

3/5/2021: First hourly Practice A

”By signing, I affirm my awareness of the standards of the Harvard College Honor Code.”

Your Name:

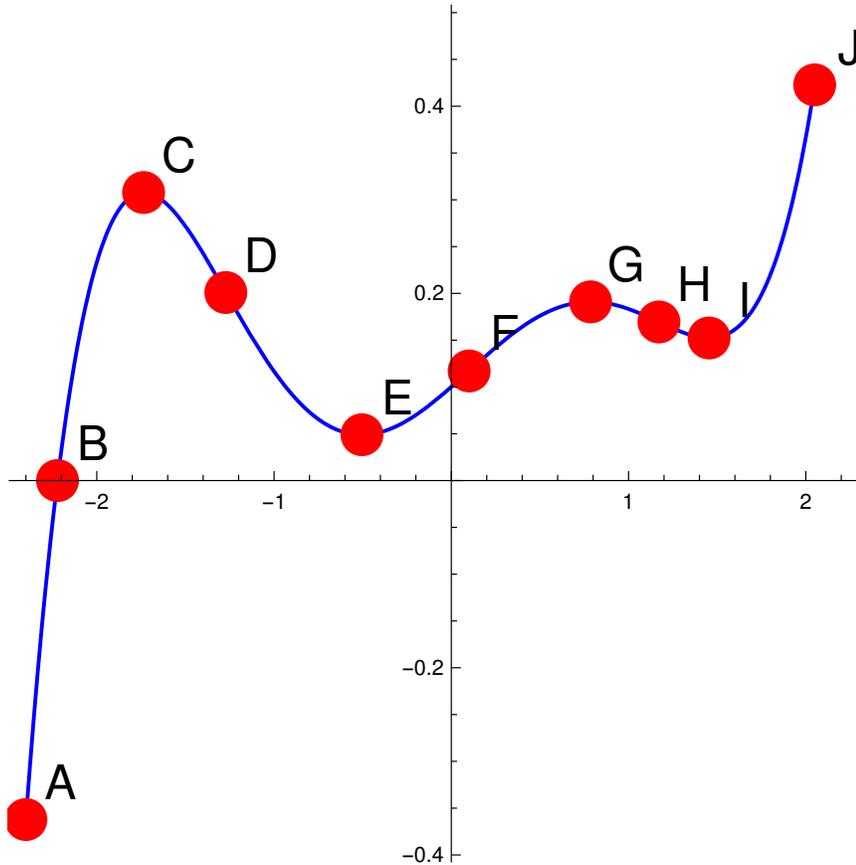
- Solutions are submitted to knill@math.harvard.edu as PDF handwritten in a file carrying your name. Capitalize the first letters like in OliverKnill.pdf. The paper has to **feature your personal handwriting** and contain no typed part. If you like, you can start writing on a new paper. For 1), you could write 1: False, 2: False ... but you then need to copy the above Honor Code statement and sign.
- No books, calculators, computers, or other electronic aids are allowed. You can use a double sided page of your own handwritten notes when writing the paper. It is your responsibility to submit the paper on time and get within that time also a confirmation. The exam is due at 6 AM on March 6th.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

Problem 1) TF questions (20 points) No justifications are needed.

- 1) T F The function $f(x) = \log(x)$ has a root at $x = 1$.
- 2) T F $f(x) = x^2 + \exp(-x^2)$ has a root on the real axes.
- 3) T F $\cos(\pi) = -1$.
- 4) T F $f(x) = \sin(|x|)$ is differentiable everywhere.
- 5) T F The chain rule assures that $\frac{d}{dx} \sin(x^5) = \cos(x)5x^4$.
- 6) T F The function $f(x) = \exp(-\sin(x))$ is continuous everywhere.
- 7) T F $\sinh(x) = (e^x - e^{-x})/2$ is positive everywhere.
- 8) T F $\cot(x)$ is the inverse function of $\tan(x)$.
- 9) T F If $f(x)$ is differentiable at 0, then $f(x)^2$ is differentiable at 0.
- 10) T F The function $x^3/(1 + x^3)$ defines an indefinite form at $x = \infty$.
- 11) T F $\sin(3\pi/4) = -1$.
- 12) T F $f(x) = \tan(x)$ has a vertical asymptote at $x = 0$.
- 13) T F e^{x^2} takes the value π at some point.
- 14) T F If $f(x) = x^2$, then $Df(x) = f(x + 1) - f(x) = 2x + 1$.
- 15) T F The intermediate value theorem implies Rolles theorem.
- 16) T F The function $\sin(1/x)$ is continuous everywhere.
- 17) T F $\frac{d}{dx} \arctan(x) = 1/(1 + x^2)$.
- 18) T F $\frac{d}{dx} \log(3 + x) = 3/(3 + x)$.
- 19) T F A continuous function on $[0, 1)$ has at least one maximum.
- 20) T F The derivative of f/g is $(fg' - f'g)/g^2$.

