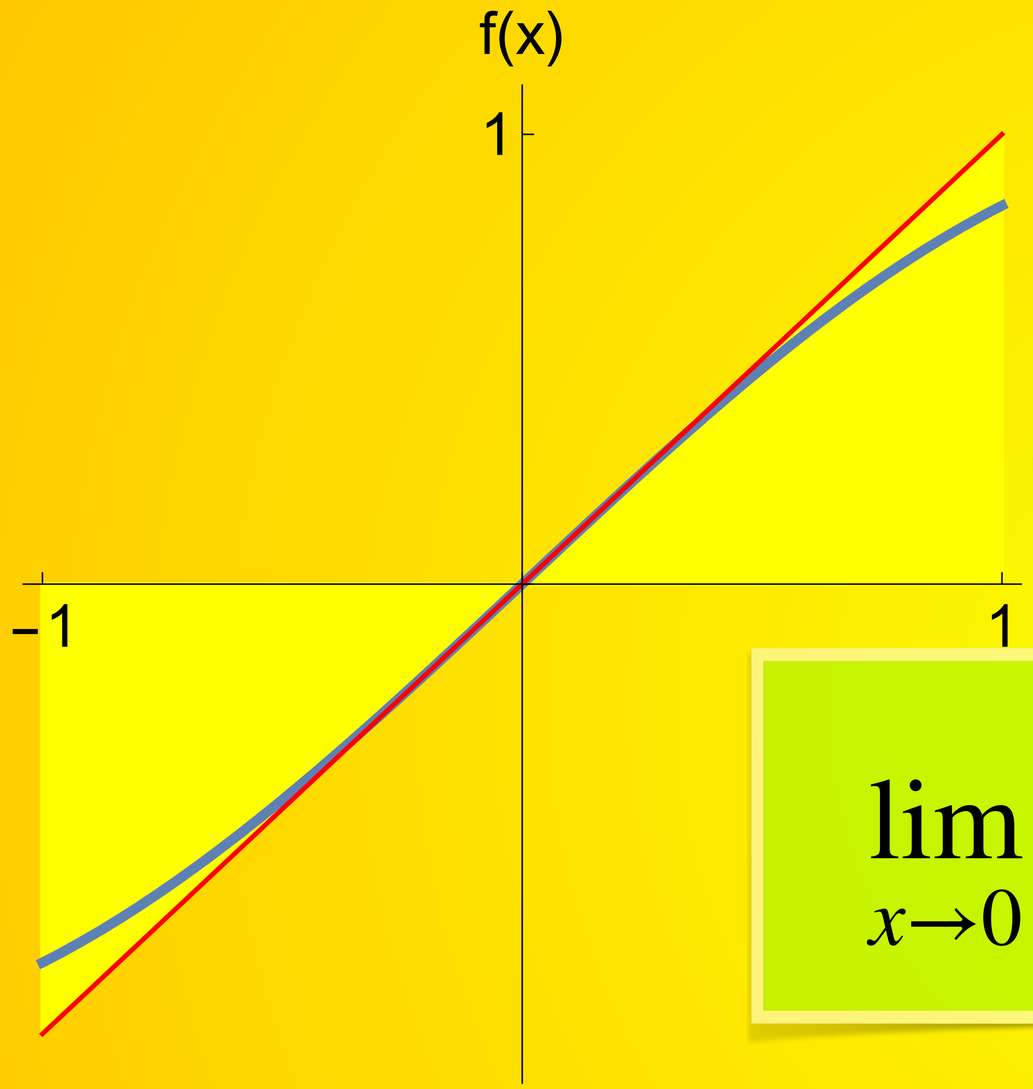
b) $$\lim_{x \to 0} \frac{\sin(x)^3 - 1}{\sin(x) - 1}$$

