The Analogies between Binomial and Trinomial Coefficients

The binomial coefficients and the trinomial coefficients are close siblings.[1] The binomial coefficient $\binom{n}{r_1,\, r_2}$ is the coefficient of $x_1^{r_1} x_2^{r_2}$ in the expansion of $(x_1 + x_2)^n$. The trinomial coefficient $\binom{n}{r_1,\, r_2,\, r_3}$ is the coefficient of $x_1^{r_1} x_2^{r_2} x_3^{r_3}$ in the expansion of $(x_1 + x_2 + x_3)^n$. Here are the analogies, arranged side-by-side.

Before gathering terms, $\big(x_1 + x_2\big)^n$ has 2^n terms.	Before gathering terms, $\big(x_1 + x_2 + x_3\big)^n$ has 3^n terms.
After gathering terms, $\big(x_1 + x_2\big)^n$ has $\left(\!\binom{2}{n}\!\right)$ terms, one for each composition of n into 2 parts.	After gathering terms, $\big(x_1 + x_2 + x_3\big)^n$ has $\left(\!\binom{3}{n}\!\right)$ terms, one for each composition of n into 3 parts.
The coefficient of $x_1^{r_1} x_2^{r_2}$ is $\binom{n}{r_1, r_2} = \frac{n!}{r_1! r_2!}$.	The coefficient of $x_1^{r_1} x_2^{r_2} x_3^{r_3}$ is $\binom{n}{r_1, r_2, r_3} = \frac{n!}{r_1! r_2! r_3!}$.
Recurrence: $\binom{n}{n, 0} = \binom{n}{0, n} = 1$	Recurrence: $\binom{n}{n, 0, 0} = \binom{n}{0, n, 0} = \binom{n}{0, 0, n} = 1$
$\binom{n}{r_1, r_2} = \binom{n-1}{r_1 - 1, r_2} + \binom{n-1}{r_1, r_2 - 1}$	$\binom{n}{r_1, r_2, r_3} = \binom{n-1}{r_1 - 1, r_2, r_3} + \binom{n-1}{r_1, r_2 - 1, r_3} + \binom{n-1}{r_1, r_2, r_3 - 1}$
(Pascal's Triangle)	(Pascal's Tetrahedron?)
$\binom{n}{r_1, r_2}$ is the number of permutations of r_1 1's and r_2 2's.	$\binom{n}{r_1, r_2, r_3}$ is the number of permutations of r_1 1's, r_2 2's, and r_3 3's.
$\binom{n}{r_1, r_2}$ is the number of ways to place r_i students in dorm i for $i = 1, 2$	$\binom{n}{r_1, r_2, r_3}$ is the number of ways to place r_i students in dorm i for $i = 1, 2, 3$

Exercise: Make another column for the $k$nomial coefficients. What are the correct generalizations?

[1] To make the closeness of the analogies even clearer, I need a new notation for binomial coefficients: in place of $\binom{n}{r}$ I will write $\binom{n}{r,\, n-r}$ or $\binom{n}{r_1,\, r_2}$ (where $r_1 + r_2 = n$). Throughout: on the left, $r_1 + r_2 = n$; and on the right, $r_1 + r_2 + r_3 = n$.