ENTRY MEASURE THEORY

[ENTRY MEASURE THEORY] Authors: Oliver Knill: 2003 Literature: measure theory

analytic set

An [analytic set] in a complete seperable metric space is the continuous image of a Borel set. Also called A-set. Any A-set is Lebesgue measurable. Any uncountable A-set topologically contains a perfect Cantor set. Suslins criterium tells that an analytic set is a Borel set if and only if its complement is also an analytic set.

atom

An [atom] is a measurable set Y of positive measure in a measure space such that every subset Z of Y has either zero or the same measure. Often an atom consists of only one point. More generally, an atom is minimal, non-zero element in a Boolean algebra.

atom

A property which holds up to a set of measure zero is said to hold [almost everywhere] (= almost surely).

Banach-Tarski theorem

The [Banach-Tarski theorem]: a ball in Euclidean space of dimension 3 can be decomposed into finitely many sets and rearranged by rigid motion to obtain two balls. The

barycentre

The [barycentre] of a Lebesgue measurable set S in an Euclidean space is the point $\int_S x dx$.

Boolean algebra

A [Boolean algebra] is a set S with two binary operations + and * which are commutative monoids $(S, +, 0)$, $(S, *, 1)$ and satisfy the two distributive laws $(x * (y + z) = x * y + y * z, x + (y * z) = (x + y) * (x + z)$ as well as the complementary laws $x * x = 1, y + y = 0$. A Boolean algebra is especially an algebra. Examples are the algebra of classes, where $+$ is the union and $*$ is the intersection or the algebra of propositions, for which $+$ is *and* and $*$ is *or*.

Boolean ring

A [Boolean ring] is a ring in which every member is idempotent.

Borel-Cantelli lemma

The [Borel-Cantelli lemma]: if Y_n are events in a probability space and the sum of their probabilities is finite, then the probability that infinitely many events occur is zero. If the events are independent and the sum of their probabilities is infinite, then the probability that infinitely many events occur is one.

Borel measure

A [Borel measure] is a measure on the sigma-algebra of Borel sets.

Borel set

A [Borel set] (=Borel measurable set) in a topological space is an element in the smallest sigma-algebra which contains all compact sets. Borel sets are also called B-sets. One can say that a B-set is a set which can be obtained of not more than a countable number of operations of union and intersection of closed open sets in a topological space. Borel sets are special cases of analytic sets.

Borel set

The smallest sigma-algebra A of subsets of a topological space (X,O) containing O is called a Borel sigma-algebra.

absolutely continuous

A measure μ is [absolutely continuous] to a measure ν if $\nu(Y) = 0$ implies $\mu(Y) = 0$.

centre of mass

The [centre of mass] (=barycentre) of a Borel measure μ in a Euclidean space X is the point $\overline{x} = \int_X x\mu(x)$. For example, if μ is supported on finitely many points x_i and $m_i = \mu(x_i)$ then $\overline{x} = \sum_i m_i x_i$. If μ is the mass distribution of a body, then its centre of mass is called the centre of gravity.

abstract integral

[abstract integral]. Denote by L, L^+ the set of measureable maps from a measure space (X, A, μ) to the real line (R, B), where B is the Borel sigma-algebra on R, R^+. For $f \in S = \{f = \sum_{i=1}^n \alpha_i \cdot 1_{A_i} \ \alpha_i \in R\ \}$, define $\int_X f\ d\mu := \sum_{a \in f(X)} a \cdot \mu\{X = a\}$. For $f \in L^+$ define $\int_X f\ d\mu = \sup_{g \in S} \int_X g\ d\mu$. For $f \in L$ finally define $\int f = \int f^+ - \int f^-$, where $f^+(x) = \max(f(x), 0)$ and $f^-(x) = -(-f)^+(x)$.

abstract integral

A sigma-additive function $\mu : A \to [0, \infty]$ on a measurable space (X,A) is called a [measure]. It is called a finite measure if $\mu(X) < \infty$.

measure

A map $f : X \to Y$ where (X, A), (Y, B) are measurable spaces is called [measurable] if $f^{-1}(B) \in A$ for all $B \in B$.

measurable space

The pair (X, A) is called a [measurable space] if

measure space

(X, A, μ) is called a [measure space] if (X, A) is a measurable space and μ is a measure on (X, A).

measure space

[sigma-additive: a real-valued function on a set A of subsets of X is called sigma-additive if for all disjoint $Y_n \in A$, one has $\mu(\bigcup_n Y_n) = \sum_n \mu(Y_n)$.

sigma-algebra

A set A of subsets of a set X is called a [sigma-algebra] if

- X is in A
- $Y \in A$ implies $Y^c \in A$. (iii) $Y_n \in A, n = 1, 2, 3, ...$ implies $\bigcup_{n=1}^{\infty} Y_n \in A$.

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