Lecture 17: Orthogonality

Consider the data points $(-1,-1),(2,5),(4,0),(-5,-4)$. The first coordinate is the x coordinate, the second coordinate is the y coordinate. These data define random variables

$$X = \begin{bmatrix} -1 \\ 2 \\ 4 \\ -5 \end{bmatrix}, Y = \begin{bmatrix} -1 \\ 5 \\ 0 \\ -4 \end{bmatrix} .$$

They are already centered and have zero mean.

1 The length of the vectors is called the **standard deviations** of X and Y:

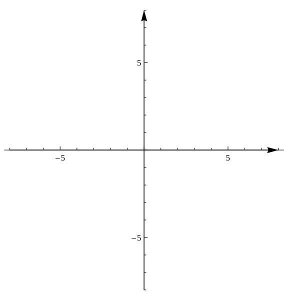
$$||X|| = \sigma(X) =$$

$$||Y|| = \sigma(Y) =$$

2 The **correlation** between X and Y is defined as

$$\frac{X \cdot Y}{||X|| \cdot ||Y||} .$$

This correlation is related to the slope of the **regression line** $y = mx + b$ as you will see in the homework.



Orthogonal matrices

Which matrices are orthogonal?

a)

$$\begin{bmatrix} \cos(2) & -\sin(2) & 0 \\ \sin(2) & \cos(2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

b)

$$\begin{bmatrix} 1/2 & 1/2 & 1/2 & 1/2 \\ 1/2 & 1/2 & 1/2 & 1/2 \\ 1/2 & 1/2 & 1/2 & 1/2 \\ 1/2 & 1/2 & 1/2 & 1/2 \end{bmatrix}$$

c)

$$\begin{bmatrix} 1/2 & -1/2 & 1/2 & -1/2 \\ -1/2 & -1/2 & 1/2 & 1/2 \\ 1/2 & 1/2 & 1/2 & 1/2 \\ -1/2 & 1/2 & 1/2 & -1/2 \end{bmatrix}$$

d)

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

e)

$$\begin{bmatrix} 1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} \\ 1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} \end{bmatrix}$$

f)

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

g)

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$