# Puzzle 2: Good Will Hunting problems

Problem: Find the generating function for the number of paths going from a node x to a node y in a graph.

Solution: If A is the adjacency matrix of the graph, where A_{ij} tells how many connections there are from i to j, then the number of paths going from x to y in n steps is $A^n x, y$. The generating function is $\sum_n A_{ij}^n x^n$ which is $(1 - xA)_{ij}^{-1}$. Using the Cramer formula, one can write this as an explicit rational function $(-1)^{i+j} \det([1 - xA_{ji}])/\det(1 - xA)$.
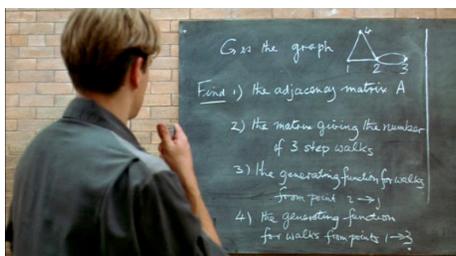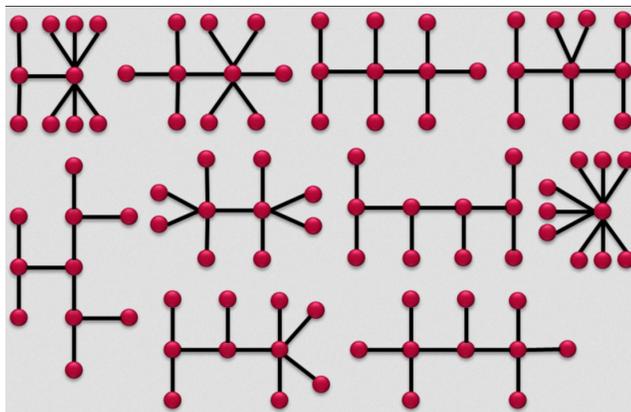

Subject: linear algebra, graph theory

Problem: Draw all connected trees having 10 vertices, such that no vertex has degree 2.

Solution:

Source: In the movie "Good Will Hunting", the main character Will Hunting (Matt Damon) solves this as a blackboard challenge problem for a concrete graph.