Rough plan:
Talk about various ways how one can attach some (usually Poisson) varieties to reductive groups + some data using the affine Grassmannian & to relate this to some varieties that were discussed in the talks of Etingof, Kontsevich & Gaiotto (Poisson snarks).
In particular, I will talk about certain varieties which "provide" a quasi-classical version of Langlands lifting (also constructed by V. Lafforgue).

Digression on Coulomb branches of 3d N=4 gauge theories (BFN)
Setup Fix G - connected reductive /C
V - a representation of G
("secretly" everything should depend only on $V \otimes V^* = T^* V$)

Physicists say that to this data one can attach certain 3d N=4 gauge theory (quiver)
can attach certain 3d $\mathcal{N}=4$ gauge theory (quantum) with $\mathcal{M}_4$, $\mathcal{M}_c$
Higgs and Coulomb branches of the moduli space of vacua. Poisson, generically symplectic affine varieties

$$\mathcal{M}_4 = T^* V \sslash G \quad \text{(potentially alg)}$$

$\mathcal{M}_c \rightarrow \quad \text{dim } \mathcal{M}_c = 2 \text{rank } (G)$

biholomorphically isomorphic to $(T^* T^V)/W$

$T^V \subset G^v$ maximal torus $W$-Weyl group

Properties: $\mathcal{M}_{c,G,v} = \text{Spec } (A_{c,v})$

1. $A_{c,v}$ is f. generated, integral & integrally closed

2. $\mathcal{M}_{c,G,v}$ is birationally isom to $(T^* T^V)/W$

3. $A_{c,v}$ is equipped with canonical non-commutative deformation, which in particular defines a Poisson alg. structure. $\mathcal{M}_c$ is gen. symplectic

A notion (due to Beauville) of a singular symplectic variety. Conj. $\mathcal{M}_c$ is always singular symp.
\[ \text{Summary. Conj. } \mathcal{M}_C \text{ is always singular symp.} \]
\[ \text{A. Weekees proved it in certain generality.} \]

4) \( \exists \mathfrak{g}_G \)-action on \( \mathcal{M}_C \) (\( \mathfrak{g}_G \)-grading on \( A_G, V \)) non-comm. deformation is graded over \( \mathbb{C}[[\hbar]] \)
\[ \text{deg } \hbar = 2 \]
If \( V \) is "sufficiently large" then \( \mathcal{M}_C \) is conical.

5) Let \( G_F \) ("flavour symmetry") be another reductive group \( G_F \to \text{Aut}_G(\mathfrak{g}) \)
Then \( \mathcal{M}_C \) will acquire a canonical symplectic deformation over \( \mathfrak{g}_F / \text{Ad } G_F = \mathfrak{t}_F / W_F \)

\[ \text{Construction:} \]
\[ A_{G', \nu} = H^*(\omega_{S_{G', \nu}}) \]

\[ \text{Ex. } V = 0 \quad A_{G, \nu} = H^*(\omega_{G^0 \mathcal{C}_G}) = H^*(\omega_{G^0 \mathcal{C}_G}) \]
\[ K = (\mathbb{C}[t]) \supset \mathcal{O} = (\mathbb{C}[t]) \]
\[ \text{Spec } A_{G, 0} = \text{"regular centralizer" in } \mathfrak{g}_V \quad e_{g, \nu} \]
\[ e_{g, \nu} = \{ (x, g) \mid x \in \mathfrak{g}_V \text{ in the Kostant section, } \}
\[ \parallel \quad g \in Z_{G^0}(x) \]
\[ g \in Z_{G^\nu}(x) \]

\[ \text{Whit} \setminus \mathbb{T}^* G^\nu / \text{Whit} \]

\text{Whit} = \text{Hamiltonian reduction w.r.t.} \ (U^\nu, \chi)

\chi - \text{generic character} \quad U^\nu \subset G^\nu \text{ max. unipotent}

\[ S_{G^\nu} = \prod (\mathfrak{g}_{1, 2} \mathfrak{g}_{1, 2}, k_1 s) / \mathfrak{g}_c - G \text{-bundle on } \]

\[ D = \text{Spec } \mathcal{O} \]

\( \kappa \)-isom. between \( \mathfrak{g}_{1, 2} / \mathfrak{g}^* \) and \( \mathfrak{g}_{2} / \mathfrak{g}^* \)

\( \mathfrak{g}^* = \text{Spec } \mathcal{O} \]

\( s \) - "common \( \nu \)-valued section of \( \mathfrak{g}_{1, 2} \) and \( \mathfrak{g}_{2} \) on \( D \).

\[ s_c \in \Gamma (\mathfrak{g}_{1, 2} \mathfrak{g}_{1, 2}, \mathcal{O}_D) \quad \kappa (\pi_1 / \mathfrak{g}^*) = \pi_2 / \mathfrak{g}^* \]

\[ S_{G^\nu} = \mathcal{R}_{G^\nu} / G(\mathcal{O}) \]

