HOMEWORK 9: MATH 124 (NUMBER THEORY)

1. Exercises

Exercise 1.1. *For each of the following periodic continued fractions, write down a quadratic equation satisfied by its value and determine its exact value.*

(1) $\langle \overline{3} \rangle$. *(5 pts)*
(2) $\langle \overline{2, 4} \rangle$. *(5 pts)*
(3) $\langle \overline{1, 2, 3} \rangle$. *(5 pts)*
(4) $\langle 1, 2, \overline{2, 4} \rangle$. *(5 pts)*

Exercise 1.2. *Compute continued fraction expansions of the following quadratic irrationals.*

(1) *Find the simple continued fraction expansion of* $\sqrt{14}$ *and the length of its shortest repeating block.* *(6 pts)*
(2) *Use the criterion involving the conjugate to determine whether the continued fraction of* $3 + \sqrt{14}$ *is purely periodic. Write its continued fraction expansion as well.* *(4 pts)*

Exercise 1.3. *Let* d *be a positive integer that is not a perfect square, and set* $m = \lfloor \sqrt{d} \rfloor$. *Determine all positive integers* c *for which*

$$\alpha = \frac{m + \sqrt{d}}{c}$$

has a purely periodic simple continued fraction expansion. *(10 pts)*

Exercise 1.4. *Use the continued fraction expansion of* $\sqrt{14}$ *from Exercise 1.2 to find good rational approximations.*

(1) *Compute the first eight convergents.* *(5 pts)*
(2) *Use two consecutive convergents to give a rational interval containing* $\sqrt{14}$ *whose length is less than* $1/10000$. *Deduce a rational approximation* p/q *satisfying*

$$\left| \sqrt{14} - \frac{p}{q} \right| < \frac{1}{10000}.$$

Justify your interval and error bound using the results on continued fractions, without using a decimal approximation to $\sqrt{14}$. *(5 pts)*