PROBABILITY THEORY

MATH 154

Final Quiz

You have 30 minutes to complete the quiz.

1) The fact that a positive left stochastic matrix A has a unique eigenvalue 1 is called the [ ] theorem.

2) The proof of the uniqueness of the existence of the eigenvalue 1 is based on a [ ] principle.

3) The random walk on the lattice $\mathbb{Z}^d$ is transient for [ ].

4) A probability vector is a vector for which all entries are non-negative and [ ].

5) A left stochastic matrix has the property that all [ ] are probability vectors.

6) The integral $\int_{|x|<\epsilon} 1/|x|^2 \, dx$ over a ball of radius ϵ in $\mathbb{R}^3$ is finite for d [ ].

7) The central limit theorem for random variables X_i taking values 1 or 0 with probability p or $(1-p)$ is called the [ ] theorem.

8) If σ is the standard deviation of a sequence of IID random variables $X_1, \ldots, X_n$, then $\sigma/\sqrt{n}$ is called the [ ].

9) A sequence of Binomial $B(n, 1/n)$ distributed random variables converges in distribution to the [ ] distribution.

10) A normalized random variable has variance [ ].

11) The map $T(X) = (X + X')/\sqrt{2}$ on normalized random variables in $\mathcal{L}^2$, where X' is an independent copy of X with the same distribution, has the unique fixed point [ ].

12) Which distribution in $\mathcal{L}^2$ maximizes differential entropy among all distributions on $\mathbb{R}$ with finite entropy?

13) What is the formula for differential entropy? [ ]

14) What was bigger: the differential entropy of the Cauchy distribution or of the normal distribution? [ ]

15) A measure preserving transformation is weakly mixing if and only if [ ] is ergodic.

16) The linear operator $U_T(X) = X(T^{-1})$ on $\mathcal{L}^2$ is called the [ ] operator.

17) A measure preserving transformation is weakly mixing if and only if the operator $U_T(f) = f(T^{-1})$ has no [ ].

18) T is weakly mixing if and only if U_T has [ ] spectrum.

19) True or false: the transformation $T(x) = x + \sqrt{2}$ on $(\mathbb{T}, \mathcal{B}, dx)$ is mixing. [ ]

20) True or false: the transformation $T(x) = x + \sqrt{2}$ on $(\mathbb{T}, \mathcal{B}, dx)$ is weakly mixing. [ ]

21) True or false: the transformation $T(x) = x + \sqrt{2}$ on $(\mathbb{T}, \mathcal{B}, dx)$ is ergodic. [ ]

22) True or false: the transformation $T(x) = 2x$ on $(\mathbb{T}, \mathcal{B}, dx)$ is mixing. [ ]

23) True or false: the transformation $T(x) = 2x$ on $(\mathbb{T}, \mathcal{B}, dx)$ is weakly mixing. [ ]

24) True or false: the transformation $T(x) = 2x$ on $(\mathbb{T}, \mathcal{B}, dx)$ is ergodic. [ ]

25) Weakly mixing means that $|P[T^{-k}(A) \cap B] - P[A]P[B]|$ converges to zero in a [ ] sense.

26) If a transformation T is ergodic, and $T(A) = A$, then the possible values of P$[A]$ are [ ].

27) True or False: the Birkhoff ergodic theorem implies the strong law of large numbers. [ ]

28) True or False: for every open interval $I = (a, b), a < b$ in $[0, 1)$ any irrational α and almost every $x \in [0, 1]$, there exists n such that $x + n\alpha \mod 1 \in I$. [ ]

29) True or False: there are ergodic billiards. [ ]

30) What method have we used to show that the squaring transformation $z \to z^2$ on $\{|z| = 1\}$ is ergodic? [ ]

31) After which mathematician is the recurrence theorem in ergodic theory named for. [ ]

32) If $A \in \mathcal{A}, \mathrm{P}[A] > 0$ is given then T is ergodic if and only if T_A is ergodic. The transformation $T_A : A \to A$ is called [ ] transformation.

33) Who proved first the maximal ergodic theorem? The first name was Eberhard [ ]

34) The strong law of large numbers assures the convergence of S_n/n of the type [ ].

35) The weak law of large numbers assures the convergence of S_n/n of the type [ ].

36) The central limit theorem assures the convergence of S_n^* of the type [ ].

37) Who proved the strong law of large numbers first in 1930? [ ]

38) We have seen a version of the strong law which only needed pairwise independence. It was named after [ ]. (A mathematician whose name starts with E.)

39) Does the weak law of large numbers need independence? It works under the assumption of pairwise [ ].

40) Which theorem was used to prove the weak law of large numbers? [ ]'s theorem.

41) We have seen 5 notions of convergence. Which one is the weakest? [ ]

42) Is there a relation between L^2 convergence and almost everywhere convergence? Yes or no? [ ]

43) Is there a relation between L^2 convergence and convergence in probability? yes or no? [ ]

44) Is there a relation between complete convergence and L^2 convergence? Yes or no? [ ]

45) Which inequality tells something about $\mathrm{E}[h(X)]$ for a monotone function $h(x)$? [ ] inequality.

46) Which inequality is used in the context of entropy. [ ]

47) The Kullback-Leibler divergence is also known as [ ] entropy.

48) Which inequality tells something about $\mathrm{E}[h(X)]$ for a convex function $h(x)$? [ ] inequality.

49) Fill in either $\subset$ or $\supset$ in $\mathcal{L}^2$ [ ] $\mathcal{L}^4$.

50) Which inequality allows to compare Banach spaces $\mathcal{L}^p$ for $p \in [1, \infty]$? [ ]

51) True or false: if a random variable X is bounded, then $X \in \mathcal{L}^\infty$. [ ]

52) Which theorem allows to show that tail σ-algebras is trivial. [ ]

53) $\sum_{n\in\mathbb{N}} \mathrm{P}[A_n] < \infty \Rightarrow \mathrm{P}[A_\infty] =$ [ ] always holds.

54) The set of automorphisms of a probability space form a [ ]

55) True or False? $\emptyset$ is independent of Ω? [ ]

56) True or False? $\emptyset$ is independent of any other set. [ ]

57) True or False? If A is independent to itself, then $P[A] = 0$ or $\mathrm{P}[A] = 1$. [ ]

58) True or False? If $(\Omega, \{\emptyset, \Omega\}, \mathrm{P})$ is the trivial probability space, then every random variable is constant. [ ]

59) True or False? If $X = c$ and $X = d$ are two random variables and c, d are different constants, then X, Y are independent. [ ]

60) In one of the homework, we looked for a given transformation at the subset of the σ algebra $\mathcal{A}$ that are of the form $A\Delta T(A)$. We called them [ ].

61) You have looked up a theorem of Denjoi-Koksma which assured that a differentiable periodic function $f(x)$ can be written as $f(x) = g(x + \alpha) - g(x)$. There was a property required for the irrational number α. It had to be [ ]

62) If A, B are independent with $\mathrm{P}[A] > 0, \mathrm{P}[B] > 0$, then $\mathrm{P}[A|B] = \mathrm{P}[B|A]$ implies that

"I affirm my awareness of the standards of the Harvard College Honor Code."

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