I. INTRODUCTION

Since 1958 measurements with particle and magnetic field detectors on spacecraft have revealed many of the details of the near-earth space environment. A simplified schematic diagram of the basic structural configuration is shown in Figure 1.

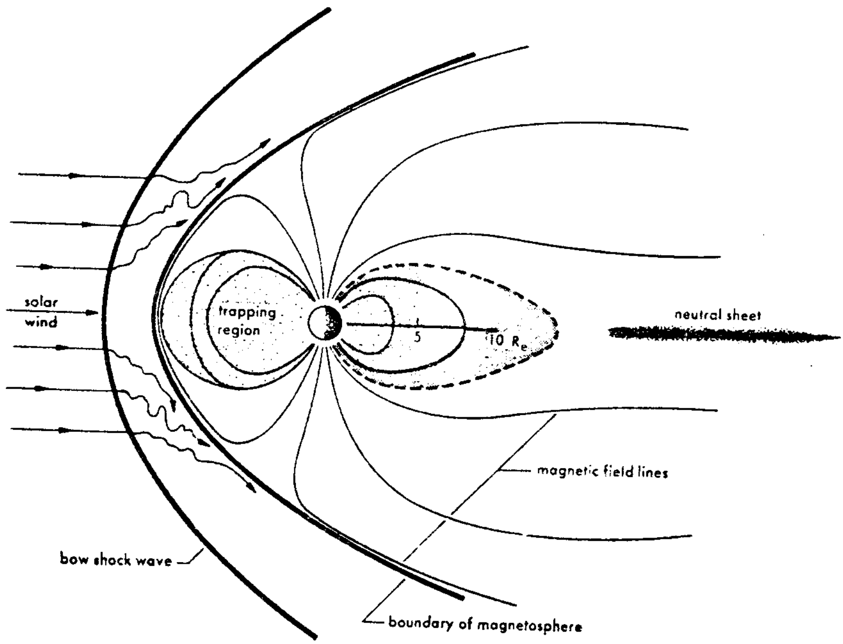

FIGURE 1 Principal features of the earth's magnetosphere.

The solar wind is a rarefied neutral plasma consisting chiefly of protons, helium nuclei and electrons. The wind originates in the solar atmosphere and streams toward the earth at an average speed of about 400 km sec^{-1}. The geomagnetic field represents a fixed obstacle, and the solar wind is diverted around it, confining the geomagnetic field to a cavity. The cavity is known as the magnetosphere, and the surface between the magnetosphere and the solar wind is known as the magnetopause.

Since the wind impinges at supersonic speed, a standing shock surface is formed on the sunward side of the earth. Within the shock the plasma is partially thermalized, allowing it to flow around the subsolar portion of the magnetopause at subsonic speed. As it flows along the flanks of the magnetosphere it speeds up again and drags the polar connecting field lines far downstream, creating the magnetotail. Between the northern and southern lobes of the tail lies a sheet of hot plasma. A so-called neutral sheet of low magnetic field lies within the plasma sheet, separating the tail lobes. While the structural features pictured in Figure 1 are supported by much experimental evidence, there still remains considerable controversy over the details of the processes leading to the observed configuration.

Deep within the magnetosphere, in the regions where the field does not drastically depart from that of a dipole, lies the Van Allen radiation belt. The belt consists of energetic charged particles—electrons, protons, alphas, and a very small fraction of heavier ions—spiraling from north to south and back along the field lines in quasi-periodic orbits. The outer zone, somewhat arbitrarily defined as that portion of the radiation belt lying beyond 2·5 earth-radii on the geomagnetic equator, is subject to extreme fluctuations in intensity associated with major geomagnetic storms which occur several times each year, and to lesser fluctuations associated with magnetospheric substorms, which usually occur several times each day. The inner zone, with which this paper is concerned, shows fewer and smaller variations. The largest changes which have been observed were the result of a nuclear bomb detonation, not to natural phenomena.

