

Math E305

Advanced Algebra and Trigonometry!

**Finding the formula
continued!**

...and sequences!

Fourth Class – Tuesday, July 1

- POTD – A fencing problem
 - *welcome to the fourth dimension?!*
- And now... what's the formula?!
 - *polynomial mysteries continued!*
- Sequences
 - *basic issues/convergence*
 - *and so we meet again, Fibonacci!*

A trip to the fourth dimension...!

Time for a fencing problem!!



Multidimensional Fence Building!

*Consider a fence in
two dimensional space...*

it's just a straight line

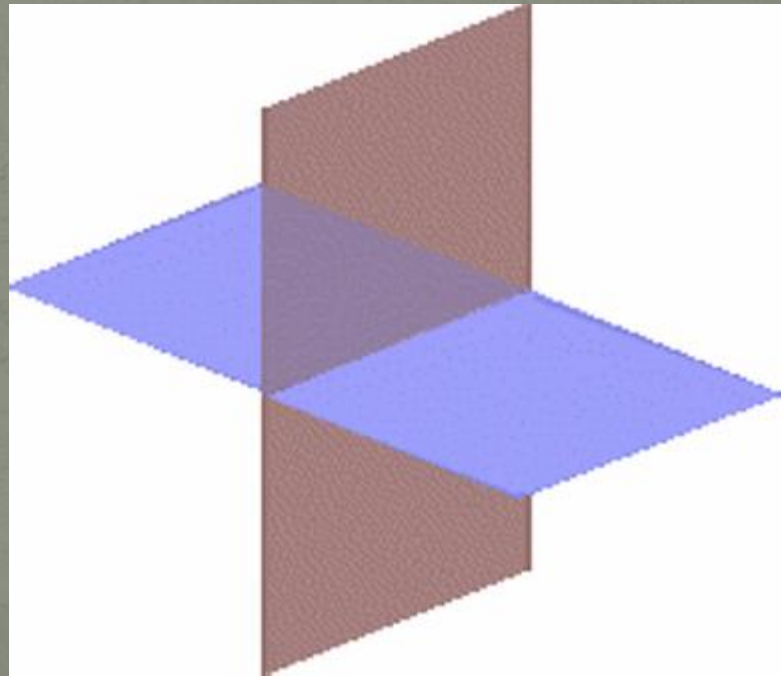
*So now what would a fence look
like in one dimensional space?*

it's just a... ?