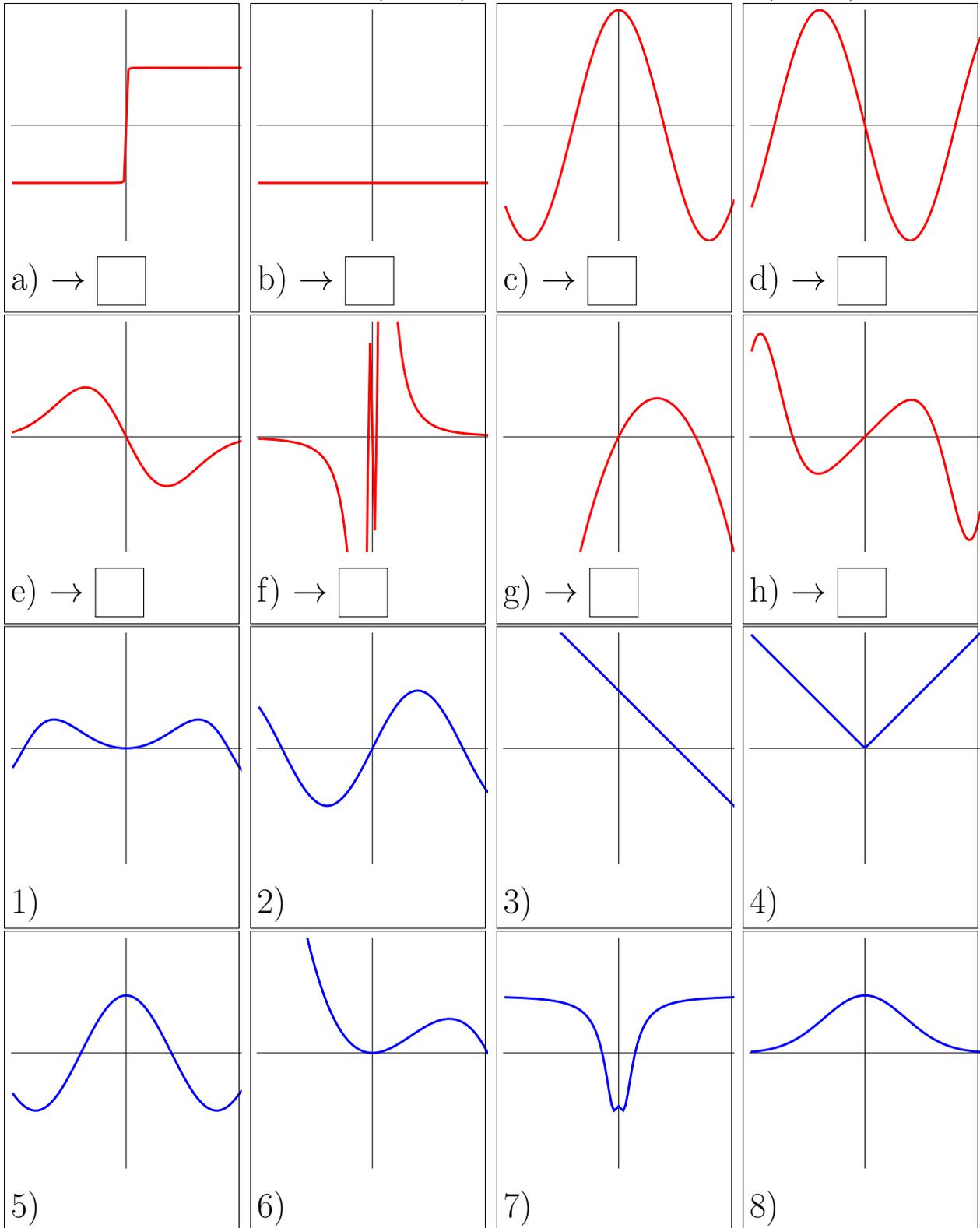
Problem 2) Choice problem (10 points) No justifications are needed.



- (2 points) List the points A-J which are roots of f .
- (2 points) List the points A-J are inflection points.
- (2 points) List the points A-J that are local maxima.
- (2 points) List the points A-J that are local minima.
- (2 points) List the points A-J that are global maxima.

Problem 3) Matching problem (10 points) No justifications are needed.

Match the derivatives f' a) to h) with the functions 1) to 8).