The proper identification of the physical mechanisms responsible for the existence and maintenance of the earth's radiation belt has proved frustratingly difficult. An optimistic argument can be constructed to demonstrate that the problem is basically a simple one. Consider some particular location within the radiation belt and some single particle species. Of all the source mechanisms that one can imagine, the odds are that one is so much more important than all the rest that the others may be ignored. If there is a second source whose strength is within an order of magnitude of the first, it is an unlikely coincidence; and surely there will not be a third. Similarly, the odds are that there will be only one important loss mechanism. This argument has been found by investigation to be substantially correct. Why then is our understanding of the radiation belt so meager? Why are we so often limited to simple qualitative descriptions?

There are two important complications. First, the strength of a source or loss mechanism may vary by orders of magnitude between two locations only a few thousand (sometimes only a few hundred) kilometers apart. Second, the two locations are usually connected by a physical mechanism which transports particles from one of the locations to the other. Furthermore, the transport rates are also strong functions of position, and there may be simultaneous transport in energy in addition to spatial transport. Therefore, a complete description of the particle intensity and its time variation at some location usually depends on a detailed knowledge of several mechanisms over a large region of space. Progress in understanding the important processes has therefore been slow.

The conceptual division of the radiation belt into an inner zone and an outer zone is based in part on the nonspecific response of the early particle detectors. These detectors responded to high-energy protons at low L values and to high-energy electrons at high L values, with an apparent minimum between. (Throughout this paper L refers to the geomagnetic parameter originated by McIlwain (1961) to indicate a position in the earth's magnetic field. The L value of a magnetic field line is approximately equal to the dimensionless ratio of the distance from the dipole to the equatorial crossing of the line in question divided by the radius of the earth.) Subsequent investigations have shown that a minimum often does exist in the electron intensity between $L = 2$ and $L = 3$; it is sometimes called the electron "slot". The proton distribution has no "slot", but the average proton energy decreases continuously with increasing L. The term "inner zone" as used in this paper refers to the region below $L \approx 2{\cdot}5$. The physical mechanisms to be discussed are those which appear to be important for electrons above about 20 keV and protons above a few MeV. This choice, while arbitrary, reflects the emphasis of published research and the rather poorly known intensity and behavior of those particles having energies between these values and the much lower thermal energy of the residual atmosphere.

The inner radiation zone is characterized by its stability. The time scale for such changes as do occur are on the order of weeks, months or years. The geomagnetic field in this region is relatively strong and little distorted by magnetospheric current systems. Time dependent perturbations of the field are therefore very weak. These conditions have made possible a more rapid advance in the theoretical description of the inner zone than has been possible for the more dynamic outer zone. The injection of electrons by nuclear explosions in the inner zone has also proved to be a valuable source of information from which many insights into the trapped electron behavior have been obtained.

These advantages have brought the study of the inner zone radiation from simple qualitative descriptions to a fairly sophisticated, but still incomplete, theory in which the effect of a substantial number of physical mechanisms on the trapped particles can be described quantitatively. It is the purpose of the important processes has therefore been slow.

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This paper is neither a review nor a presentation of new research results. It rests in large measure on the published work of others, often rewritten here with the benefit of hindsight to simplify the presentation, to emphasize what is currently thought to be important, and to standardize notation and terminology. The brief historical reviews are limited to one specific aspect of the important processes has therefore been slow.

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The reader is expected to be familiar with the basic aspects of the adiabatic theory of charged particle motion in a magnetic field. For convenience, a brief summary of the knowledge required is given in the following section.

II. TRAPPED PARTICLE MOTION IN THE EARTH'S MAGNETIC FIELD

The motion of an energetic charged particle in a static magnetic field **B** is described by

$$m\frac{d\mathbf{w}}{dt} = q(\mathbf{w} \times \mathbf{B}) \tag{2-1}$$

where m, q, and **w** are the mass, charge and velocity of the particle. If **B** is uniform the solution to Eq. (2-1) is a helix; the velocity parallel to **B** is unaffected by the field, and the motion perpendicular to **B** is a circle with radius of gyration

