Math 305

Advanced Algebra and Trigonometry!

Finding the formula...

...is there always a formula?!

Third Class – Monday, June 30

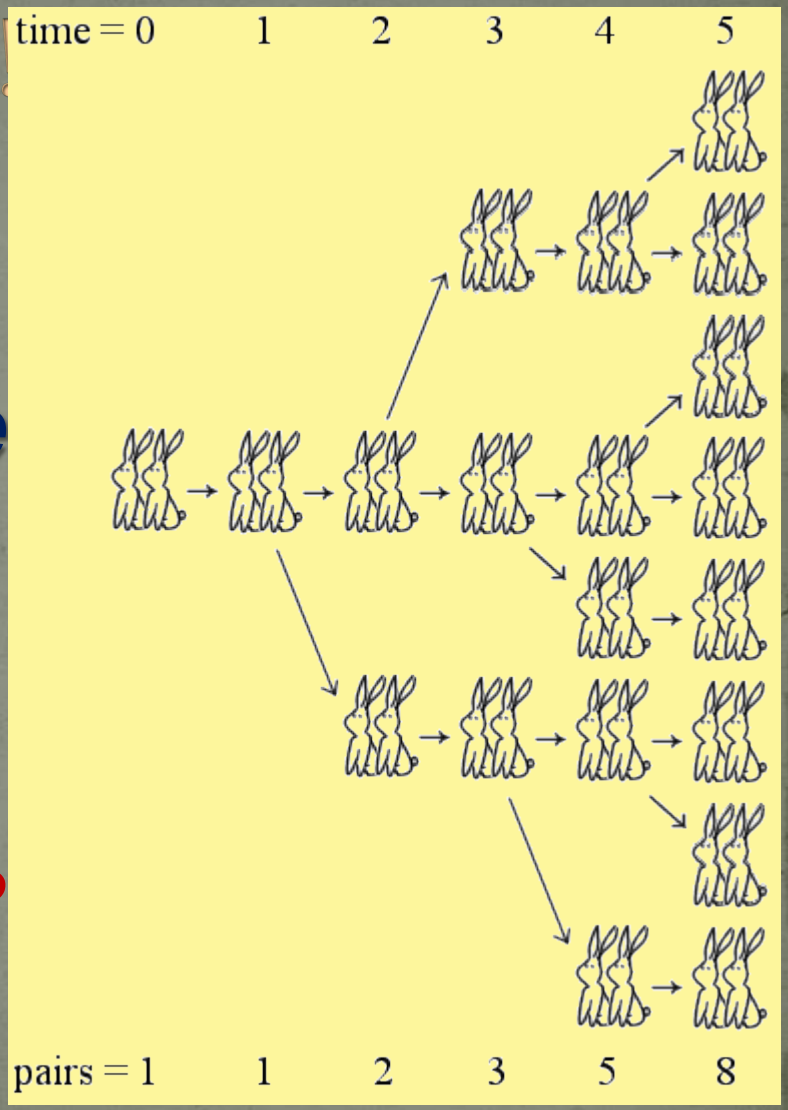
- POTD
  - *Summing Fibonacci!*
- And now… what's the formula?!
  - *polynomial mysteries!*
- Sequences
  - *what's next?!*
- A fencing problem!
  - *welcome to the fourth dimension!*

POTD –
an old favorite!

You probably all
(and love

1 1 2 3

*(quick history question –
where did it come from?
what context?)*



And now the POTD!

Sum it! Now what is…

1 + 1 + 2 + 3 + 5 + 8 + 13 + …

well that's obviously not going to work out!

…how about a slightly modified question!

What is 1 + .1 + .02 + .003 + .0005 +…?

POTD – continued…

How does this connect to what we saw in our last class?…

What is 1 / (1 – x)? *(formally…!)*

→ *Generating functions!*

now try to find something similar for…

$$1 + 2x + 4x^2 + 8x^3 + 16x^4 + \ldots$$

Aha – the sequence connection!

Leaping out a bit…

So the Geometric Sequence

$$1 \quad a \quad a^2 \quad a^3 \quad a^4 \quad a^5 \ldots$$

corresponds to the generating function

$$1 + a x + a^2 x^2 + a^3 x^3 + a^4 x^4 + a^5 x^5 + \ldots$$

which is (formally) equivalent to …

$$1 / (1 - ax)$$

…and Arithmetic Sequences?!

Now can you work out a simple rational polynomial corresponding to the generating function

$$1 + 4x + 7x^2 + 10x^3 + 13x^4 + 16x^5 + \ldots$$

time for a side(?) path…

we looked at $1 / (1 - x)$

what is the generating function equivalent to $1 / (1 - x)^2$ *?*

Now bringing it back around…!

Can you work out a simple rational polynomial corresponding to the (familiar?!) generating function

$$1 + x + 2x^2 + 3x^3 + 5x^4 + 8x^5 + \ldots$$

aka $\sum_{k=1}^{\infty} F_k x^{k-1}$

or which can be written as $\sum_{k=0}^{\infty} F_{k+1} x^k$

…and now the surprising punchline –

what can you use this result for?!!

Now let's connect sequences and polynomials

Suppose you have the following input/output table for a function…

input	output	Δ
0	1	3
1	4	3
2	7	3
3	10	3
4	13	3
5	16	3

What's a possible polynomial function matching this table?

Finding a mystery function…

Suppose the following gives an input/output table for a polynomial function… ***see if you can discover it!***

input	output	Δ	Δ^2
0	1	-1	4
1	0	3	4
2	3	7	4
3	10	11	4
4	21	15	
5	36		

Finding a mystery function…

first, look at Δ…

so Δ = **4n - 1**

next… what is f(3)?

input	output	Δ	Δ^2
0	1	-1	4
1	0	3	4
2	3	7	4
3	10	11	4
4	21	15	
5	36		

Finding a mystery function…

$f(3) = 1 + (-1) + 3 + 7$

$= 1 + (4 \cdot 0 - 1) + (4 \cdot 1 - 1) + (4 \cdot 2 - 1)$

…which can be rewritten as…?

input	output	Δ	Δ^2
0	1	-1	4
1	0	3	4
2	3	7	4
3	10	11	4
4	21	15	
5	36		

Finding a mystery function…

so f(n) = ?

$= 1 + 4(0+1+2+\ldots+(n-1)) + n \cdot (-1)$

wait… *it's triangle numbers…*

no… handshake numbers!!

input	output	Δ	Δ^2
0	1	-1	4
1	0	3	4
2	3	7	4
3	10	11	4
4	21	15	
5	36		

Finding a mystery function…

so $f(n) = 1 + 4(0+1+2+\ldots+(n-1)) + n \cdot (-1)$

and so $f(n) = 1 + 4[n(n-1)/2] - n$

so $f(n) = 2n^2 - 3n + 1$

input	output	Δ	Δ^2
0	1	-1	4
1	0	3	4
2	3	7	4
3	10	11	4
4	21	15	
5	36		

and as a check, what is $\Delta f(n)$?

Sequences – what's next?!

Here's a simple one…

1 2 4 8 16…

What's next?

Answer = that's a pretty ~~stupid~~ silly question!

It could be anything you'd like!

…Moser's Circle Problem!

Online Encyclopedia of Sequences

Sequences – what are they?

Here's an irritating one!

3 3 5 4 4 3 5 5 4 …

So what exactly ***is*** a sequence?

1 2 4 8 16…

other examples?

What about

Karina Mike Sarah Andy

…is this a sequence?