

## 3/5/2021: First hourly Practice D

”By signing, I affirm my awareness of the standards of the Harvard College Honor Code.”

Your Name:

- Solutions are submitted to knill@math.harvard.edu as PDF handwritten in a file carrying your name. Capitalize the first letters like in OliverKnill.pdf. The paper has to **feature your personal handwriting** and contain no typed part. If you like, you can start writing on a new paper. For 1), you could write 1: False, 2: False ... but you then need to copy the above Honor Code statement and sign.
- No books, calculators, computers, or other electronic aids are allowed. You can use a double sided page of your own handwritten notes when writing the paper. It is your responsibility to submit the paper on time and get within that time also a confirmation. The exam is due at 6 AM on March 6th.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

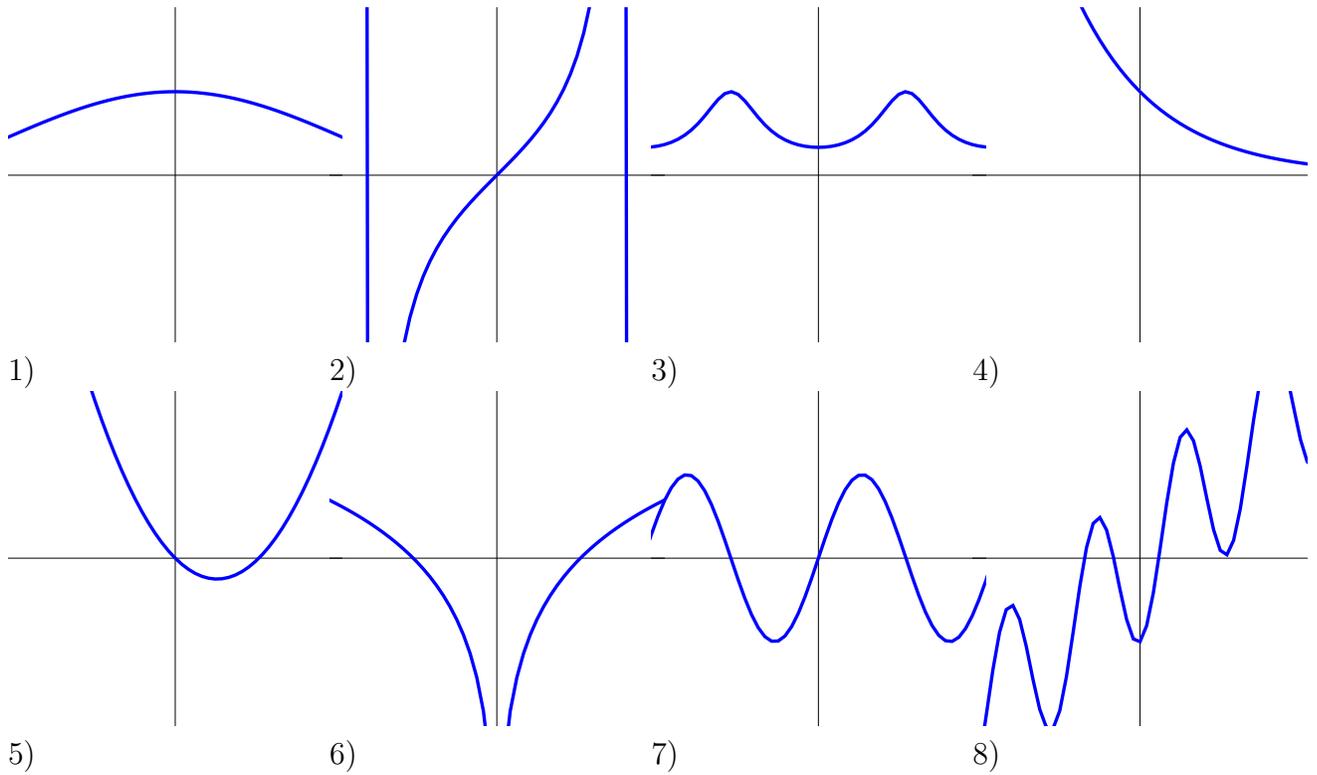
Problem 1) True/False questions (20 points) No justifications are needed.