$$\text{radius} = \frac{mw_\perp}{qB} \tag{2-2}$$

In a nonuniform electric and magnetic field, if the gyration radius of the particle is much less than the scale size of the fields and if the electric field magnitude is small, the particle trajectory will be very nearly a helix. However, the small deviations from helical motion are crucial and lead directly to the ability of a dipole-like field to trap charged particles. Under these conditions the trajectory can be approximated as a circular motion about a moving guiding center, and for most purposes only the location of the guiding center is of interest. The equations of motion of the guiding center can be decomposed into components perpendicular and parallel to **B** (Alfvén and Fälthammar, 1963)

$$\mathbf{v}_\perp = c\frac{\mathbf{E}\times\mathbf{B}}{B^2} + \frac{mc}{qB^3}\left(\frac{w_\perp^2}{2} + w_\parallel^2\right)\mathbf{B}\times\nabla B \tag{2-3}$$

$$\frac{dv_\parallel}{dt} = -\frac{w_\perp^2}{2B}\frac{\partial B}{\partial s} + qE_\parallel \tag{2-4}$$

where $v_\perp$ and $v_\parallel$ are the components of the guiding center velocity perpendicular and parallel to **B** and $\partial B/\partial s$ is the derivative of the magnetic field with respect to distance along a magnetic field line. From Eqs. (2-3) and (2-4) the general characteristics of particle motion are immediately apparent. Neglecting the electric field terms it is clear from Eq. (2-4) that the guiding center will be repelled from a region of increasing B. In the geomagnetic field this force leads to the reflection of particles as they move along field lines toward higher latitudes. From Eq. (2-3) we see that both the perpendicular and parallel velocity components contribute to $\mathbf{v}_\perp$, the guiding center drift. In a pure dipole field $\mathbf{B} \times \nabla B$ will be in the azimuthal direction. In the geomagnetic field $\mathbf{B} \times \nabla B$ will be largely azimuthal also, but the magnetic irregularities cause some deviation from purely azimuthal drift. Thus, the overall motion of a charged particle in the geomagnetic field will be a superposition of a gyration about a field line, a bounce motion between the northern and southern hemispheres, and an eastward or westward drift in longitude dependent on the sign of q. Positive particles move westward while for negative particles $\mathbf{v}_\perp$ is eastwards.

Except for special geometries it is not possible to integrate Eqs. (2-3) and (2-4) to derive the long-term motion of trapped particles. However, these guiding center equations are used to calculate instantaneous particle velocities and to derive the effects of perturbing fields on trapped particles. To understand the long term motion one must resort to a perturbation theory describing the motion in an almost uniform, almost stationary field. This theory is developed in detail elsewhere (Northrop, 1963; Northrop and Teller, 1960) and only the results will be given here. As described above the trajectory of a trapped particle in the geomagnetic field can be conveniently decomposed into three almost periodic motions, a gyration about magnetic field lines, a bounce along field lines between northern and southern hemispheres, and a drift in longitude. (As will be seen later the longitudinal drift path closes on itself so that the overall trajectory of the guiding center is a closed magnetic shell whose elements are field lines.) Each of these motions has associated with it an adiabatic invariant, a quantity which is conserved throughout the particle trajectory. The adiabatic invariants are not true constants of motion; each is constant only under the conditions that (a) time changes in the fields are small during the period associated with that motion and (b) spatial variations are small between successive cycles. Table I gives the definitions of the adiabatic invariants and the approximate periods required for each component of the motion. The periods given here are typical of particles in the inner radiation belt but depend on particle type, energy, equatorial pitch angle, and location. The important point to note is the large ratio between the periods associated with each invariant, a fact which makes practical the decomposition of motion into gyration, bounce, and drift.

TABLE I

Motion	Adiabatic invariant	Typical periods for 1 MeV protons	1 MeV electrons
Gyration about field line	$\mu = p_\perp{}^2/2Bm_0$	2×10^{-2} sec	10^{-4} sec
Bounce between hemispheres	$J = \oint p_\parallel \, ds$	3·6 sec	0·2 sec
Longitudinal drift	$\Phi = \oint \mathbf{B} \cdot d\mathbf{A}$	0·5 h	0·5h

In the definitions of the invariants $p_\perp$ and $p_\parallel$ are the momentum components perpendicular and parallel to the magnetic field, and m_0 is the particle rest mass. The integral for J is taken along the magnetic field over a complete bounce, and the integral for Φ is over the area enclosed by the drift path of the particle.

