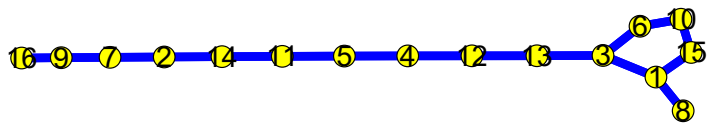
Puzzle 1: Neighboring square numbers

Problem: Write down the numbers 1-16 in a row so that the sum of two arbitrary neighbors is a square number.

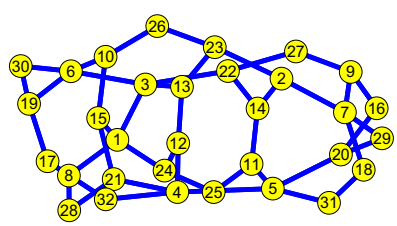
Solution: The number 16 must be at an end. Only 9 can be a neighbor. Now everything is forced. We get a solution to a Hamiltonian path problem in a square graph 16, 9, 7, 2, 14, 11, 5, 4, 12, 13, 3, 6, 10, 15, 1, 8



Subject: Arithmetic, square numbers, graph theory

Problem: Write down the numbers 1-32 in a circle so that the sum of two arbitrary neighbors is a square number.

Solution: This asks for a Hamiltonian cycle in a graph. 32 appears is the smallest number for which one can solve the problem. To investigate this, we can for every n write down a graph with vertex set $\{1, \ldots, n\}$ such that (a, b) are an edge if $a + b$ is a square. Lets call them the **square number graphs** G_n. Now G_{32} is the smallest of these graphs which is a **Hamiltonian graph**.



Source: Original question 1: Anna Beliakova, Uni Zürich, und Dmitrij Nikolenkov, High school Trogen, They also ask for the cyclic problem with 16 numbers, where no solution exists.