c) $$\lim_{x \to 0} \frac{\tan(x)}{\sin(x)}$$

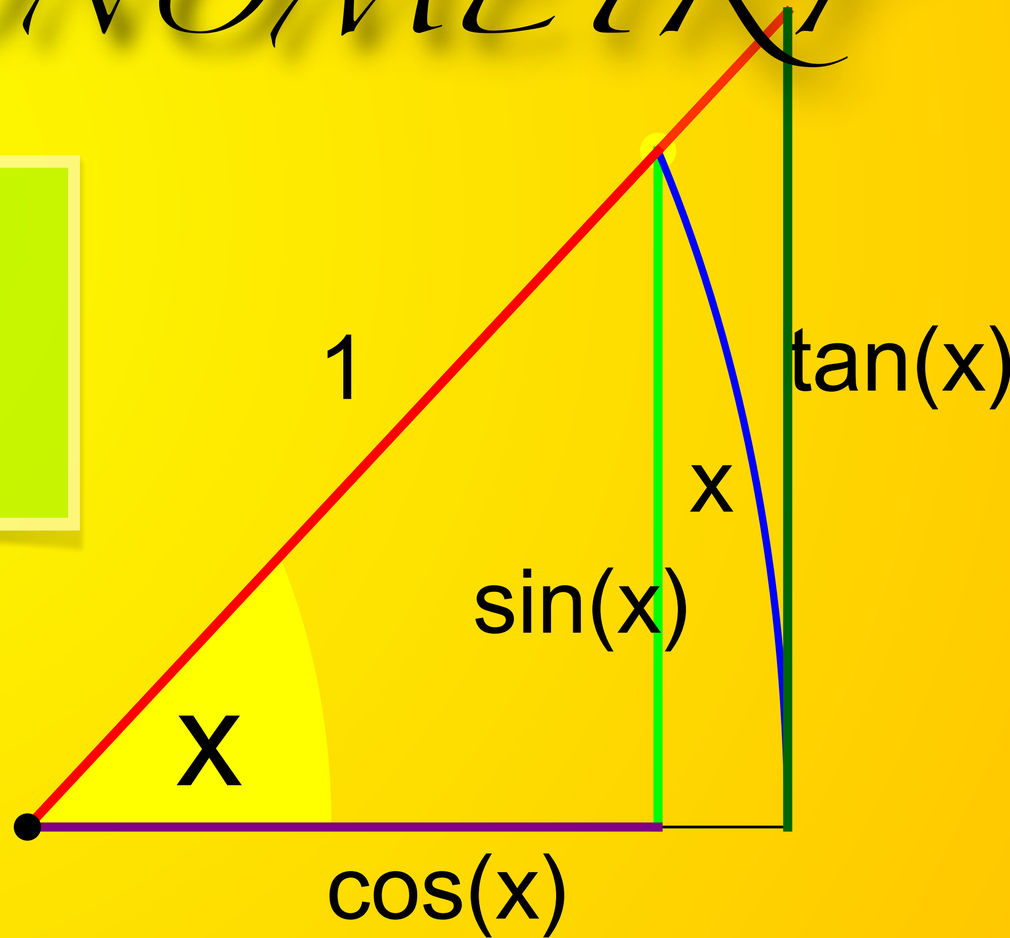
LIMITS



FUNDAMENTAL THEOREM TRIGONOMETRY



$$\lim_{x \to 0} \frac{\sin(x)}{x} = 1$$



$$1 = \frac{\sin x}{\sin x} \leq \frac{x}{\sin x} \leq \frac{\tan(x)}{\sin(x)} = \frac{1}{\cos(x)}$$