Because the drift path of a geomagnetically trapped particle surrounds the earth and the magnetic dipole, the enclosed magnetic flux is equal to the northward directed flux of the field minus the southward return flux passing through the dipole position. This net flux is also equal to the northward flux outside the drift shell, and it is generally more convenient to calculate Φ by integrating B over the equatorial plane outside the particle drift shell.

The quantities μ, J and Φ are frequently called the first, second, and third adiabatic invariants. Note that μ differs from the classical magnetic moment for relativistic particles due to the fact that the rest mass, rather than the total mass, appears in the denominator. The conservation of the first invariant, μ, describes the change in pitch angle, or $p_\perp$, accompanying changes in B as the particle moves along the field line. As the particle moves toward either pole, B increases causing $p_\perp$ to increase. If $\mathbf{E} = 0$ the total momentum p cannot change, so an increase in $p_\perp$ implies that the pitch angle has increased. When $p_\perp = |\mathbf{p}|$ the particle will be moving at right angles to B and will be reflected. The value of B at which the particle mirrors is obtained immediately as

$$B_m = \frac{p^2}{2m_0\mu} \tag{2-5}$$

The second adiabatic invariant J specifies the trace of a particle during its longitudinal drift. At each longitude, the particle will mirror at a field value B_m, and it will be located on that field line for which the integral J is equal to the required value. The constancy of J and μ requires that the particle guiding center returns to the initial field line after drifting completely around the earth, thus closing the magnetic shell.

experimental data to determine the exact role of the IMF in geophysical phenomena it appears likely that it will be a crucial one.

When the solar wind encounters an obstacle such as the earth the resulting interaction depends on the characteristics of the object. In the case of the earth an upstream shock, the Bow Shock, is formed.

3. Bow shock

Various theoretical arguments have been presented to justify the assumption that, for certain calculations, the solar wind may be treated as a continuum fluid (e.g. Levy et al. 1964). With this assumption and the knowledge that the solar wind gas is in supersonic flow, the formation of a collision free standing shock upstream of the magnetosphere was predicted theoretically and later observed experimentally. The standoff distance for this shock shown in Figure 4 is of the order of several R_e.

In the vicinity of the shock the solar wind flow changes from supersonic to subsonic, the particle density and temperature and the magnetic field strength increase. The shock therefore converts most of the solar wind kinetic energy, which was in the form of bulk motion outside the shock, to random thermal motion inside.

4. Magnetosheath

The region between the shock and the compressed geomagnetic field, which is populated by the shocked solar wind plasma, is called the magnetosheath. Near the nose of the magnetosphere, magnetosheath ions and electrons have typical energies of 500 eV and 100 eV respectively and the bulk flow velocities are low compared to the random (thermal) motion. As the plasma flows from the nose the random motion is converted into bulk motion and the plasma once again passes into supersonic flow. The position of the transition region from subsonic to supersonic flow, shown in Figure 4 as the "sonic surface", is not well known but is expected to occur near the region indicated.

The earthward boundary of the magnetosheath or equivalently the outer boundary of the magnetosphere is called the magnetopause (see Figure 4).

5. Magnetopause

The measured position and shape of the magnetopause are in reasonable agreement with theoretical predictions based on a continuum gas-dynamics approach (Spreiter and Alksne, 1969). In this approximation the magnetopause is defined as the surface where the magnetosheath particle pressure P is balanced by the geomagnetic field pressure $B^2/8\pi$ (in c.g.s. units), B being the magnetic field strength on the earthward side of the magnetopause. This approximation is justified since the magnetic field pressure in the magnetosheath and the particle pressure inside the magnetopause are usually small compared to the above terms.

