

Unit 23

Applications

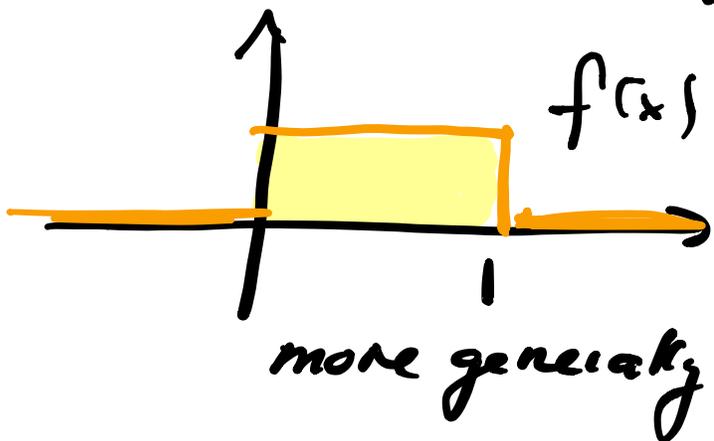
1. Probability density

A function f is called a probability density function PDF if

$$f(x) \geq 0$$
$$\int_{-\infty}^{\infty} f(x) dx = 1$$

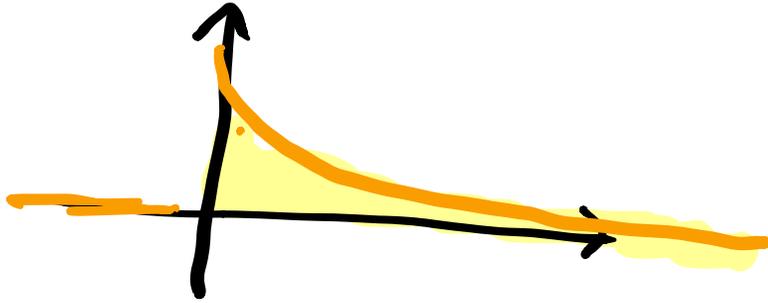
Example:

$$a) f(x) = \begin{cases} 1 & x \in [0, 1] \\ 0 & \text{else} \end{cases}$$



$\frac{1}{b-a}$ on $[a, b]$

$$b) f(x) = e^{-x} \\ \text{on } [0, \infty)$$



check: $f(x) \geq 0$

$$\int_0^{\infty} e^{-x} dx = 1$$

2. Normal distribution

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

Standard normal
distribution.

is positive.

$$\int_{-\infty}^{\infty} f(x) dx = 1$$

needs
some
multivariate

See movie "gifted".

3. Probability of an event

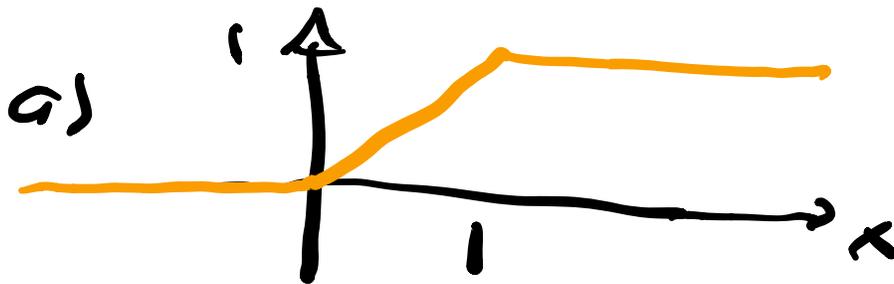
$$P([a, b]) = \int_a^b f(x) dx$$

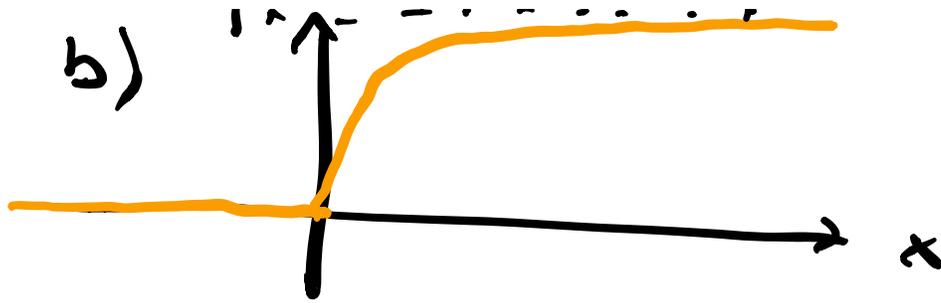
Probab. that
an experim. takes values
in $[a, b]$

Useful:

$$F(x) = \int_{-\infty}^x f(x) dx$$

Cumulative
distribution function CDF.





4. Mean

f is a PDF

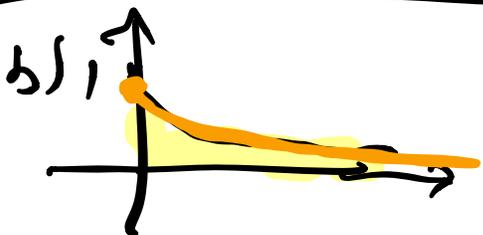
Its mean is defined

$$\mu = \int_{-\infty}^{\infty} x f(x) dx$$

Example:



$$\int_0^k x \cdot 1 dx = \frac{1}{2}$$



$$\int_0^{\infty} x e^{-x} dx$$

$$= -e^{-x}(1+x) \Big|_0^{\infty}$$

