INTRODUCTION TO CALCULUS

MATH 1A

Unit 6: Worksheet

Examples

Here are some worked out examples, similar to what we expect you to do for the homework. When finding the value of $Sf(x)$, we want to add a constant such that $Sf(0) = 0$. In general, you will not need to evaluate functions and can leave terms like $\sin(5 \cdot x)$ as they are. If you have seen calculus already, then you could do this exercise by writing

$$\frac{d}{dx} f(x), \int_0^x f(t)\, dt$$

instead of $Df(x)$ or $Sf(x)$. Since we did not introduce the derivative df/dx nor the integral $\int_0^x$ yet, for now, just use the differentiation and integration rules you have on the handout.

1: Find the derivative $Df(x)$ of the function

$$f(x) = \log(1 + x) + \sin(5 \cdot x) + x^7 + 3.$$

2: Find $Df(x)$ for

$$f(x) = \exp(3x) + \sin(17x) + \log(5 + x).$$

3: Find the integral $Sf(x)$ of the function

$$f(x) = 1/(x + 5) + \cos(3x).$$

4: Find the integral $Sf(1)$ of the function

$$f(x) = \exp(-4 \cdot x)$$

The exponential function

The exponential function $\exp(x) = (1+h)^{x/h}$ is a **compound interest formula** where h is the interest rate. In the case $h = 1$ we have $\exp(x) = 2^x$. It maps positive integers to positive integers and as you have checked in the first homework,

$$D\exp(x) = \exp(x) \ .$$

From the fundamental theorem, we get $SD\exp(x) = S\exp(x) = \exp(x) - \exp(0)$ we see

$$S\exp(x) = \exp(x) - 1 \ .$$

We can sum up therefore. For example, $D\exp(3) = 2^4 - 2^3 = 8 = \exp(3)$. We also have $S\exp(3) = \exp(3) - 1$. In numbers, this means $1 + 2 + 4 = 8 - 1$.

1: What does $D\exp(5) = \exp(6) - \exp(5) = \exp(5)$ mean in numbers?
2: What does $S\exp(5) = \exp(5) - 1$ mean in numbers?
3: Explain $DS\exp(x) = \exp(x)$ with the left picture.
4: Explain $SD\exp(x) = \exp(x) - 1$ in the right picture.