Although this approach is generally successful in predicting the position it does not yield any information on the detailed structure of the magnetopause. For example it implies that there is no interconnection between the interplanetary and geomagnetic fields which, as mentioned previously, is probably of prime importance. The determination of the small scale magnetopause structure requires a much more sophisticated approach to the problem. A reasonably complete review of the present theoretical understanding of the magnetopause was presented by Willis (1972).

6. The plasma sheet and neutral sheet

The interaction of the solar wind with the geomagnetic field causes the magnetosphere to be distorted into a comet like shape with an extended downstream tail (Figure 4). The core of the geomagnetic tail is populated by an energetic plasma, the Plasma Sheet, with typical energies of the order of keV's. This plasma is observed near the equatorial plane and extends typically from $\sim 8\ R_e$ to large distances in the midnight sector and from $\sim 5\ R_e$ to the magnetopause in the noon sector (Vasyliunas, 1968). It will be shown in the following that the particles in the plasma sheet are similar to auroral primaries and are probably the auroral particle source.

The low magnetic field region in the tail which separates the oppositely directed magnetic field lines is referred to as the neutral sheet and is the region in which the cross-tail currents are concentrated producing the extended tail configuration.

Earthward of and overlapping the inner portion of the plasma sheet, the geomagnetic field is relatively undistorted and stable. It is in this region that the Van Allen radiation belts are found.

7. The radiation belts

The radiation belts are formed by very energetic particles, typically tens to hundreds of keV and higher, which are trapped in the dipole field. Most of these particles are thought to have diffused radially inward from the plasma sheet and to have been energized in the process. Another possible source for these particles is the so-called CRAND (Cosmic Ray Albedo Neutron Decay) process which might supply a significant fraction of the very energetic inner radiation belt particles.

Since this region of the magnetosphere is reasonably well shielded from solar wind induced perturbations, trapped particles have long lifetimes. Some inner radiation belt particles have lifetimes of the order of many years. A convenient review of radiation belt characteristics is given by Roederer (1970).

A brief summary of particle motions in a magnetic field is given below.

If a particle is injected into a static uniform magnetic field ($\bar{B}$) at an angle α (pitch angle) to the field it will execute a helical motion with a radius of curvature of gyroradius ρ given by

$$\rho = \frac{P \sin \alpha}{Bq} = \frac{P_\perp}{Bq}$$

where m and q are the mass and charge of the particle and p is the momentum. The angular frequency of the motion ω_c, or the cyclotron angular frequency is

$$\omega_c = \frac{Bq}{m}.$$

The magnetic moment μ, sometimes referred to as the first adiabatic invariant, defined as $\mu = P_\perp^2/2mB$ is obviously a constant of motion under these conditions. It can be shown that μ is also conserved in a time varying non-uniform field provided spatial variations in B are small over one gyro-radius ρ and temporal fluctuations are slow compared to ω_c.

Assuming the μ is conserved and that no external forces act on the particle, meaning that the kinetic energy is constant, a particle's velocity parallel to $\bar{B}$ ($v_\parallel$) is given by

$$v_\parallel = v\left[1 - \frac{B_f \sin^2\alpha_i}{B_i}\right]^{\frac{1}{2}}$$

where i and f refer to the initial and final values of the parameters. Therefore as a particle moves towards a region of increasing B, $v_\parallel$ decreases and eventually the particle will reverse its motion at a value of B given by B_m (the value of B at the mirror point) where

$$B_m = \frac{B_i}{\sin^2\alpha_i}.$$

Now consider charged particle motions in the earth's geomagnetic field which is approximately dipolar in the region occupied by the radiation belts (Figure 7). As the particle approaches the earth it will encounter an increasing B and will eventually mirror at a point M. It will then travel back

