

## Lecture 2: Worksheet

In this worksheet, we look at some combinatorial formulas for 4 important combinatorial tools:

- **Permutations** In how many ways can we permute  $n$  objects Example: There are 6 ways to permute  $ABC$ .
- **Combinations** How many words of length  $k$  in an alphabet of  $n$  elements are there: Example: there are 9 words of length 3 with the letters  $MA$
- **Selection with order** How many ways can we select  $k$  elements from  $n$  elements if the order does not matter. Example: there are 6 ways to select a set of 2 elements from 4.
- **Selection without order** In how many ways can we select  $k$  elements from  $n$  elements if the order does matter. Example: there are  $4 * 3 = 12$  ways to select 2 ordered pairs from 4

### How many ways are there to:

permute  $n$  different elements

write word of length  $k$  in alphabet of  $n$  letters

pick  $k$  different from  $n$  if order matters

pick  $k$  different from  $n$  without looking at order

### The answer is:

$$n! = n(n-1)\dots 2 \cdot 1$$

$$n^k$$

$$\frac{n!}{(n-k)!}$$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

**1 Problem 1)** You play "scrabble" and have the letters  
1, 9,  $A, M, T, B, H$  .

What is the probability to obtain the word  $MATH19B$ ?

**2 Problem 2)** You randomly type 6 letters on your computer using the alphabet  $A - Z$ . What is the probability that the text will show

$HARVARD$  .

**3 Problem 3:** If we pick 2 letters from the group of letters

$LINEAR$  .

What is the chance to get the two letters  $AR$ ?

**4 Problem 4:** A combination lock has 40 numbers 0 – 39. A lock combination consists of 3 different numbers, where the order matters. You try 10 times. What is the probability that you pick the lock?