*Now consider the fence problem in
three dimensional space...*

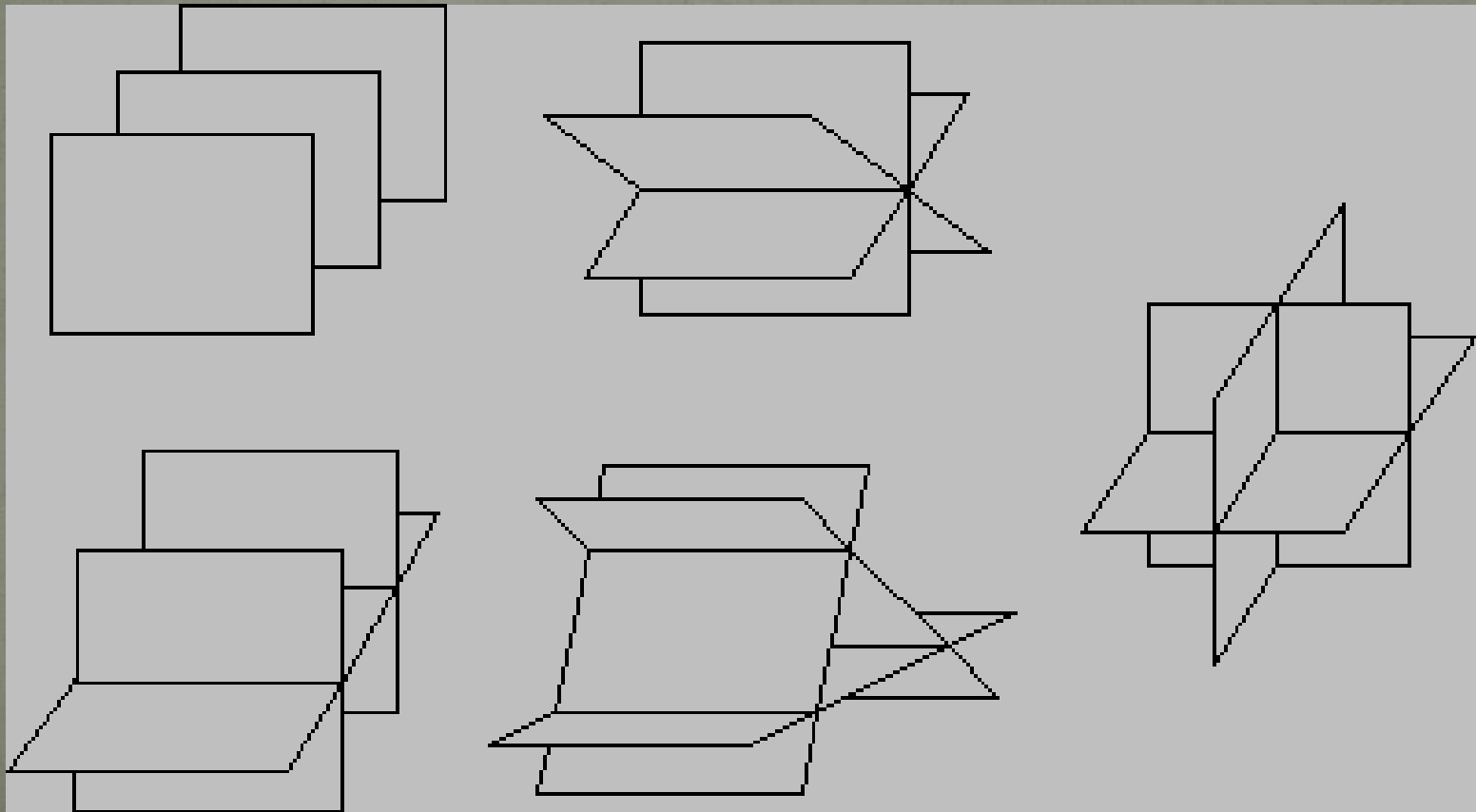
the fences are planes...

*One plane splits
space into two
zones and two
fences (planes)
split space into...*



What about three fences (three planes)?

*How many possible arrangements
are there?...*



What about four fences (four planes)?

How can we picture this?



...not the maximum possible!

Back to mystery functions...

so $f(n) = 1 + 4(0+1+2+\dots+(n-1)) + n \cdot (-1)$

and so $f(n) = 1 + 4[n(n-1)/2] - n$

so $f(n) = 2n^2 - 3n + 1$

input	output	Δ	Δ^2
0	1	-1	4
1	0	3	4
2	3	7	4
3	10	11	4
4	21	15	
5	36		

*and as
a check,
what is
 $\Delta f(n)$?*

The fence problem...

Fences	Dimension			
	1	2	3	4
0	1	1	1	1
1	2	2	2	2
2	3	4	4	4
3	4	7	8	8
4	5	11	15	16
5	6	16	26	31
6	7	22	42	57

The fence problem continued...

Fences	Regions			
	in 3 rd dim.	Δ	Δ^2	Δ^3
0	1	1	1	1
1	2	2	2	1
2	4	4	3	1
3	8	7	4	1
4	15	11	5	1
5	26	16	6	1
6	42	22	7	1

The fence problem continued...

Fences	Regions in 3 rd dim.	Δ	Δ equals
0	1	1	0 + 1
1	2	2	1 + 1
2	4	4	3 + 1
3	8	7	6 + 1
4	15	11	10 + 1
5	26	16	15 + 1
6	42	22	21 + 1

The fence problem continued...

Fences	Regions in 3 rd dim.	Δ	Δ equals
0	1	1	$\frac{1}{2} \cdot 0^2 + \frac{1}{2} \cdot 0 + 1$
1	2	2	$\frac{1}{2} \cdot 1^2 + \frac{1}{2} \cdot 1 + 1$
2	4	4	$\frac{1}{2} \cdot 2^2 + \frac{1}{2} \cdot 2 + 1$
3	8	7	$\frac{1}{2} \cdot 3^2 + \frac{1}{2} \cdot 3 + 1$
4	15	11	$\frac{1}{2} \cdot 4^2 + \frac{1}{2} \cdot 4 + 1$
5	26	16	$\frac{1}{2} \cdot 5^2 + \frac{1}{2} \cdot 5 + 1$
6	42	22	$\frac{1}{2} \cdot 6^2 + \frac{1}{2} \cdot 6 + 1$

The fence problem continued...