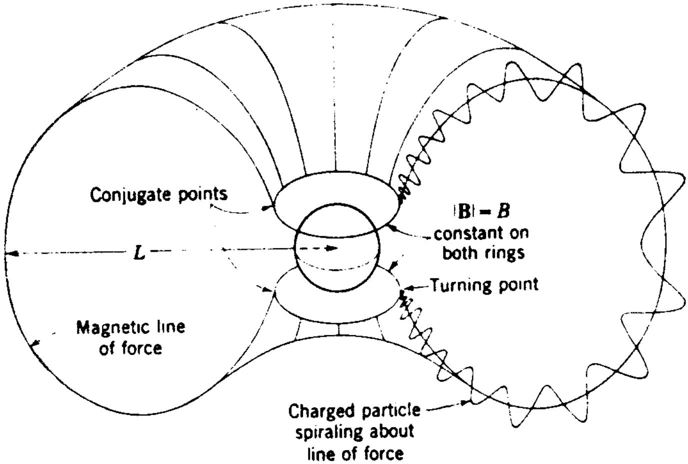

FIGURE 7 Motion of a charged particle in the geomagnetic field (Space Science, C. A. Londquist ed., 1966, McGraw-Hill, N. Y.).

through the equatorial plane along approximately the same field line and mirror at the conjugate point M^*. Thus the particle will bounce between mirror points, the time required to complete one cycle being the bounce period (T_b).

If a particle is injected into the radiation belts such that one of its mirror points is below the atmosphere, usually assumed to be 100 km above the earth's surface, the particle will be lost from the trapping region when it strikes the atmosphere. The pitch angle associated with a mirror point at 100 km is therefore called the loss cone angle (α_{LC}) and particles with angles less than α_{LC} are considered to be in the loss cone.

The quantity J associated with this bounce motion and defined as

$$J = \int_{M}^{M^*} p_{\|} \, ds$$

where the integral is taken along the magnetic field lines between mirror points (M and M^*), is also an invariant of motion as long as the magnetic field changes are small over one bounce period. This quantity, the second adiabatic invariant, is related to the more intuitive quantity $I=J/P$ which has units of length and a value approximately equal to the distance between mirror points measured along the field line.

If there are gradients in the magnetic field, such as in the geomagnetic field, first order drifts in the particle guiding centres will result (see e.g. Roederer). For radiation belt particles the dominant drift is caused by the

$$J = 2\int_{s_m}^{s'_m} \{2m[W - U(s) - M\,B(s)]\}^{\frac{1}{2}}\,ds = \text{const}. \tag{2.48}$$

The mirror points s_m, s'_m have to be determined as a solution of (2.17). *Parallel electric fields can thus have a profound effect on particle shell geometry.* The case of a time-dependent magnetic field will be discussed in Chapter III.

II.6 Application I: Particle Drifts in the Dipole Field

It is important to study in detail the motion of a particle* injected into a *dipolar magnetic field*, because this latter represents the zeroth approximation to the real geomagnetic field. We shall use the geomagnetic East longitude φ, the geomagnetic latitude λ and the radial distance r as coordinates. $\boldsymbol{u}_\varphi$, $\boldsymbol{u}_\lambda$, $\boldsymbol{u}_r$ are the corresponding unit vectors (Fig. 39). The

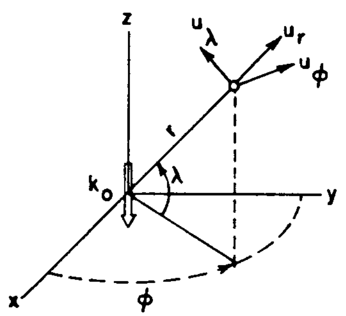

Fig. 39

magnetic dipole moment k_0 of the earth is assumed to be located at its center, directed southward. Its value is $k_0 = 8.02 \times 10^{15}$ Weber $\cdot\, m = 0.311$ Gauss R_e^3.