\( \mathcal{R}_{G^\nu} \) - same data but \( \mathfrak{g}_2 \) is trivialized on \( D \).

\[ \mathcal{R}_{G^\nu} \rightarrow \text{Gr}_6 \]

ind-scheme

\[ \mathcal{R}_{G^\nu} \subset \text{Gr}_6 \times V(\mathcal{O}) \]
$A_{G_1V} = H^* \left( \mathcal{O}(0), R_{G_1V} \right) \right.$

Algebra structure is convolution

$\mathbb{G}_m$-action = homological grading

Quantization = make everything equivariant with respect to loop rotation

Let $\pi : R_{G_1V} \to \text{GR}_C$

$A_{G_1V} = \pi_\ast \mathcal{O} \in \mathcal{D} \left( \text{GR}_C \right)$

Ring object

$A_{G_1V} = H^* \left( \mathcal{O}(0), A_{G_1V} \right)$

Apply the derived Satake functor to $A_{G_1V}$

$\mathcal{D}_{\mathcal{O}(0)} \left( \text{GR}_C \right) \xrightarrow{\Phi} \mathcal{G}^\mathcal{U}$ - equivariant dg-module over $\text{Sym} \cdot \mathcal{O}^\mathcal{U}[-2]$

$\Phi(A_{G_1V})$ - an algebra over $\text{Sym} \cdot \mathcal{O}^\mathcal{U}[-2]$ with a compatible $\mathcal{G}^\mathcal{U}$-action

$A_{G_1V} = \text{Restriction of } \Phi(A_{G_1V})$ to Kostant section
A_G,V = Restriction of \( \Phi(A_G,V) \) to Kostant section

Poisson, generically symp.

Remark How to prove that \( A_G,V \) is \( f \)-generated integral, integrally closed etc.

Define a filtration on \( A_G,V \) by dominant coweights of \( G \)

\[ \pi : R_{G,V} \rightarrow \mathcal{O}_G \]
\[ A_G,V = H_{x, BM}^{G(\partial)} (R_{G,V}) \]
\[ H_{x, BM}^{G(\partial)} (\pi^{-1} (G^{\lambda})) = A_G^A \leftarrow A_G \rightarrow A_G,V \]

This is a filtration \( A_G^A \cdot A_G^M \cdot A_G^M \rightarrow A_G,V \)

\[ \text{gr } A_G,V \text{ is something very explicit} \]

(can be described explicitly by generators and relations)

Remark You can ask if the same works for \( \Phi(A_G,V) \) - algebra over \( \text{Sym}_G \) with \( G \)-action

Probably the same argument will prove finite generatedness of \( \Phi(A_G,V) \).

Plan Fix two groups \( L, M \) with a homomorphism
Plan Fix two groups \( L, M \) with a homomorphism \( L^\nu \to M^\nu \).

Want To define an affine Poisson variety
\[ X_{L, M} = \text{Spec} \, A_{L, M} \] which is "quasi-classical version" of local Langlands lifting.

Poisson action of \( L \times M \)
\[ \text{Whit}_M (X_{L, M}) = \text{Whit}_L (T^* L) \, \, (\star) \]

Example \( M = L^n = L \times \ldots \times L \)
\[ L^\nu \to M^\nu \] is the diagonal embedding

\( X_{L, M} \) - variety with action of \( L^{n+1} \)

Poisson

\( X_{L, n} \) - variety with action of \( L^n \)

\[ \text{Whit}_L (X_{L, n}) = X_{L, n-1} \]

\[ \xrightarrow{\phi} \]

\[ L^\nu \to M^\nu \]

\[ A_{L, M} \]

\[ \text{Whit}_L (\otimes_{L, M}) \]

\[ B_{L, M} \in D_p (\mathcal{O}_{L^\nu}) \]

endowed with \( M \)-action
endowed with $M$-action

$i : \text{Gr}_{L^u} \to \text{Gr}_{M^u}$

$M \in \text{Reg}_{M^u} \in \mathcal{D}_{M^u(\mathfrak{o})} (\text{Gr}_{M^u})$ - perverse sheaf which corresponds to $\mathbb{C}[M]$ under geometric Satake

$\mathcal{O}_{L^u(\mathfrak{o})} (\text{Gr}_{L^u})$

$\mathcal{O}_{L^u}(\mathcal{O}_{L^u,M})$ - algebra over $\text{Sym } L$ with an action of $L \times M$

( it also has a natural action of $\text{Sym } M$)

Conjecture $\Phi_{L^u}(\mathcal{O}_{L^u,M})$ is finitely generated integral, integrally closed etc...

Theorem True when $L, M$ are of type $A$

(in this case this turns out to be a special case of the Coulomb branch construction)