Lecture 13: Worksheet

Image and Kernel

1 What is the kernel $\ker(A)$ of $A = [5, 1, 4]$? What is the image $\text{im}(A)$?

2 What is the kernel and image of a projection of onto a plane Σ?

We call the kernel **trivial**, if it contains only the 0 vector.

3 Can the kernel of a $2x2$ matrix A be trivial if A^{10} is invertible?

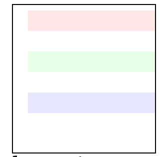
4 Can you find a 3x3 matrix A with the property that $\ker(A^2) = \mathbf{R}^3$ but that A is not the zero matrix?

Row and column picture

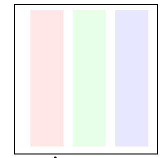
5 Kernel and image are somehow dual to each other. The kernel is associated to row vectors. The image is associated to column vectors. How?

$$\ker(A) = \{x \in \mathbf{R}^m \mid Ax = 0\}$$

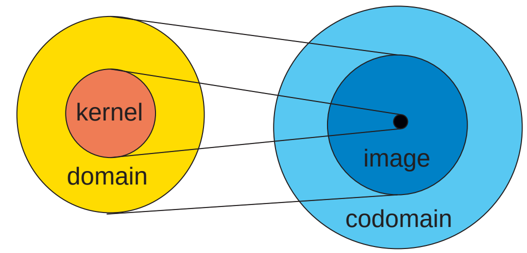
$$\text{im}(A) = \{Ax \mid x = \mathbf{R}^m\}$$



ker picture
$\perp$ to rows



im picture
columns



Why do we look at the kernel of a matrix?

- It is useful to understand linear maps. To which degree is a map non-invertible? If x is a solution and y is in the kernel of A, then also $A(x + y) = b$, so that $x + y$ solves the system also.
- In statistics it will allow us to determine whether fitting works. We will see that the fitting works if and only if the kernel of the data matrix is nontrivial.

Why do we look at the image?

- A solution $Ax = b$ can be solved if and only if b is in the image of A.
- The image will be pivotal for understanding data fitting. A general fitting problem will consist of projecting onto the image of a matrix.

In general, the abstraction helps to understand topics like error correcting codes. where two matrices H, M with the property that [.... homework ...] appear. The encoding $x \mapsto Mx$ is robust in the sense that adding an error e to the result $Mx \mapsto Mx + e$ can be corrected: $H(Mx + e) = He$ allows to find e and so Mx.