The magnetic field of a dipole is, in vector form:

$$\boldsymbol{B} = \frac{1}{r^3}\left[3(\boldsymbol{k}_0 \cdot \boldsymbol{u}_r)\boldsymbol{u}_r - \boldsymbol{k}_0\right].$$

In polar coordinates:

$$\begin{aligned} B &= -\frac{2k_0}{r^3}\sin\lambda, \\ B_\lambda &= \frac{k_0}{r^3}\cos\lambda, \\ B_\varphi &= 0. \end{aligned} \tag{2.49}$$

* In most of what follows, *when we say "particle" we really mean guiding center!*

The differential equations of a field line are given by:

$$\frac{r\,d\lambda}{B_\lambda}=\frac{dr}{B_r}\,; \quad d\varphi=0$$

which, integrated for (2.49), give (Fig. 40)

$$r=r_0\cos^2\lambda \qquad (2.50)$$
$$\varphi=\varphi_0=\text{const}\,.$$

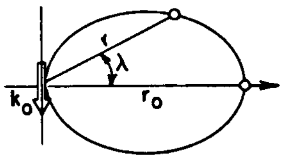

Fig. 40

The element of field line arc length ds follows from (2.50)

(2.51) $$ds=(dr^2+r^2\,d\lambda^2)^{\frac{1}{2}}=r_0\cos\lambda(4-3\cos^2\lambda)^{\frac{1}{2}}\,d\lambda\,.$$

Field lines lie in meridian planes and each one is completely specified by the parameter r_0 (the distance to its equatorial point) and the longitude φ. In radiation belt physics *a dimensionless parameter* is introduced, designated by the letter L:

(2.52) $$L=\frac{r_0}{R_e} \qquad (R_e=6.371\ \text{Km}).$$

The intersection of a field line of parameter L with the earth's surface occurs at a latitude λ_e given by:

(2.53) $$\cos^2\lambda_e=\frac{R_e}{r_0}=\frac{1}{L}\,.$$

The B-value as a function of latitude along a given field line is:

(2.54) $$B(\lambda)=\frac{k_0}{r_0^3}\,\frac{[4-3\cos^2\lambda]^{\frac{1}{2}}}{\cos^6\lambda}=B_0\,\frac{[4-3\cos^2\lambda]^{\frac{1}{2}}}{\cos^6\lambda}$$

where

(2.55) $$B_0=\frac{k_0}{r_0^3}=\frac{0.311}{L^3} \qquad \text{(Gauss)}$$

is the field intensity at the equatorial point. The radius of curvature of a given field line at a point of latitude λ turns out ot be

(2.56) $$R_c(\lambda)=\frac{r_0}{3}\cos\lambda\,\frac{[4-3\cos^2\lambda]^{\frac{3}{2}}}{[2-\cos^2\lambda]}\,.$$

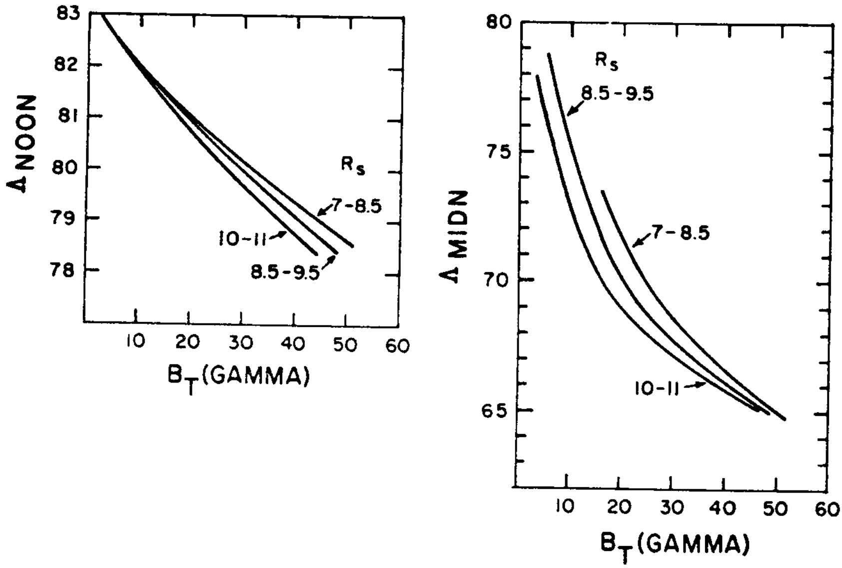

Fig. 46. Latitude of the earth intersection of the noon and midnight limit of "closed" field lines [12] as a function of the tail field intensity B_T and for different intervals of stand-off distance R_s, for the Mead-Williams model