- 1)  T  F The function  $\cot(x)$  is the inverse of the function  $\tan(x)$ .
- 2)  T  F We have  $\cos(x)/\sin(x) = \cot(x)$
- 3)  T  F  $\sin(3\pi/2) = -1$ .
- 4)  T  F The function  $f(x) = \sin(x)/x$  has a limit at  $x = 0$ .
- 5)  T  F For the function  $f(x) = \sin(\sin(\exp(x)))$  the limit  $\lim_{h \rightarrow 0} [f(x+h) - f(x)]/h$  exists.
- 6)  T  F If a differentiable function  $f(x)$  satisfies  $f'(3) = 3$  and is  $f'$  is odd then it has a critical point.
- 7)  T  F The l'Hospital rule assures that the derivative satisfies  $(f/g)' = f'/g'$ .
- 8)  T  F The intermediate value theorem assures that a continuous function has a derivative.
- 9)  T  F After healing, the function  $f(x) = (x+1)/(x^2-1)$  is continuous everywhere.
- 10)  T  F If  $f$  is concave up on  $[1, 2]$  and concave down on  $[2, 3]$  then 2 is an inflection point.
- 11)  T  F There is a function  $f$  which has the property that its second derivative  $f''$  is equal to its negative  $f$ .
- 12)  T  F The function  $f(x) = [x]^4 = x(x-h)(x-2h)(x-3h)$  has the property that  $Df(x) = 4[x]^3 = 4x(x-h)(x-2h)$ , where  $Df(x) = [f(x+h) - f(x)]/h$ .
- 13)  T  F The quotient rule is  $d/dx(f/g) = (f'g - fg')/g^2$  and holds whenever  $g(x) \neq 0$ .
- 14)  T  F The chain rule assures that  $d/dxf(g(x)) = f'(g(x)) + f(g'(x))$ .
- 15)  T  F If  $f$  and  $g$  are differentiable, then  $(3f + g)' = 3f' + g'$ .
- 16)  T  F The function  $(e^x - 1)/(\cos(x) - 1)$  defines an indefinite form at 0.
- 17)  T  F One can rotate a four legged table on an arbitrary surface such that all four legs are on the ground.
- 18)  T  F The fundamental theorem of calculus relates integration  $S$  with differentiation  $D$ . The result is  $DSf(x) = f(x)$ ,  $SDf(x) = f(x) - f(0)$ .
- 19)  T  F The product rule implies  $d/dx(f(x)g(x)h(x)) = f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$ .
- 20)  T  F Euler and Gauss are the founders of infinitesimal calculus.

Problem 2) Matching problem (10 points) No justifications are needed.

Match the following functions with their graphs.

