Final Quiz

1) The first, second and third fundamental form are [_____]) tensors.

2) If $x(t)$ is a curve, then $\dot{x}(t) = dx(t)$ is a [_____] tensor.

3) If $f : M \to \mathbb{R}$ is function, then $df(t)$ is a [_____] tensor.

4) The or false: the first fundamental form is symmetric.

5) True or false: The Einstein equations are $R - \frac{1}{2}Sg = \kappa T$.

6) The Schwarzschild metric describes the gravity of a [_____] hole.

7) The Christoffel symbol Γ_{ijk} is a tensor. True or false?

8) True or False: the Christoffel symbols satisfy $\Gamma_{ijk} = \Gamma_{jik}$.

9) If F is a vector field which has $\operatorname{curl}(F) = 0$, and G is the unit disk $x^2 + y^2 \leq 1, z = 0$ with boundary C then the line integral is $\int_C F\,dr = $ [_____].

10) The shape operator matrix A is is a [_____] tensor.

11) The equations $\ddot{x} = -\sum_{i,j} \Gamma_{ij}^k \dot{x}^i \dot{x}^j$ describe [_____].

12) The Riemann curvature is a [_____] tensor.

13) The covariant derivative maps a vector field to a [_____].

14) Vectors in a Lorentzian manifold of negative length are [_____] like.

15) Vectors in a Lorentzian manifold of positive length are [_____] like.

16) A discrete manifold has the property that every unit sphere is a [_____].

17) The empty graph is a [_____] sphere.

18) The 0-dimensional sphere has [____] vertices and [____] edges.

19) A connected 2-manifold with Euler characteristic 2 must be a [____].

20) True or False? There are 2 manifolds different from the sphere with Euler characteristic 2 [____].

21) The principle that action and length functional have the same critical points is called the [____] principle.

22) It is possible that the projective plane has constant zero curvature. True or false?

23) The Möbius strip has Euler characteristic [____].

24) A torus has Euler characteristic [____].

25) The Klein bottle has Euler characteristic [____].

26) The Euler characteristic of a 2-manifold in terms of V, E, F is given by the formula [____].

27) Recursively, a graph is contractible if $S(v)$ and $G-v$ are both [____].

28) The Euler characteristic of a d-sphere is [____].

29) If G is a discrete m manifold and $f : V(G) \to \{0, \ldots, k\}$ is a map, then M_f is a [____] manifold or [____].

30) The discrete Gauss-Bonnet tells that $\chi(G)$ is [____] if $K(v) = 1 - \frac{f_0(S(v))}{2} + \frac{f_1(S(v)))}{3} + \ldots$.

31) A **Riemannian manifold** (M, g) is a smooth manifold M with a positive definite symmetric [____] tensor field g.

32) A non-critical point is called a [____] point.

33) True or false: the Ricci tensor is symmetric: [____].

34) True or false: the Ricci tensor is positive definite: [____].

35) If $f : M \to \mathbb{R}^k$ is smooth and M a m-manifold, and if $\text{rank}(df(x)) < m$ then x is called a [____].

36) For a compact 2-manifold, $\iint_M K dV = $ _____ .

37) A key in the proof of the local Gauss-Bonnet theorem is _____ 's theorem.

38) If M is $x^2/9 + y^2/4 + z^2/16 = 1$, then $\iint_M K \, dV = $ _____ .

39) A 3-dimensional manifold has Euler characteristic _____ .

40) The Euler's gem formula is _____ .

41) The geodesic curvature of a geodesic is _____ .

42) True or False: The geodesic curvature is always non-negative _____ .

43) True or false: there are infinitely many closed periodic geodesics for a torus _____ .

44) The half sphere $x^2 + y^2 + z^2 = 1, z \geq 0$ has a boundary curve for which the geodesic curvature is _____ .

45) The normal curvature κ_n the geodesic curvature κ_g and the curvature κ satisfy the equation _____ .

46) The fact that geodesics have zero geodesic curvature is called _____ ladder.

47) The normal curvature of a grand circle in a sphere of radius 3 is _____ .

48) The geodesic curvature of a grand circle in a sphere of radius 3 is _____ .

49) In a triangle on a sphere the sum of the angles is _____ than π.

50) In a triangle on a pseudo sphere, the sum of the angles is _____ than π.

51) Lambert's theorem deals with the sum of the angles of a triangle on a _____ .

52) The theorema _____ tells that K only depends on g.

53) A 1-form is a _____ tensor.

Differential Geometry

54) The Euler handshake lemma tells that the vertex degrees d_i of a graph satisfy
$\sum_{i=1}^{V} d_i =$ _____ .

55) The sum of the angular defects $K(p)$ of a convex polyhedron is equal to _____ .

56) The fact that the exponential map exists for all times is called the _____ theorem.

57) On a Clifford torus, the geodesics are _____ lines.

58) The method of evolving geodesics by gluing them to the manifold.

59) Geodesics extremize the _____ functional.

60) The Einstein equations come from extremizing the _____ functional.

61) The Einstein equations tell that the Ricci tensor is _____ to the metric tensor.

62) For 2-manifolds, the relation between scalar curvature S and curvature K is _____ .

63) True or False: in a discrete 2-manifold the vertices and edges and faces satisfy $2E = 3F$.

64) There are manifolds with different differential structures _____ .

65) The mathematician who solved the Poincaré conjecture is called _____ .

66) If a curve $x(t)$ is contained in the plane $3x + 5y + 2z = 1$, then the normal curvature is _____ .