Midterm Part I (in class October 16, 2025)

Sign your name. You have 30 minutes. Each question is one point. No half points.

1) If $r(u,v)$ parametrizes a surface, then $|r_u \times r_v|^2$ is ⬚ in terms of g.

2) If $r(t)$ parametrizes a curve, then $|r'(t)| = \sqrt{\det(g)}$ is called the ⬚

3) The integral $\int_a^b \sqrt{\det(g)}\, dt$ for a curve $r(t)$ was called ⬚ integral.

4) The curvature of a circle of radius $1/7$ is ⬚

5) The result that the shortest connection between two points in the plane is a line is the ⬚ theorem.

6) The curve with name ⬚ has constant curvature and non-zero constant torsion.

7) The curve $[0, \cos(3t), \sin(3t), 0]$ has curvature ⬚ and torsion ⬚

8) The Frenet frame in 3 dimensions is also called the ⬚ frame.

9) For a curve in $\mathbb{R}^n$, the curvature κ_{n-1} is called ⬚

10) The curve with name ⬚ has infinitely many vertices.

11) The evolute of a curve is $r(t) + N(t)/\kappa(t)$, where $\kappa(t)$ is the ⬚

12) Tennis ball: a curve dividing the erea of S^2 equally has ____ vertices at least.

13) The Euler characteristic of a projective plane is ____

14) The Euler characteristic of a Klein bottle is ____

15) The Euler characteristic of a discrete 2-manifold is defined as ____

16) The constant curvature discrete 2-sphere with 12 vertices is also called a ____

17) We needed the ____ process to produce general Frenet frames.

18) The matrix $I^{-1}II$ is always a symmetric matrix. True or False? ____

19) The mean curvature of the Clifford Klein bottle is ____

20) The Gauss curvature of the round unit sphere is ____

21) Which blind mathematician first looked at discrete curvature? ____

22) If $e_k(t) \cdot e_k(t) = 1$ for all t then $e_k(t)'$ is ____ to $e_k(t)$.

23) $so(5)$ consists of orthogonal matrices. True or False? ____

24) $SU(3)$ is associated to the strong force. It is a special ____ group.

25) It is possible that a matrix entry of $SO(5)$ is 5. True or False? ____

26) It is possible that a matrix entry of $so(5)$ is 5. True or False? ____

27) The fundamental theorem of linear algebra tells $\ker(A)^{\perp} =$ ____

28) If r parametrizes an ellipsoid then $\iint_R |n_u \times n_v| \, dudv =$ ____

29) True or false? If you know I and II, then you know III. ____

30) The curvature of a discrete 2-manifold is given as ____