EXAMPLES

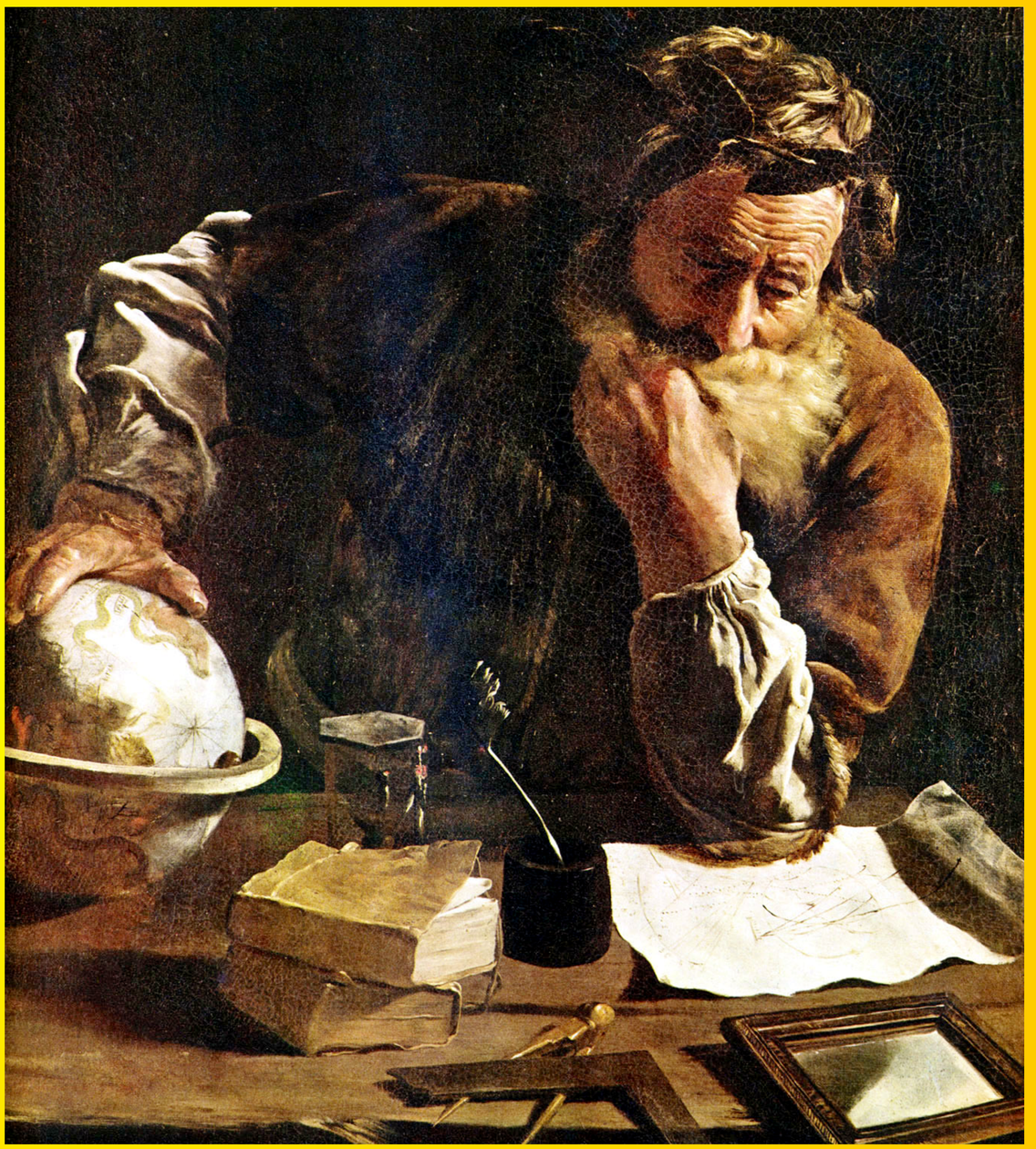
a) $\lim_{x \to 0} \dfrac{5x}{\sin(7x)}$