Problem 4) Continuity (10 points)

Which of the following functions are continuous on $[-1, 1]$? As usual we extend the domain of definition to points, where a continuation is possible. In each case make the decision “continuous” or “not continuous” and point to the x value which needs special attention.

a) (2 points) $f(x) = \frac{x^6-1}{x^2-1}$

b) (2 points) $f(x) = \frac{\sin(\sin(x))}{\sin(\sin(\sin(x)))}$.

c) (2 points) $f(x) = \frac{\sin^2(x)}{2+\sin(x^2)}$

d) (2 points) $f(x) = \log|x|e^x$

e) (2 points) $f(x) = \frac{\sin(\tan(x))}{\sin(x)}$

Problem 5) Derivatives (10 points)

Find the derivatives of the following functions. In each case, in-

indicate which differentiation rule you use.

a) (2 points) $f(x) = \frac{1}{1+e^x}$.

b) (2 points) $f(x) = \cos(x) \sin(x)$.

c) (2 points) $f(x) = \frac{1+x^3}{1+x^2}$.

d) (2 points) $f(x) = \arctan(\sin(x))$.

e) (2 points) $f(x) = \log(\log(5x))$.

Problem 6) Limits (10 points)

Find the limits $\lim_{x \rightarrow 0} f(x)$ for the following functions:

a) (2 points) $f(x) = \frac{\exp(7x)-1}{\exp(4x)-1}$.

b) (2 points) $f(x) = \frac{x-1}{x+1}$.

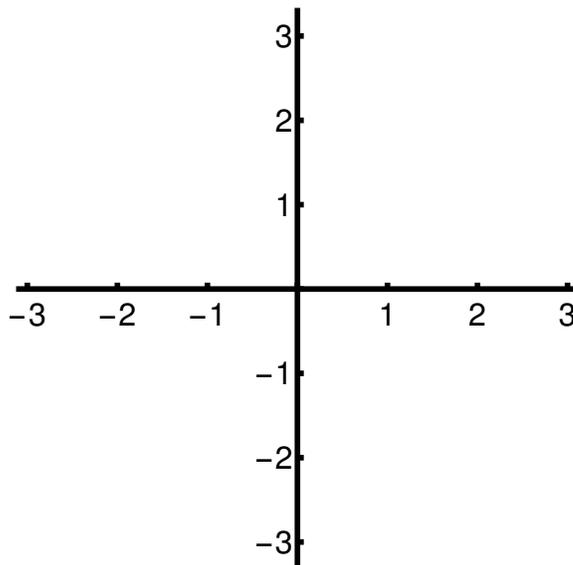
c) (2 points) $f(x) = \frac{\arctan(x)}{\sin(x)}$.

d) (2 points) $f(x) = \frac{\log(x^3)}{\log(x^2)}$.

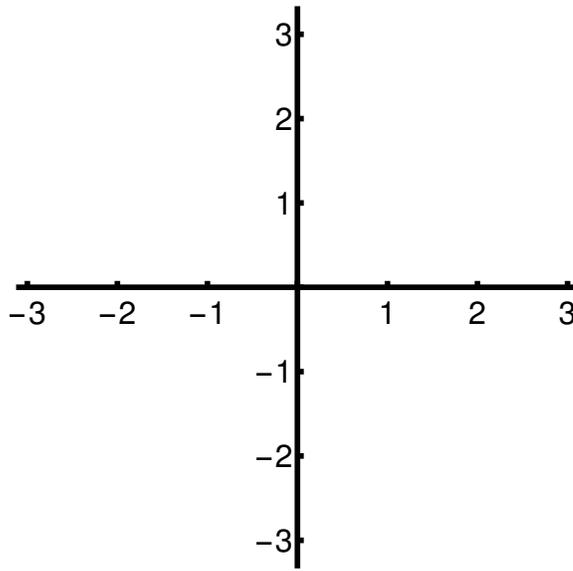
e) (2 points) $f(x) = \frac{\sin(3x)\sin(5x)}{\sin(7x)\sin(2x)}$.

Problem 7) Functions (10 points)

a) (5 points) Draw the graph of the natural log function $f(x) = \log|x| = \ln|x|$. Make sure you get the roots and asymptotes correct.



b) (5 points) Draw the graph of the arctan function $f(x) = \arctan(x)$. Make sure you get the roots and asymptotes correct.



Problem 8) Extrema (10 points)

a) (3 points) Find all the critical points of the function

$$f(x) = x^3 - 3x + 1 .$$

b) (3 points) Use the second derivative test to classify the critical points of f .

c) (2 points) On the interval $[-3, 3]$, where is the global maximum, and where is the global minimum?

d) (2 points) Which theorem assures that there is a global maximum and a global minimum on $[-3, 3]$?

Problem 9) Algebra rules (10 points, 2 points each)

a)	$(e^x)^y$	
b)	e^{x+y}	
c)	$\log(xy)$	
d)	$\frac{\tan(x)}{\sin(x)}$	
e)	$\frac{x^9}{x^3}$	

Choose from the following expressions.

- $1/\cos(x)$
- $\cos(x)/\sin^2(x)$.
- $e^x e^y$
- $\log(x + y)$
- $e^x + e^y$
- $e^{(xy)}$
- x^6
- $\log(x) - \log(y)$
- $e^{(x^y)}$
- $\log(x) + \log(y)$
- x^3
- $\cos(x)$