Take a particle that starts at a given longitude φ, circling around a given field line and mirroring at a value B_m. The integral (2.38) computed along the field line between the two mirror points has a value I. This means that when drifting through any other longitude, for example 180° away, this particle will bounce along a field line that passes through the intersection of the corresponding I=const* and B_m=const surfaces

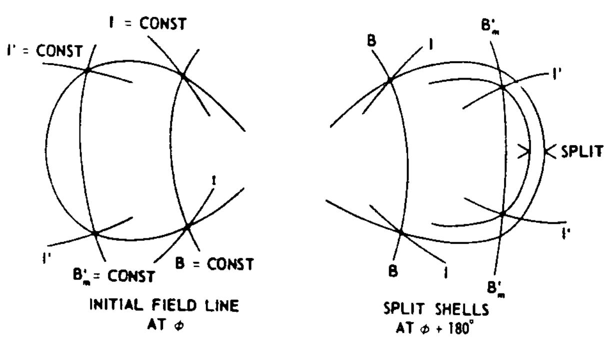

Fig. 47

* We define an I=const surface as the locus of all mirror points of particles whose integral (2.38) has a given value I.

(Fig. 47). Now take a particle which starts on the *same* initial field line but mirrors at a lower value $B'_m < B_m$. Its integral I' will also be smaller, $I' < I$. After a 180° longitudinal drift this second particle will be travelling along a field line that passes through the intersection of the surfaces $I' = \text{const}$ and $B'_m = \text{const}$. *Only* in case of perfect azimuthal symmetry (as in the pure dipole) will these surfaces intersect *exactly* on the same line as that of the first particle, and thus be coincident. This is called *shell degeneracy* [*13*]. In the general case, particles starting on the same field line at a given longitude will populate different shells, according to their initial mirror point fields or, which is equivalent, according to their initial equatorial pitch angles. (Of course, all these different shells would be tangent to each other at the initial field line.) This effect is called *shell splitting* [*13*].

The quiet time Mead-Williams model has been used [*14*] to compute magnetic shells (see Appendix III) of particles initially mirroring on a common field line and having equatorial pitch angles with cosines μ_0 equal to 0.2, 0.4, 0.6, 0.8, and nearly 1 (mirroring close to the earth's surface). Fig. 48 shows how particles, starting on a common line in the *noon*

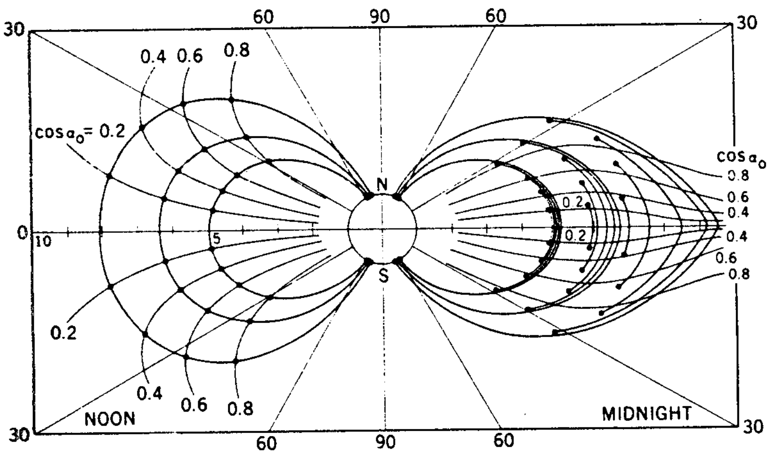

Fig. 48. Computed shell splitting for particles starting on common field lines in the noon meridian [*14*]. Dots represent particles' mirror points. Curves giving the position of mirror points for constant equatorial pitch angle α_0 are shown

meridian, do indeed drift on different shells which intersect the midnight meridian along different field lines. The dots represent particles' mirror points. Curves giving the position of mirror points for constant equatorial pitch angles are traced for comparison (in a dipole field they would be constant latitude lines [c.f. Section II.6]). Notice the change