b) $\lim_{x \to 0} \dfrac{\sin^2(x^3)}{x^6}$

c) $\lim_{x \to 0} \dfrac{5x(x^2 - 1)}{\sin(7x)(x - 1)}$

ARCHIMEDES

Archimedes
288 BC -212BC



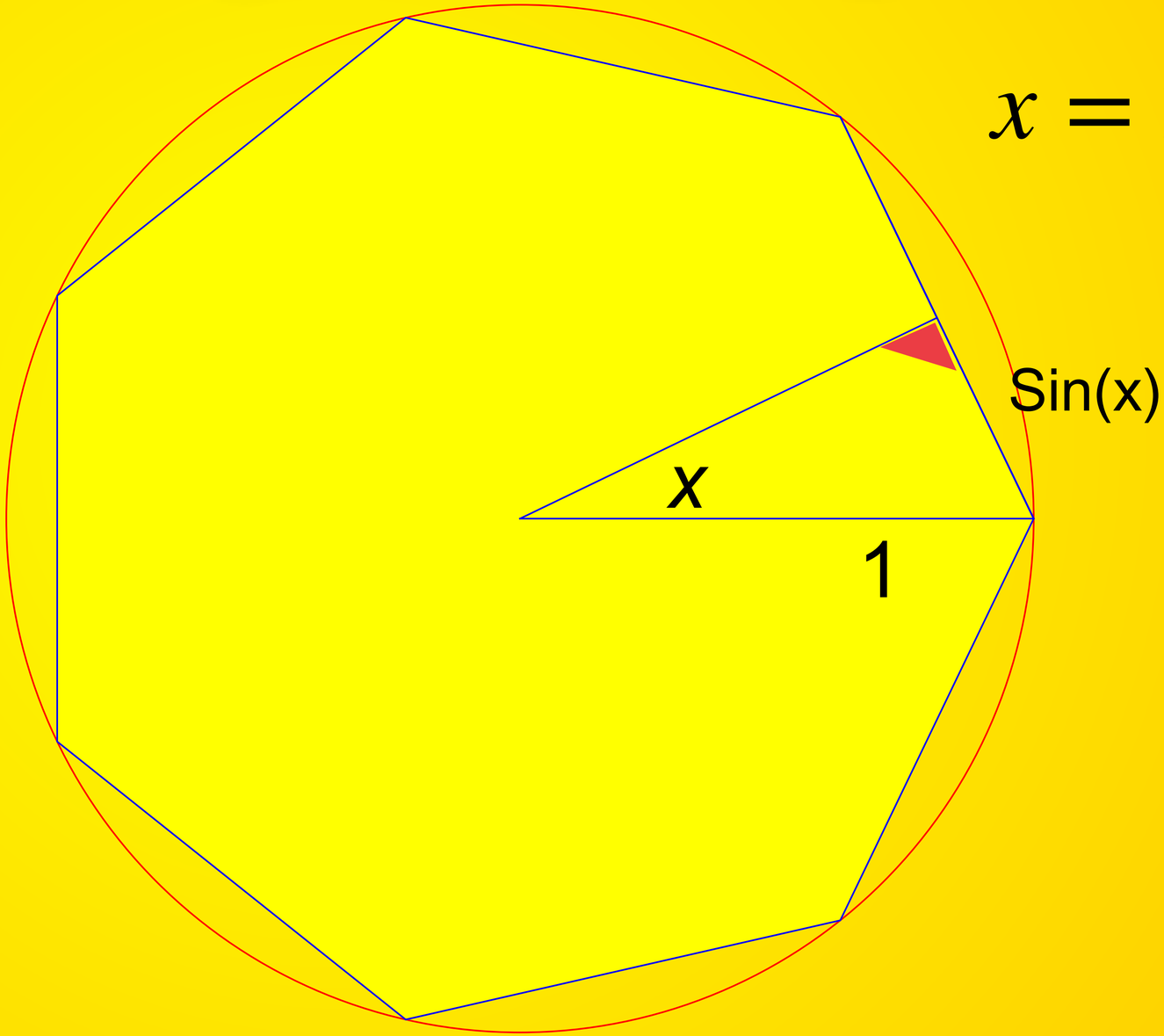
CIRCUMFERENCE

$$\frac{\sin(x)}{x} = \frac{\sin(2\pi/n)}{2\pi/n} \to 1$$

$$x = 2\pi/n$$

so:

$$n\sin(2\pi/n) \to 2\pi$$



Archimedes
288 BC -212BC

Set Marker
Hover Mode
Boost

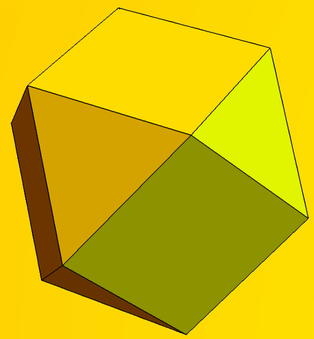
JAM

a) $\lim_{x \to 0} \dfrac{e^{2x} - 1}{e^{x} - 1}$

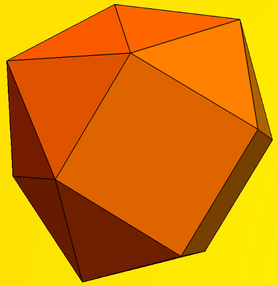
b) $\lim_{x \to 0} \dfrac{\sin(\sin(x))}{\sin(x)}$

c) $\lim_{x \to 1} \dfrac{\log^2(x)}{\log(x)}$

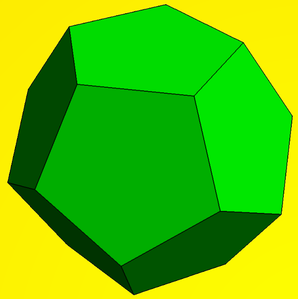
GROUPS



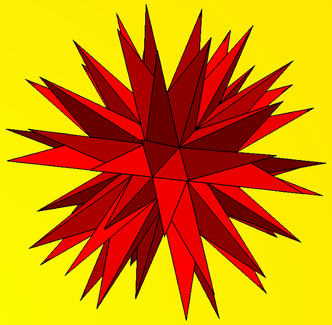
Cubeoctahedron



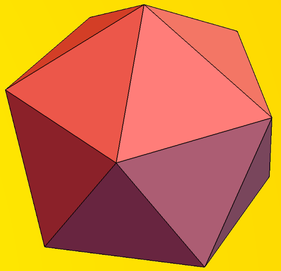
Disphenoid



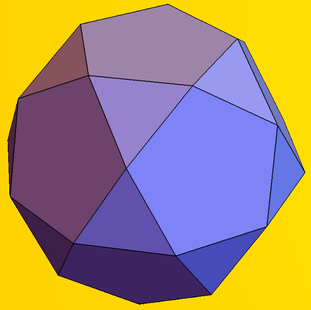
Dodecahedron



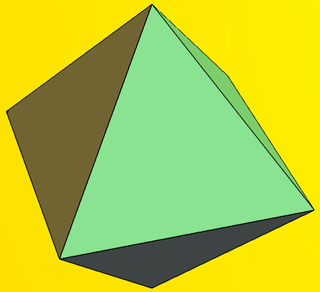
Echidnahedron



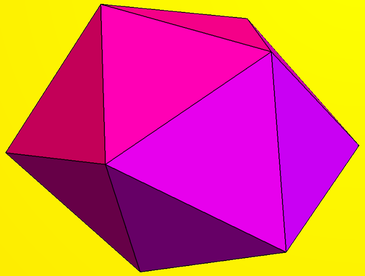
Icosahedron



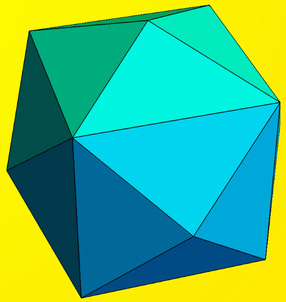
Icosidodecahedron



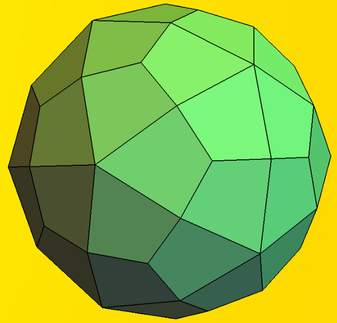
Octahedron



Sphenocorona



Tetraxishexahedron



Trapezohedron

The End