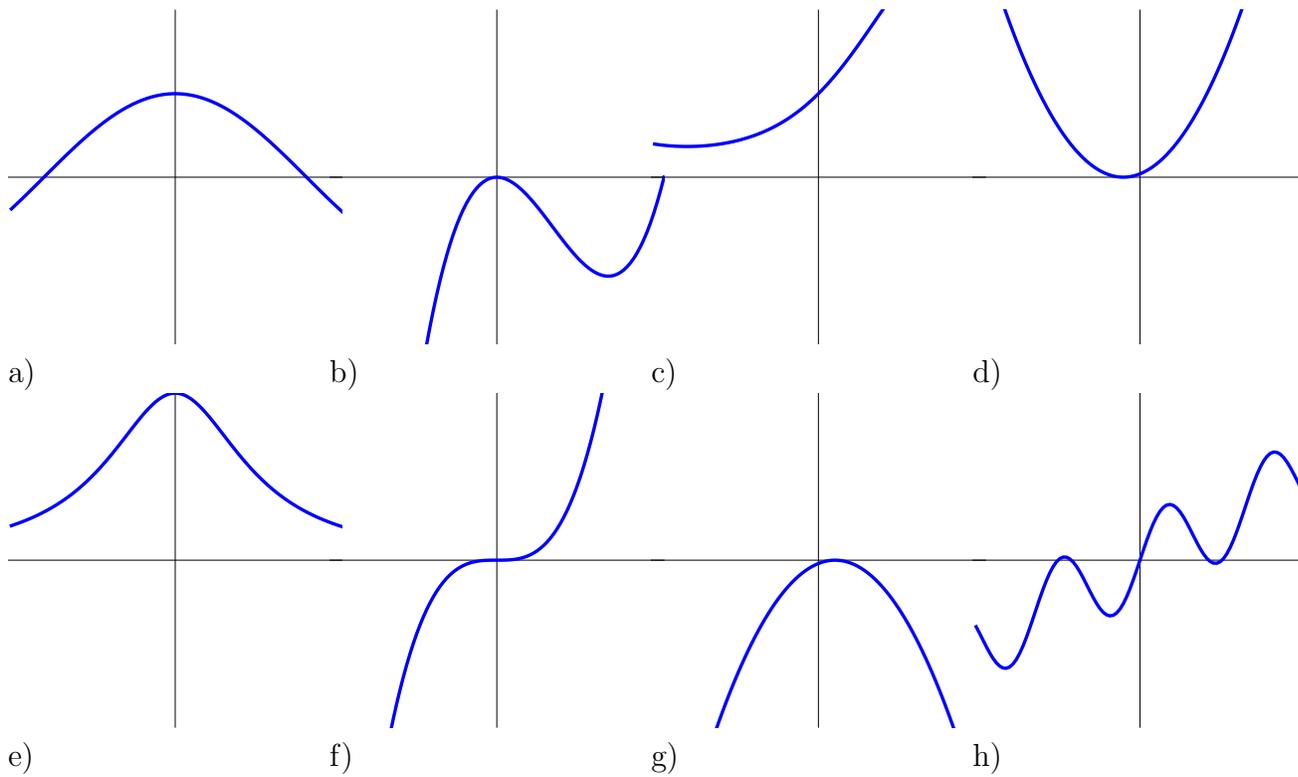
Function	Fill in 1-8
$x^2 - x$	
$\exp(-x)$	
$\sin(3x)$	
$\log( x )$	
$\tan(x)$	
$1/(2 + \cos(x))$	
$x - \cos(6x)$	
$\sin(3x)/x$	

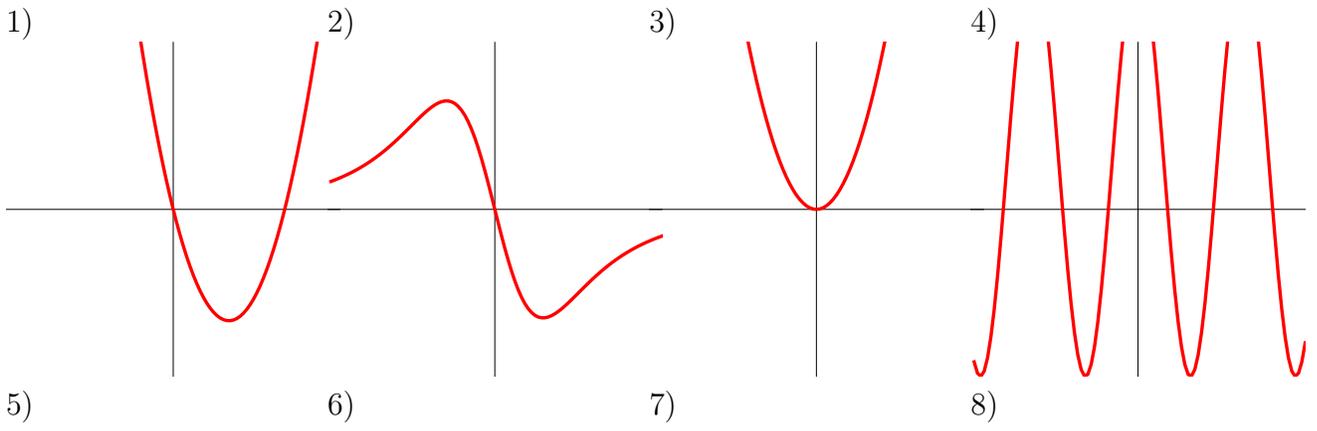
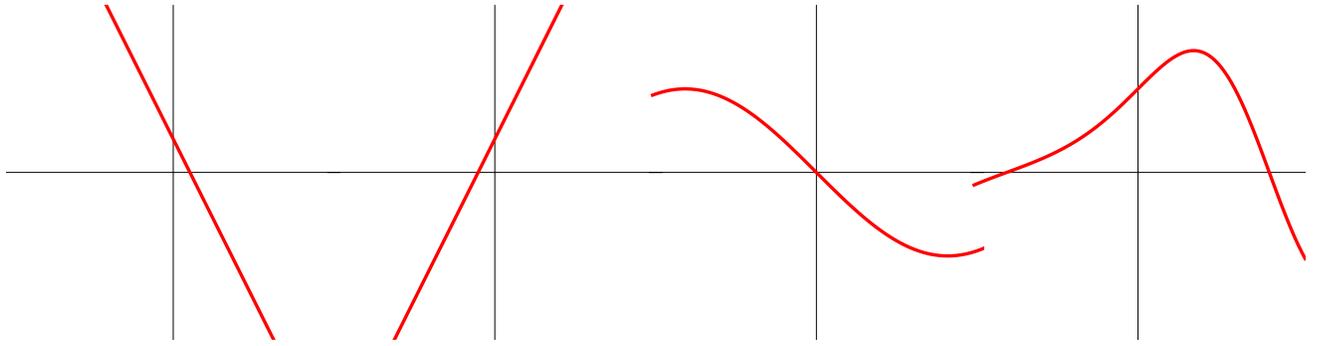