Fences	Regions in 3 rd dim.	Δ	Δ equals
0	1	1	$\frac{1}{2} \cdot 0^2 + \frac{1}{2} \cdot 0 + 1$
1	2	2	$\frac{1}{2} \cdot 1^2 + \frac{1}{2} \cdot 1 + 1$
2	4	4	$\frac{1}{2} \cdot 2^2 + \frac{1}{2} \cdot 2 + 1$
3	8		

so, for example, $f(3) = 8 = 1 + 1 + 2 + 4$

$$= 1 + \frac{1}{2} \cdot 0^2 + \frac{1}{2} \cdot 0 + 1$$

$$+ \frac{1}{2} \cdot 1^2 + \frac{1}{2} \cdot 1 + 1$$

$$+ \frac{1}{2} \cdot 2^2 + \frac{1}{2} \cdot 2 + 1$$

The fence problem continued...

so, for example, $f(3) = 8 = 1 + 1 + 2 + 4$

$$\begin{aligned} &= 1 + \frac{1}{2} \cdot 0^2 + \frac{1}{2} \cdot 0 + 1 \\ &\quad + \frac{1}{2} \cdot 1^2 + \frac{1}{2} \cdot 1 + 1 \\ &\quad + \frac{1}{2} \cdot 2^2 + \frac{1}{2} \cdot 2 + 1 \end{aligned}$$

$$= 1 + \frac{1}{2} (0^2 + 1^2 + 2^2) + \frac{1}{2} (0 + 1 + 2) + (1 + 1 + 1)$$

and in general...

$$f(n) = 1 + \frac{1}{2} (0^2 + 1^2 + \dots + (n-1)^2) + \frac{1}{2} (0 + 1 + \dots + (n-1)) + n \cdot 1$$

The fence problem continued...

$$f(n) = 1 + \frac{1}{2} (0^2 + 1^2 + \dots + (n-1)^2) + \frac{1}{2} (0 + 1 + \dots + (n-1)) + n \cdot 1$$

and now using the formula for the sum of the first n squares equaling $n(n+1)(2n+1)/6$ we get...

(remembering to sub in “(n-1)” for “n” in this formula!)

$$f(n) = 1 + \frac{1}{2} [(n-1)n(2(n-1)+1)/6] + \frac{1}{2} [n(n-1)/2] + n$$

which simplifies as...

$$f(n) = [n^3 + 5n + 6]/6$$

which works – i.e. gives the values we want – phew!

So now how about the fourth
dimension?! *good luck!*

	Dimension			
Fences	1	2	3	4
0	1	1	1	1
1	2	2	2	2
2	3	4	4	4
3	4	7	8	8
4	5	11	15	16
5	6	16	26	31
6	7	22	42	57

Let's reverse the table...

4 Dim.

Fences	Regions	Δ	Δ^2	Δ^3	Δ^4	Δ^5
0	1	1	1	1	1	0
1	2	2	2	2	1	0
2	4	4	4	3	1	0
3	8	8	7	4	1	0
4	16	15	11	5	1	0
5	31	26	16	6	1	0
6	57	42	22	7	1	0

But notice we already know a formula for the first differences...

4 Dim.

Fences	Regions	Δ
0	1	1
1	2	2
2	4	4
3	8	8
4	16	15
5	31	26
6	57	42

$$\Delta(n) = [n^3 + 5n + 6]/6$$

so what, for example, does $f(3)$ equal?

A sequence encore –
class participation!

Find any interesting ones?!

1 2 3 4 10 19 36 69...

(and now what's next?!)

Sequences – descriptions

What about something this simple?

.9 .99 .999 .9999 .99999 ...

Looks suspiciously like it's heading towards...

Okay, so define a new term!

Convergence!

A sequence $\{a_n\}_{n \in \mathbb{N}}$ converges to L if...

$$\forall \epsilon > 0 \quad \exists B \in \mathbb{N} \text{ s.t. } \forall n \geq B \quad |a_n - L| < \epsilon$$

Sequences – descriptions

What about

3 3.1 3.14 3.141 3.1415 ...

Describe as many aspects of this sequence as possible...

convergence,
(strictly) increasing,
bounded...

