HOMEWORK 9: MATH 223B (GALOIS COHOMOLOGY AND CLASS FIELD THEORY)

1. Exercises

Exercise 1.1. *Recall the Potryagin duality functor* $(-)^\vee$ *from Exercise 2.6 in the course notes. Let* $K/\mathbb{Q}$ *be a number field. Show the following.*

(1) ***(3 pts)*** *Show that we have an isomorphism*

$$(\mathbb{A}/\iota(\mathbb{Q}))^\vee \simeq \mathbb{Q}$$

of abelain groups (Hint: use the explicit description of the LHS provided in class and embed $\mathbb{Q}/\mathbb{Z}$ *into* S^1 *as* n*th roots of unity).*

(2) ***(3 pts)*** *Use Part (1) and Lemma 4.49 of the course notes, to show that we have an isomorphism*

$$(\mathbb{A}_K/\iota_K(K))^\vee \simeq K$$

of abelian groups.

Exercise 1.2. ***(3 pts)*** *Show that a fundamental domain for* $\mathbb{I}/\iota(\mathbb{Q}^*)$ *is given by*

$$D^* := \mathbb{R}_{>0} \times \prod_p \mathbb{Z}_p^*.$$

(Hint: follow the basic structure of the analogous proof for the adeles (Corollary 4.52 in the course notes); however, note that the analogue of weak approximation is very easy in this case!).