Problem 3) Matching problem (10 points) No justifications are needed.

Match the following functions with their derivatives.

Function	Fill in the numbers 1-8
graph a)	
graph b)	
graph c)	
graph d)	
graph e)	
graph f)	
graph g)	
graph h)	





Problem 4) Functions (10 points) No justifications are needed

Match the following functions with simplified versions. In each of the rows, exactly one of the choices A-C is true.

Function	Choice A	Choice B	Choice C	Enter A-C
$\frac{x^4-1}{x-1}$	$1 + x + x^2 + x^3$	$1 + x + x^2$	$1 + x + x^2 + x^3 + x^4$	
$2^x$	$e^{2\log(x)}$	$e^{x\log(2)}$	$2^{e\log(x)}$	
$\sin(2x)$	$2\sin(x)\cos(x)$	$\cos^2(x) - \sin^2(x)$	$2\sin(x)$	
$(1/x) + (1/(2x))$	$1/(x + 2x)$	$3/(2x)$	$1/(x + 2x)$	
$e^{x+2}$	$e^x e^2$	$2e^x$	$(e^x)^2$	
$\log(4x)$	$4\log(x)$	$\log(4)\log(x)$	$\log(x) + \log(4)$	
$\sqrt{x^3}$	$x^{3/2}$	$x^{2/3}$	$3\sqrt{x}$	

Problem 5) Roots (10 points)

Find the roots of the following functions

- a) (2 points)  $7\sin(3\pi x)$
- b) (2 points)  $x^5 - x$ .
- c) (2 points)  $\log|ex|$ .
- d) (2 points)  $e^{5x} - 1$
- e) (2 points)  $8x/(x^2 + 4) - x$ .

Problem 6) Derivatives (10 points)

Find the derivatives of the following functions:

- a) (2 points)  $f(x) = \cos(3x)/\cos(10x)$
- b) (2 points)  $f(x) = \sin^2(x)\log(1 + x^2)$
- c) (2 points)  $f(x) = 5x^4 - 1/(x^2 + 1)$

d) (2 points)  $f(x) = \tan(x) + 2^x$

e) (2 points)  $f(x) = \arccos(x)$

Problem 7) Limits (10 points)

Find the limits  $\lim_{x \rightarrow 0} f(x)$  of the following functions:

a) (2 points)  $f(x) = (x^6 - 3x^2 + 2x)/(1 + x^2 - \cos(x))$ .

b) (2 points)  $f(x) = (\cos(3x) - 1)/(\cos(7x) - 1)$ .

c) (2 points)  $f(x) = \tan^3(x)/x^3$ .

d) (2 points)  $f(x) = \sin(x) \log(x^6)$

e) (2 points)  $f(x) = 4x(1 - x)/(\cos(x) - 1)$ .

Problem 8) Extrema (10 points)

a) (5 points) Find all local extrema of the function  $f(x) = 30x^2 - 5x^3 - 15x^4 + 3x^5$  on the real line.

Problem 9) Extrema (10 points)

(10 points) A cup of height  $h$  and radius  $r$  has the volume  $V = \pi r^2 h$ . Its surface area is  $\pi r^2 + \pi r h$ . Among all cups with volume  $V = \pi$  find the one which has minimal surface area. Find the global minimum.

