Welcome to Math 305 *Advanced Algebra and Trigonometry!*

When it's the Form that matters…

Second Class – Friday, June 27, 2014

- Warm-up
  - *cheap calculations!*
- POTD – Counting coins
  - *infinite polynomials?*
  - *Hello Fibonacci?!*
- Teaching approaches to algebra
  - *The Model Method – Singapore Math*

Advanced Algebra – it's about ***Form*** and Function

How about this somewhat strange question?

calculating f(18) cheaply!

…back to medieval times... a traveling merchant has need of evaluating the polynomial $12\,t^4 + 7\,t^3 + 5\,t^2 + 6\,t + 11$ at $t = 18$. The merchant takes this extravagantly difficult problem to a professional mathematician/computer (back in those days "computers" were actually people who'd solve sums and other problems for others). The computer charges 10 farthings for each multiplication and 5 farthings for each addition...

POTD - Counting coins!

Suppose there's a pile of coins containing 3 pennies, 2 nickels, 1 dime, 1 quarter, and 1 half dollar coin…

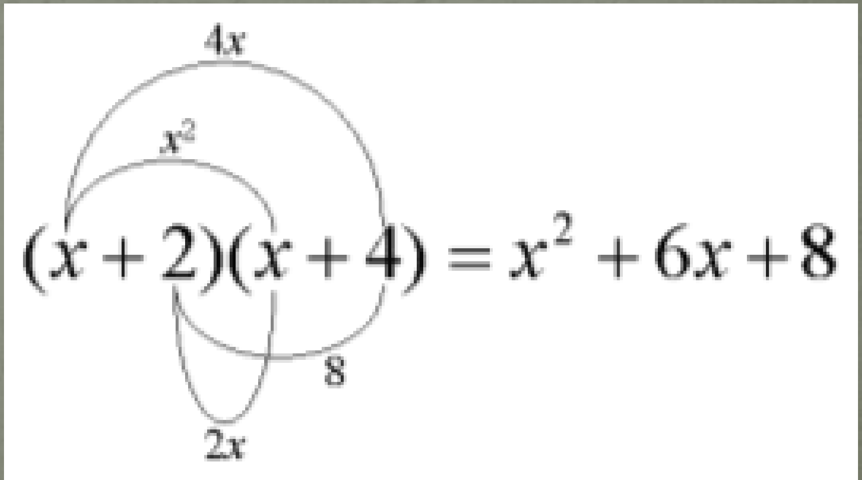
How many ways can you select 4 coins from the pile?

POTD…counting coins!

Suppose you have a pile of coins of various denominations… *so how many ways are there of putting together a pile of 4 coins?*

Think about the distributive property…

Time to use Mathematica?!



POTD – Counting coins …continued!

Suppose there's a pile of coins containing pennies, nickels, dimes, quarters, and half dollar coins…

How many ways can you select 8 coins from the pile?

Something new?!

Can we connect this to the following…

What is 1 / (1 – x)? *(formally….!)*

→ *Generating functions!*

now try to find something similar for…

$$1 + 2x + 4x^2 + 8x^3 + 16x^4 + \dots$$

Aha – a sequence connection!

Leaping out a bit…

*So the **Geometric Sequence***

1 a a^2 a^3 a^4 a^5 …

corresponds to the generating function

$$1 + a x + a^2 x^2 + a^3 x^3 + a^4 x^4 + a^5 x^5 + \dots$$

which is (formally) equivalent to …

$$1 / (1 - ax)$$

...and Arithmetic Sequences?!

Now can you work out a simple rational polynomial corresponding to the generating function

$1 + 4x + 7x^2 + 10x^3 + 13x^4 + 16x^5 + \ldots$

time for a side(?) path…

we looked at $1 / (1 – x)$

what is the generating function equivalent to $1 / (1 – x)^2$ *?*

Take it up a notch!!

What about the generating function corresponding to

$$1 / (1 - x)^3$$

…or the generating function corresponding to

$$1 / (1 - x)^4$$

ugh!

But wait,

those numbers look familiar too!!

					1					
				1		1				
			1		2		1			
		1		3		3		1		
	1		4		6		4		1	
1		5		10		10		5		1

Pascal's Triangle

The triangle is connected with many different mathematical concepts,
such as to combinations (from probability),
as well as the binomial theorem in algebra…

Recall that…

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

Pascal's Triangle

This can be "seen in action" by…

$(x+y)^0 = 1$ — 0th row

$(x+y)^1 = 1x + 1y$ — 1st row

$(x+y)^2 = 1x^2 + 2xy + 1y^2$ — 2nd row

$(x+y)^3 = 1x^3 + 3x^2y + 3xy^2 + 1y^2$ — 3rd row

$(x+y)^4 = 1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$ — 4th row

$(x+y)^5 = 1x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + 1y^5$ — 5th row

POTD – Counting coins

…so now can we do this one?

Suppose there's a pile of coins containing pennies, nickels, dimes, quarters, and half dollar coins…

How many ways can you select 8 coins from the pile?