PROBABILITY THEORY

MATH 154

Midterm Quiz

You have 30 minutes to complete the quiz.

1) The win expectation in the Petersburg paradox is [ ].
2) Bertand's paradox showed that when modeling a random process, the [ ] is important.
3) True or False? (Simpson's paradox) If $a/b > c/d$, then $(a+c)/(b+d) > 1$. [ ]
4) Complete $\mathrm{P}[A \cup B] = \mathrm{P}[A] + \mathrm{P}[B]$ - [ ]
5) If Alice has 2 kids one one of them is a girl. What is the probability that the other is a girl? [ ]
6) There is a bijection between $[0,1]$ and $\mathbb{Z}$. [ ]
7) True or False: Every σ algebra is countable. [ ]
8) If Ω is countable, then every σ algebra on Ω is countable [ ]
9) True or False. For every $(\Omega, \mathcal{A})$, where $\mathcal{A}$ is a non-empty σ algebra, there is a measure P which makes $(\Omega, \mathcal{A}, \mathrm{P})$ a probability space. [ ]
10) There is a theorem about the tail σ-algebra of a sequence of σ algebras. Which mathematician is involved in this? [ ]

11) Probability theory started when investigating questions appearing in [ ]. The first book of Cardano was about this.

12) True or False? If you know the moment generating function of a bounded random variable X, then you know all the moments of X. [ ]

13) If $(\Omega, \mathcal{A}, \mathrm{P})$ is a probability space, then Ω is called the [ ]

14) True or False? If A is an event of probability $\mathrm{P}[A] = 0$, then A is the empty set. [ ]

15) Let A be an event in a σ algebra. What is $A + A$? [ ]

16) What is $1 + A + A^2 + A^3 + \cdots + A^{100}$. Simplify. [ ]

17) If $(\Omega, \mathcal{A}, \mathrm{P})$ is a probability space, then $A \in \mathcal{A}$ is called an [ ]

18) True or False? Any function $X : \Omega \to \mathbb{R}$ is a random variable. [ ]

19) A random variable on the trivial σ algebra $\{0, 1\} = \{\emptyset, \Omega)$ is [ ]

20) True or False? If every function $\Omega \to \mathbb{R}$ is a random variable, then the σ algebra is trivial. [ ]

21) True or False? If A is an event in a probability space then $A^c = \Omega \setminus A$ is an event. [ ]

22) A Π-system $\mathcal{I}$ has the property that if A, B are in $\mathcal{I}$ then [ ] is in $\mathcal{I}$.

23) The smallest Λ system that contains a Π-system is a [ ].

24) If A, B are disjoint events, then $\mathrm{P}[A|B] =$ [ ].

25) If A, B both have the same positive probability, then $\mathrm{P}[A|B]/\mathrm{P}[B|A] =$ [ ].

26) True or False? If A is independent of B and B is independent of C, then A is independent of C. [ ]

27) True or False? If A, B, C are independent then A is independent of C [ ]

28) True or False? $\mathrm{E}[X+Y] = \mathrm{E}[X] + \mathrm{E}[Y]$. [ ]

29) True or False? $\mathrm{Var}[X+Y] = \mathrm{Var}[X] + \mathrm{Var}[Y]$. [ ]

30) True or False? $\mathrm{E}[\lambda X] = \lambda \mathrm{E}[X]$. [ ]

31) True or False? $\mathrm{Cov}[\lambda X, Y] = \lambda \mathrm{Cov}[X, Y]$. [ ]

32) True or False? If $X \leq Y \Rightarrow \mathrm{E}[X] \leq \mathrm{E}[Y]$. [ ]

33) True or False? $\mathrm{Var}[X] = 0$ implies $X = 0$. [ ]

34) True or False? $\mathrm{Var}[\lambda X] = \lambda \mathrm{Var}[X]$. [ ]

35) True or False? The correlation satisfies $-1 \leq \mathrm{Corr}[X, Y] \leq 1$. [ ]

36) True or False? If X, Y are independent, then they are uncorrelated. [ ]

37) True or False? There are random variables for which the expectation is finite but the variance does not exist. [ ]

38) True or False? There are independent random variables for which the correlation does not exist. [ ]

39) The moment generating function $M_X(t)$ is defined if X is in $\mathcal{L}^{XXX}$. What is XXX? [ ]

40) True or False? The characteristic function $\phi_X(t)$ is defined for any random variable. [ ]

41) A step function is a random variable that takes [ ] many values.

42) In order that the expectation of a random variable exists, we need that it in $\mathcal{L}^{XXX}$, where $XXX =$ [ ].

43) In order that the variance of a random variable exists, we need it to be in $\mathcal{L}^{XXX}$, where $XXX =$ [ ].

44) Borel-Cantelli states that if A_n is a sequence of independent events for which $\sum_n P[A_n] < \infty$ then A_∞ has measure [ ].

45) True or False? The tail σ algebra is defined as $\bigcap_{J \subset I, J \text{ finite}} \mathcal{A}_J$. [ ]

46) Assume that you have texts of length n_k for which $\sum_n 1/26^{n_k} < \infty$, then a monkey typing randomly will type [ ] many of these texts.

47) Every measure can be decomposed into μ_{ac}, μ_{pp} and [ ]

48) True or False? There are measures on $[0,1]$ which are pure point but which have $[0,1]$ as support. [ ]

49) True or False? If A, B are independent events then, then $A, 1+B$ are independent. [ ]

50) True or False? If A, B are independent events then, then $A, A+B$ are independent. [ ]

51) True or False? If $\mathrm{P}[B] > 0$, and A, B are independent, then $\mathrm{P}[A|B] = \mathrm{P}[B|A]$ [ ]

52) True or False? If A, B are independent and B, C are independent then A, C are [ ]

53) True or False? If A, B, C are independent, then $A+B, B+C$ are independent. [ ]

54) True or False? If A, B, C are independent then $A \cup B$ is independent of C

55) True or False? Two disjoint sets A, B are independent if and only if $P[A] = 0$ or $P[B] = 0$

56) True or False? $\emptyset$ is independent of Ω?

57) True or False? $\emptyset$ is independent of any other set.

58) True or False? If A is independent to itself, then $P[A] = 0$ or $\mathrm{P}[A] = 1$.

59) True or False? If $(\Omega, \{0, 1\}, \mathrm{P})$ is the trivial probability space, then every random variable is constant.

60) True or False? If $X = c$ and $X = d$ are two random variables and c, d are different constants, then X, Y are independent.

"I affirm my awareness of the standards of the Harvard College Honor Code."

Name: