Puzzle 3: Miller puzzle

Problem: Find a solution to the system or equations

$$a_1(10a_2 + a_3) = 100a_4 + 10a_5 + a_6) = 100a_4 + 10a_5 + a_6 = (10a_7 + a_8) * a_9$$

where $(a_1, \ldots, a_9)$ is a permutation of the 9 numbers $1, \ldots, 9$. In short $a_1 * a_2 a_3 = a_4 a_5 a_6 = a_7 a_8 * a_9$.

Solution: Brute force search shows that there are exactly 4 solutions:
$\vec{a} = (2, 7, 8, 1, 5, 6, 3, 9, 4)$
$\vec{a} = (3, 5, 8, 1, 7, 4, 2, 9, 6)$
$\vec{a} = (4, 3, 9, 1, 5, 6, 7, 8, 2)$
$\vec{a} = (6, 2, 9, 1, 7, 4, 5, 8, 3)$

Subject: arithmetic, addition and multiplication