

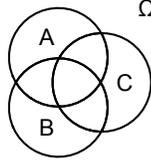
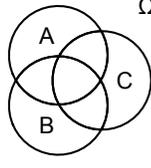
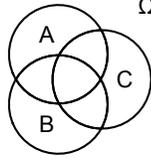
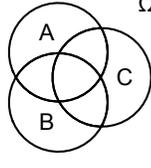
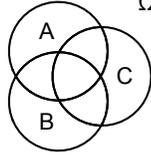
# PROBABILITY THEORY

MATH 154

## Inclass Midterm

You have 40 minutes. No material, no tools. Each problem is 10 points.

Problem 1)  $A, B, C$  are events. Shade the regions in the Venn diagram. Shade the regions. We use the notation from class for the Boolean algebra.

	$(A \setminus B) \cdot C$
	$A^c \cdot B$
	$A \cdot A \cdot A + B + C$
	$(\Omega + A^c) \cap A$
	$(A \setminus B) + (B \setminus A)$

Problem 2) A random variable  $X$  takes the value  $a_1 = 7$  with probability  $p = 1/4$  and the value  $a_2 = -7$  with probability  $p_2 = 1/4$  and  $a_3 = 0$  with probability  $p_3 = 1/2$ .

- a) What is the expectation  $m$  of  $X$ ?
- b) What is the standard deviation  $\sigma[X]$  of  $X$ ?
- c) What is the skewness  $E[(X - m)^3]/\sigma^3$ ?
- d) Write down the moment generating function  $M_X(t)$ .
- e) And finally give the characteristic function  $\phi_X(t)$ .

Problem 3) Let  $(\Omega, \mathcal{A}, P)$  be the Lebesgue probability space on  $\Omega = [-1, 1]$  with the standard measure  $P[[a, b]] = (b - a)/2$ . Let  $X$  be the random variable  $X(x) = x^3$  and  $Y$  the random variable  $Y(x) = x$ .

- a) What are the expectations of  $X$  and  $Y$ ?
- b) What are the standard deviations  $\sigma[X], \sigma[Y]$  of  $X$  and  $Y$ ?
- c) What is the moment generating function  $M_Y(t)$ ?
- d) What is the CDF  $F_X(t)$ ?
- e) What is the correlation  $\text{Corr}[X, Y]$  between  $X$  and  $Y$ ?

Problem 4) True/False Problems.

- 1) True or False? There is a  $\sigma$  algebra with 16 events.
- 2) True or False? There is a bijection between the complex numbers and the natural numbers
- 3) True or False? If you know the moment generating function of a bounded random variable  $X$ , then you know all the moments of  $X$ .
- 4) True or False? If  $A$  is an event of probability  $P[A] = 0$ , then  $A$  is the empty set.
- 5) True or False? On a Lebesgue probability space there are random variables taking only finitely many values.
- 6) True or False? If every function  $\Omega \rightarrow \mathbb{R}$  is a random variable, then the  $\sigma$ -algebra is trivial.
- 7) True or False? If  $A$  is an event in a probability space, then  $A^c = \Omega \setminus A$  is always also an event.
- 8) True or False? For every  $(\Omega, \mathcal{A})$ , where  $\mathcal{A}$  is  $\sigma$ -algebra, there is a probability measure  $P$  making  $(\Omega, \mathcal{A}, P)$  a probability space
- 9) True or False?  $\text{Cov}[\lambda X, \lambda Y] = \lambda^2 \text{Cov}[X, Y]$ .
- 10) True or False? If  $X^n, Y^n$  are uncorrelated for all integers  $n \geq 0$ , then they are independent.

Problem 5) One word answer questions.

1) A  $\Pi$ -system  $\mathcal{I}$  has the property that if  $A, B$  are in  $\mathcal{I}$  then the  is in  $\mathcal{I}$ .

2) The Lebesgue integral uses the space  $\mathcal{S}$  of  for defining the expectation of random variables in  $\mathcal{L}^1$  via  $E[X] = E[X^+] - E[X^-]$ .

3) If  $(\Omega, \mathcal{A}, P)$  is a probability space, then  $\Omega$  plays the role of the .

4) A  $\sigma$  algebra for which every event has probability 0 or 1 is called .

5) The smallest  $\Lambda$ -system that contains a  $\Pi$ -system must be a .

6) Simplify  $A + A^2 + A^3 + \dots + A^{20}$ . .

7) The doubling strategy in a Casino is called the  strategy in probability theory.

8) Let  $A$  be an event in a  $\sigma$ -algebra. What is  $A + A^c + A \cdot A$ ? .

9) If  $A, B$  both have the same positive probability, then  $P[B|A]/P[A|B] =$  .

10) If  $A = B$  then  $P[A|B] =$  .

**"I affirm my awareness of the standards of the Harvard College Honor Code."**

Name: