Math 305

Advanced Algebra and Trigonometry!

Goodbye Circles – hello Conics!

time to get hyperbolic!

Last Class – Wednesday, July 16th

- POTD
  - *Playing with wax paper…!*
- Following up on the new shapes…
  - *hyperbolic trigonometry!*
  - *…along with some new formulas!*
- And what about that final…?
  - *reviewing for the final*

Last POTD!

This one's a bit mysterious….!

Take a piece of wax paper…

Draw a circle in the middle, and trace the circle out with a marker, marking the center of the circle as well

…one person in your group should draw an "infinite" circle on their paper instead!

Use the marker to pick another point on the wax paper… (where?!)

Playing with wax paper continued…

Now fold the wax paper so that your new point (or just the circle center) lies on the circle.

Do this a bunch of times, each time using the marker (and ruler) to draw along the fold lines you've made

Each person in your group should choose one of the four possible point/circle combinations

Playing with wax paper continued…

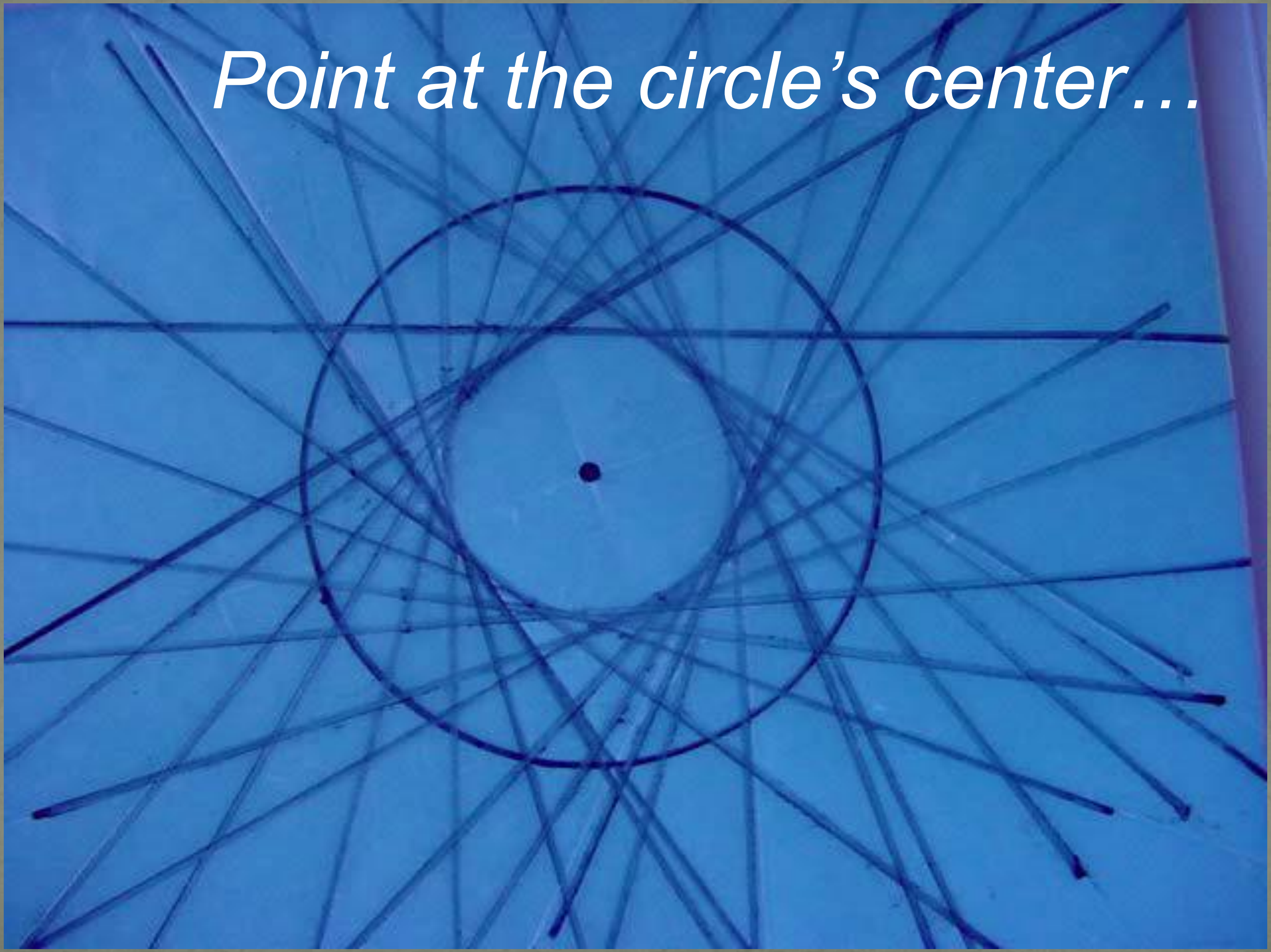
Okay – now, what did you get? What shape do your lines "show off"?

Now try to figure out why that's the case…

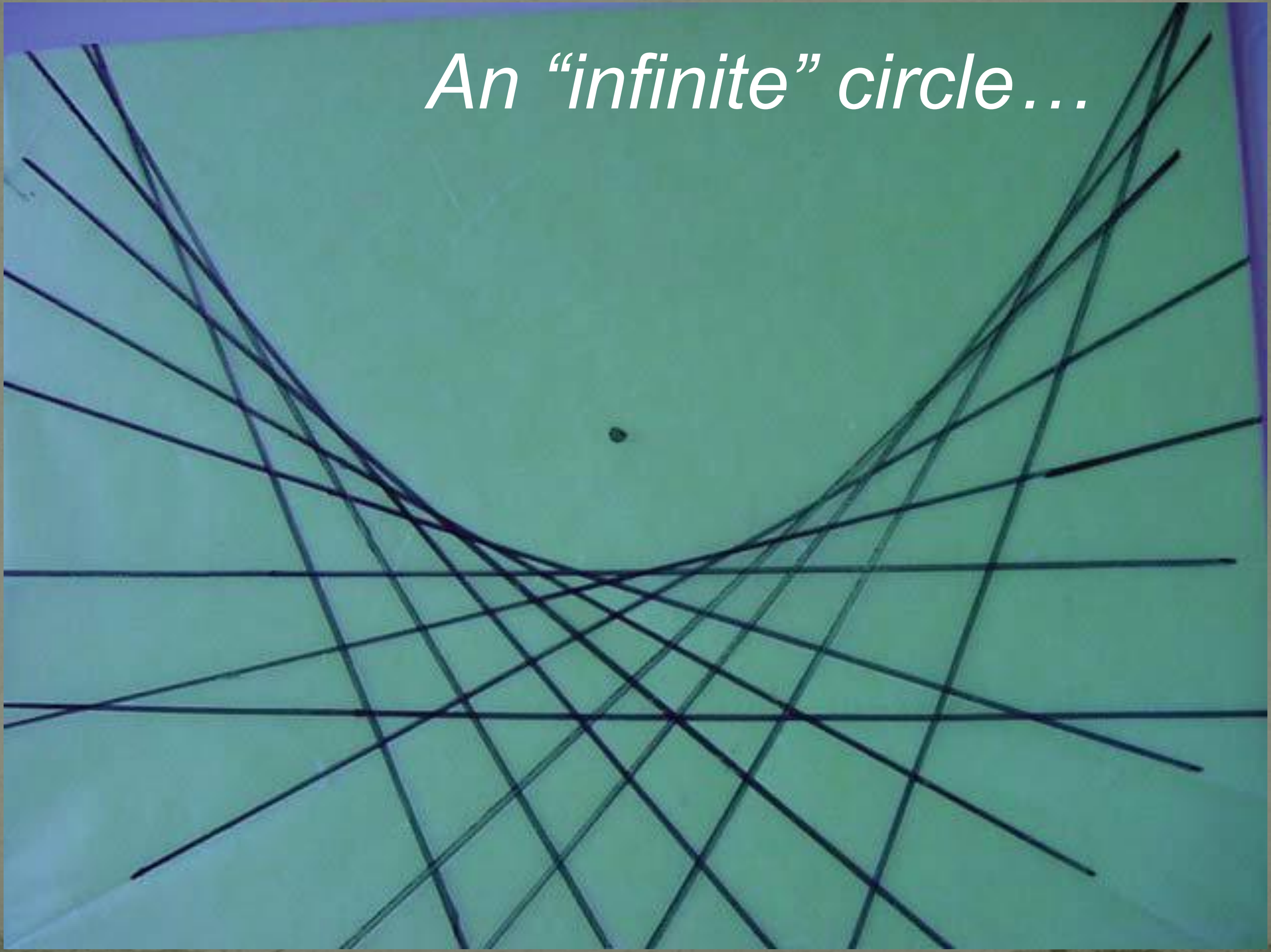
good luck!

Help each other out as soon as you've figured out your shape!

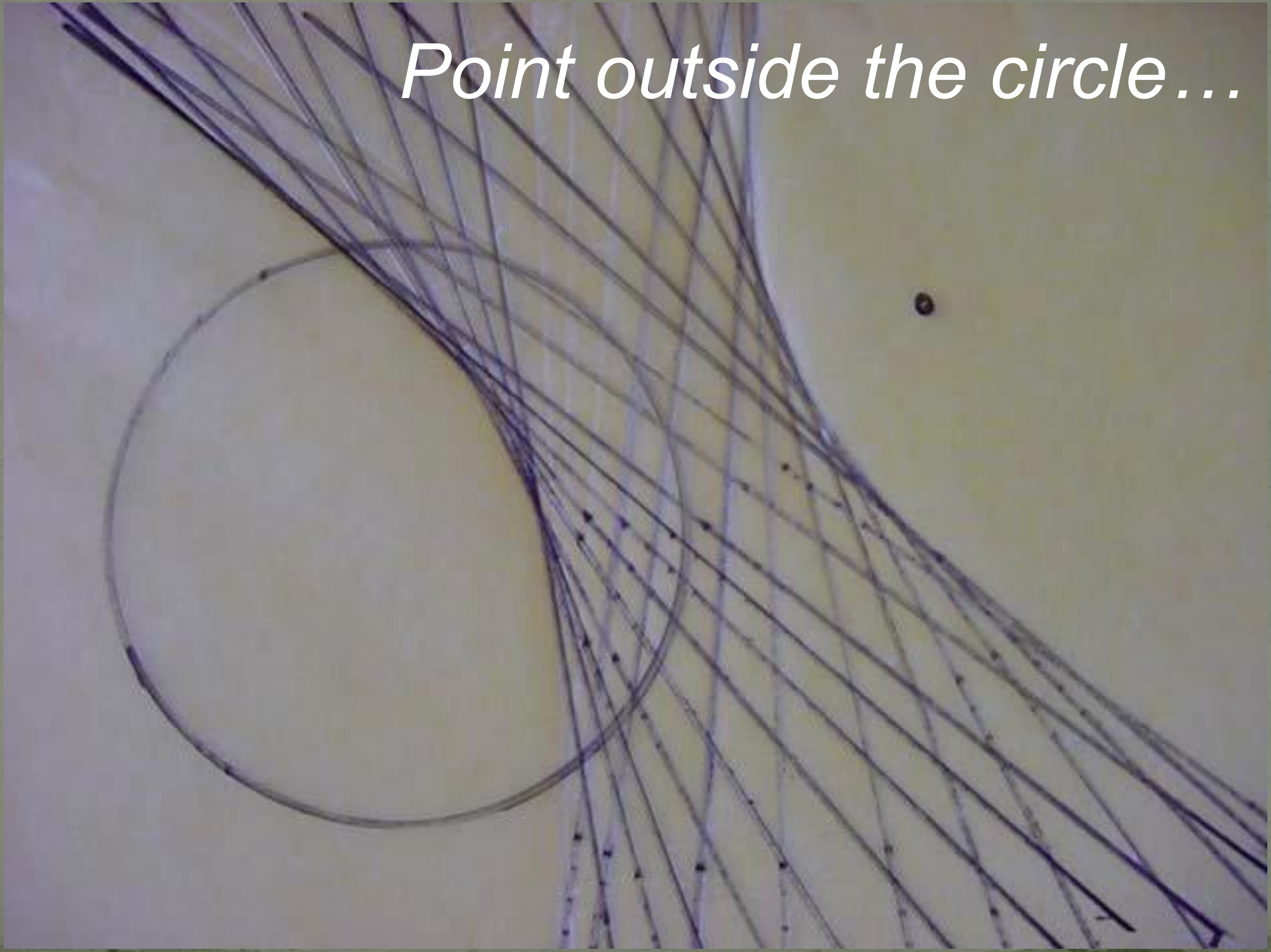
Conics!



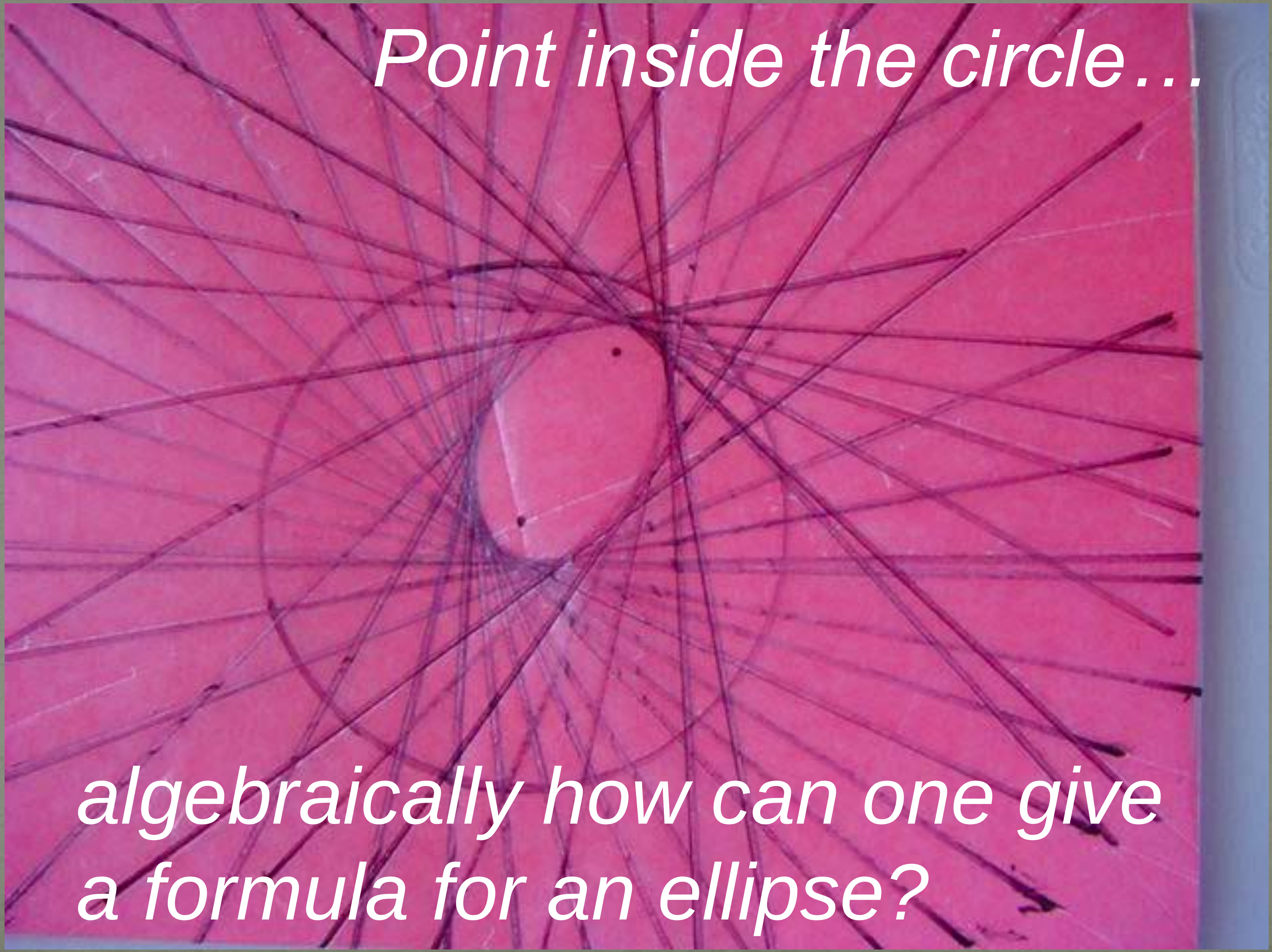
Conics!



Conics!

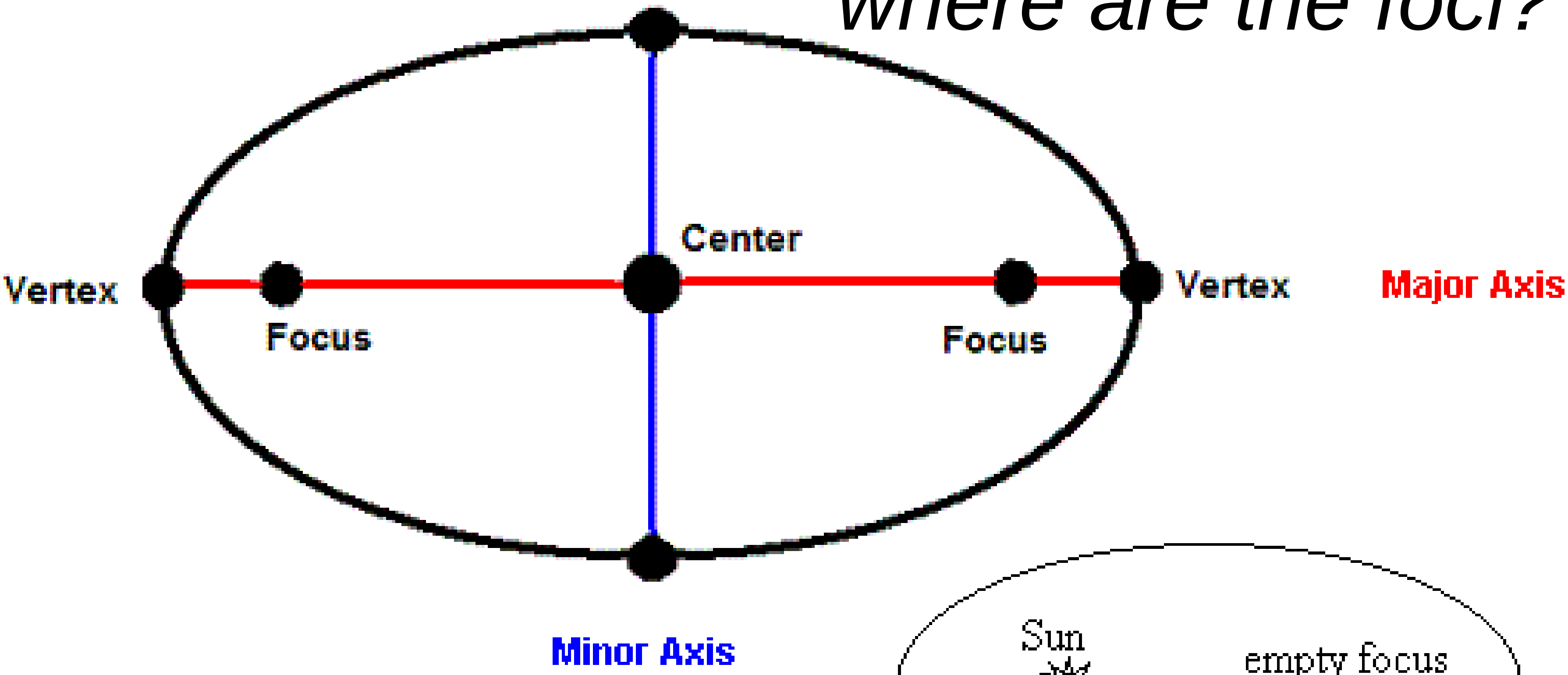


Conics!

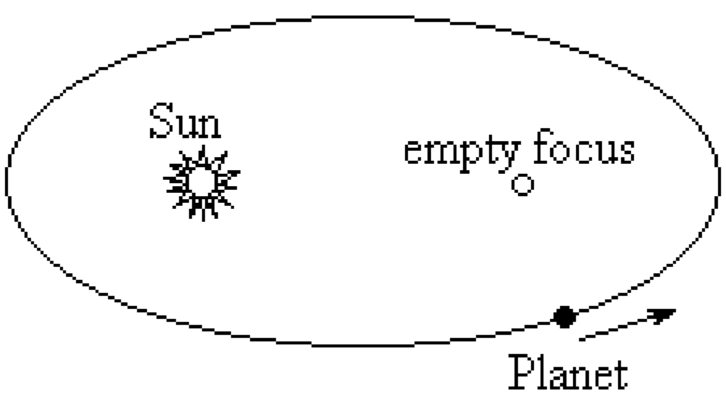

Point inside the circle…
algebraically how can one give a formula for an ellipse?

Ellipses – just "stretched out" circles!

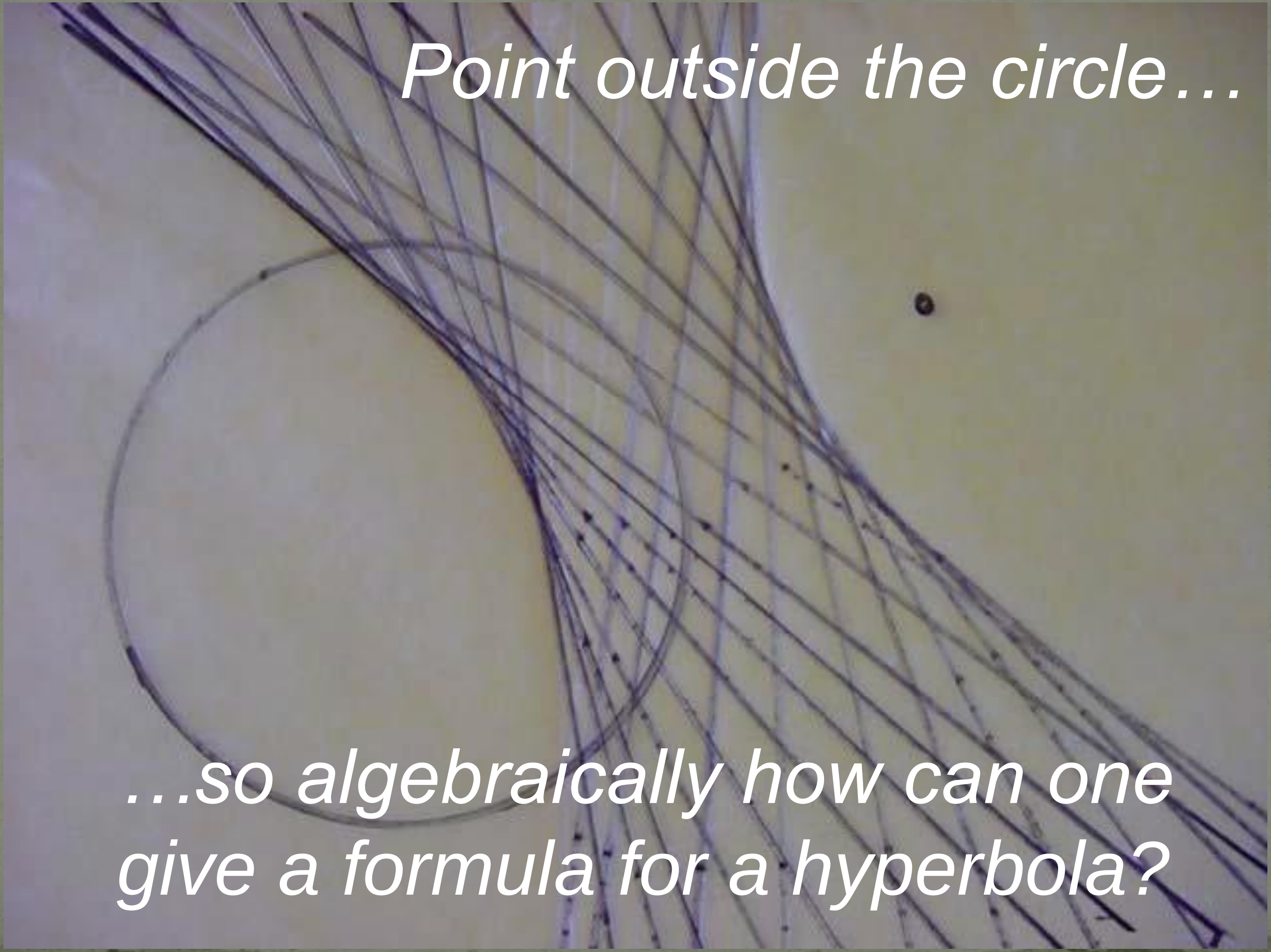
where are the foci?



what's it "good for"?

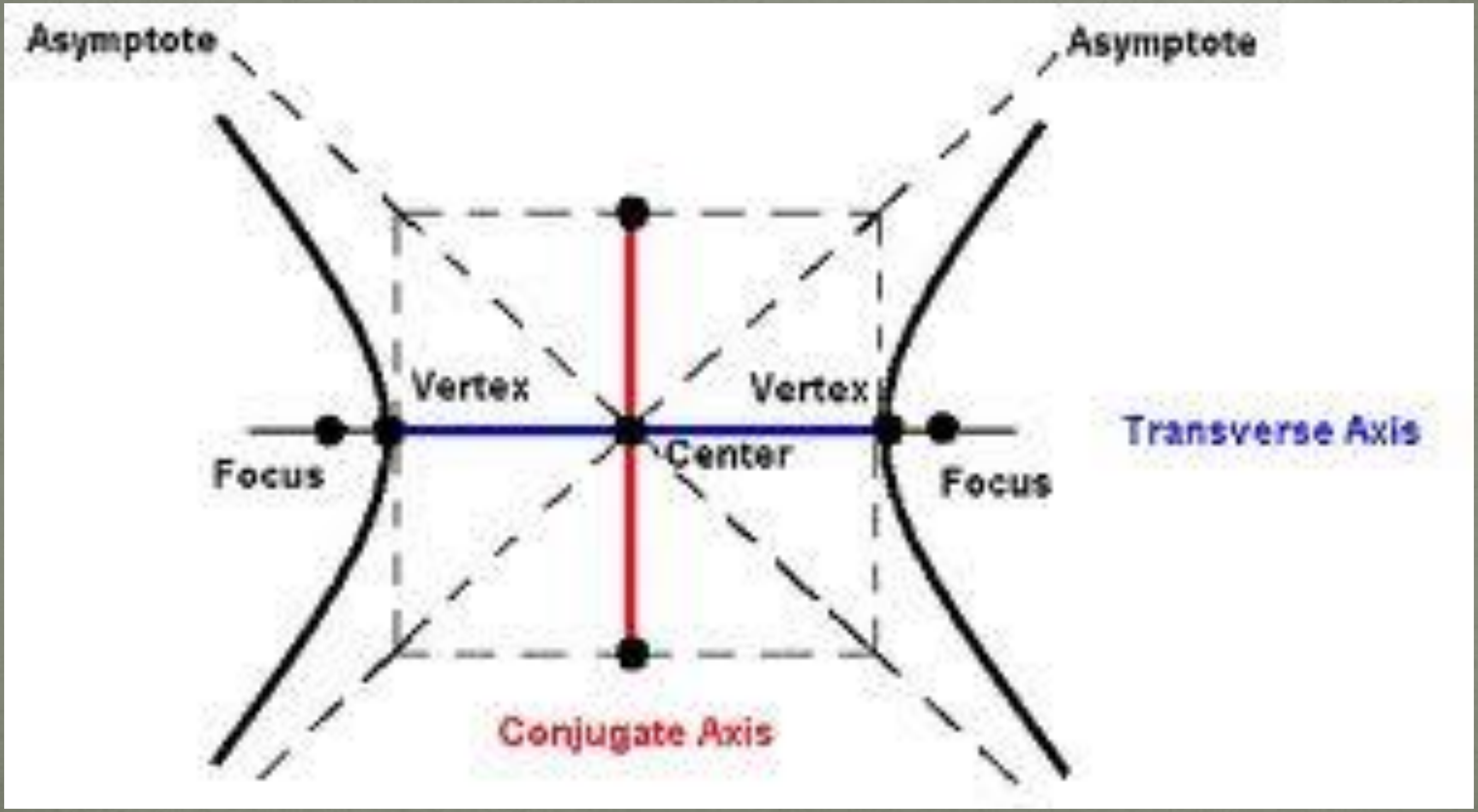


Conics!


Point outside the circle…
…so algebraically how can one give a formula for a hyperbola?

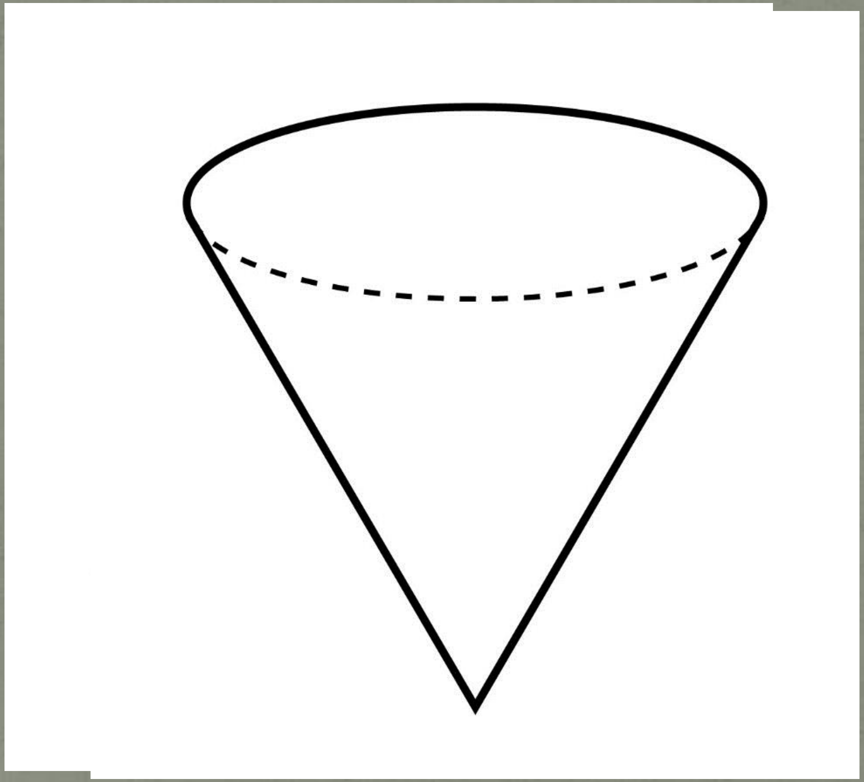
Hyperbolas – just ellipses turned "inside out"?!

now where are the foci?



Cones!

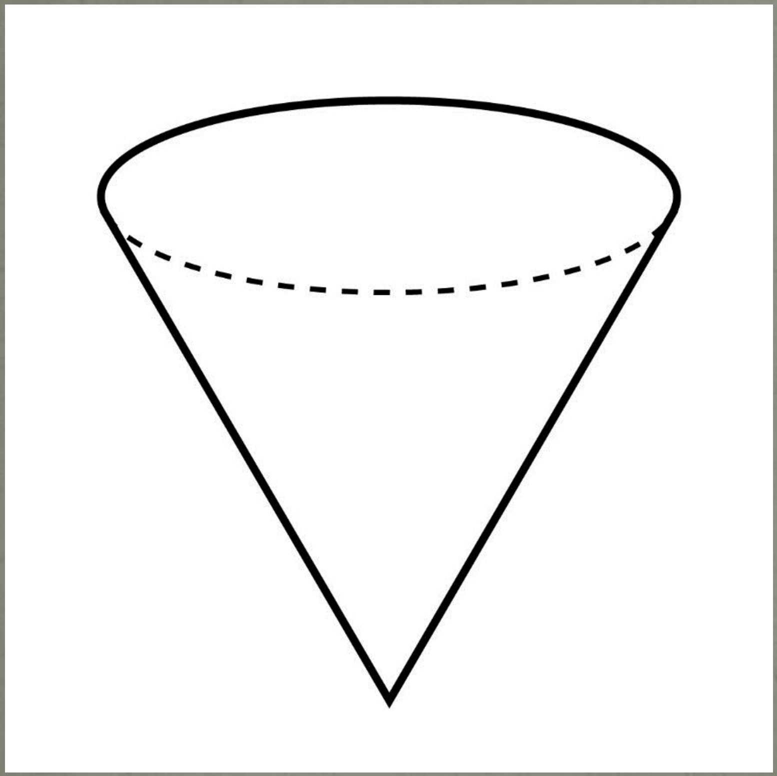
Try to make a cone in space…

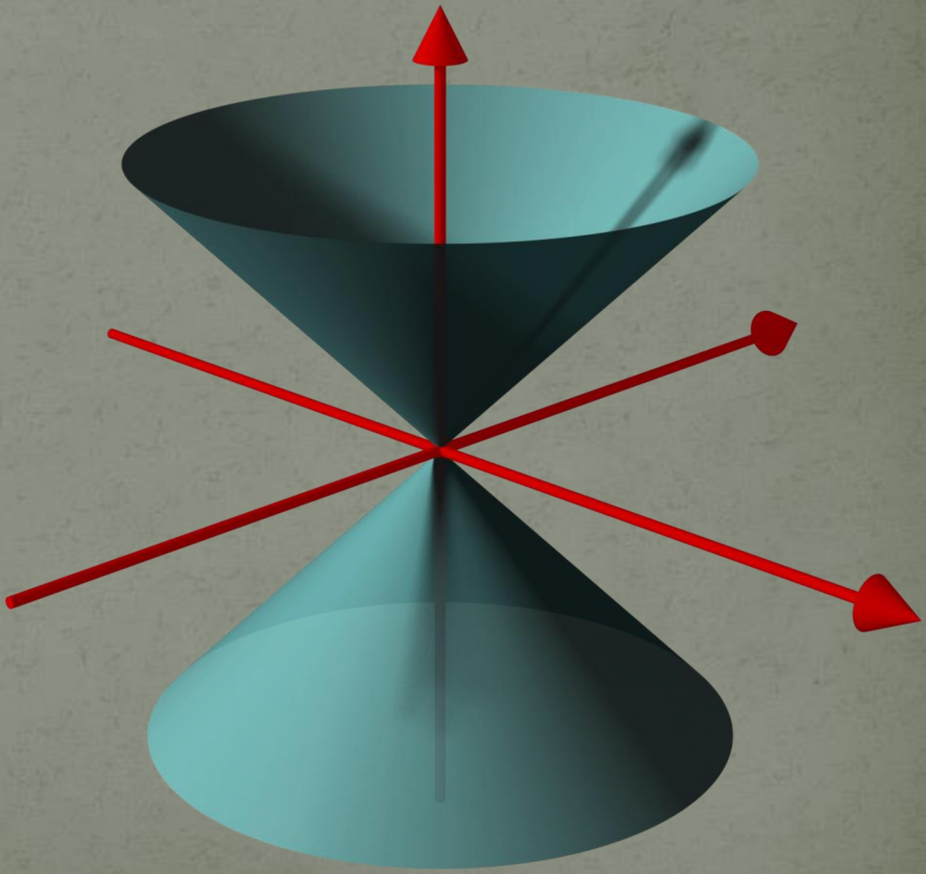


Let's flip it first…

Cones!

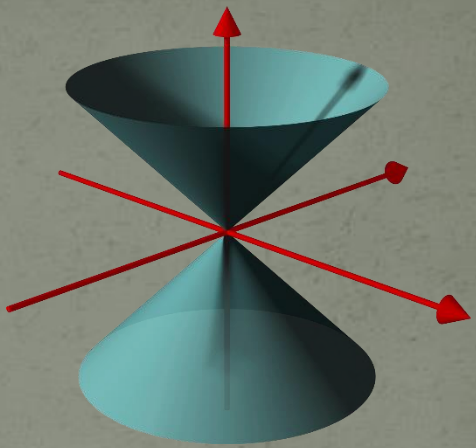
Try to make a cone in space…

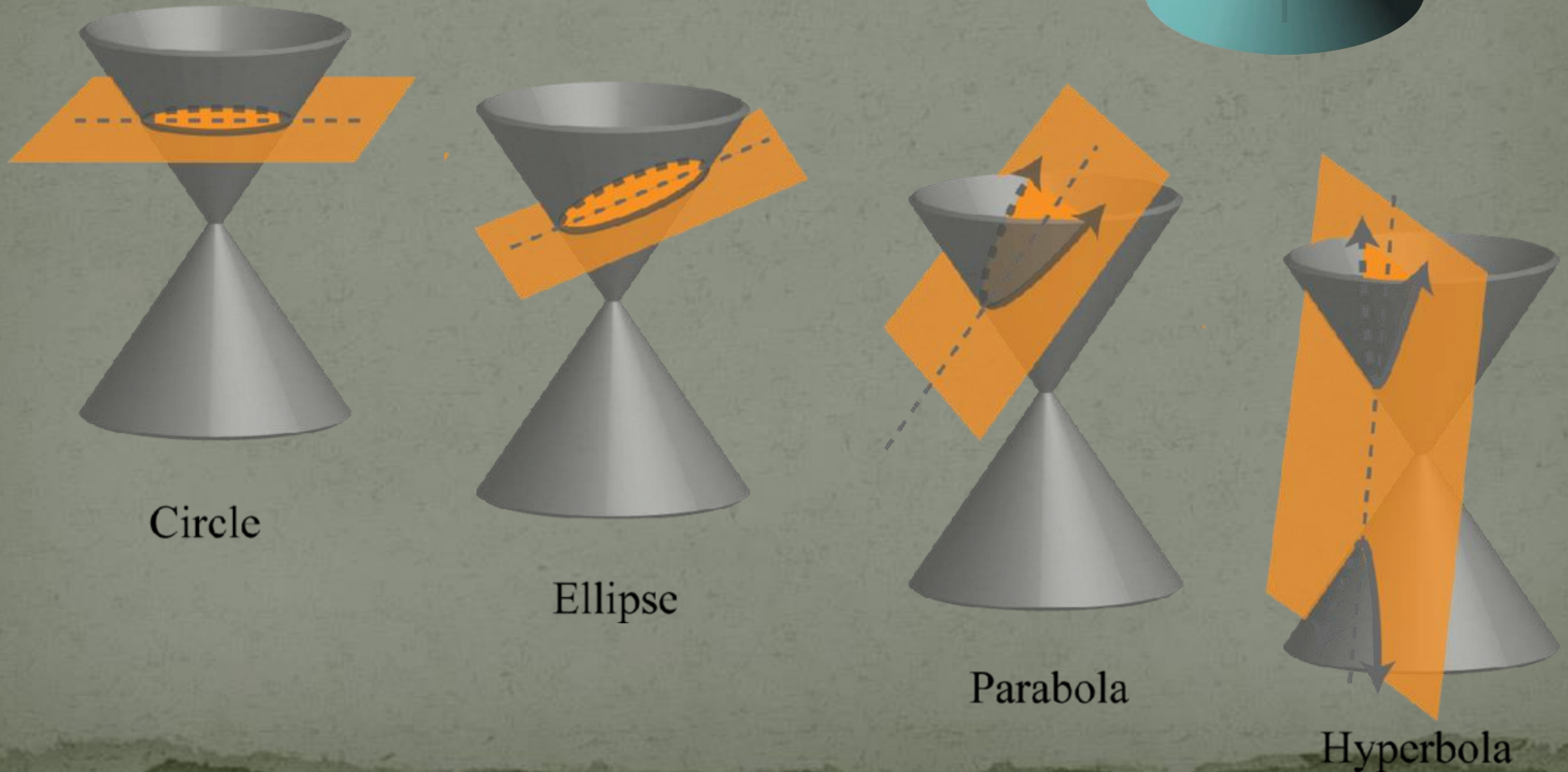




Slicing the cone – different results!

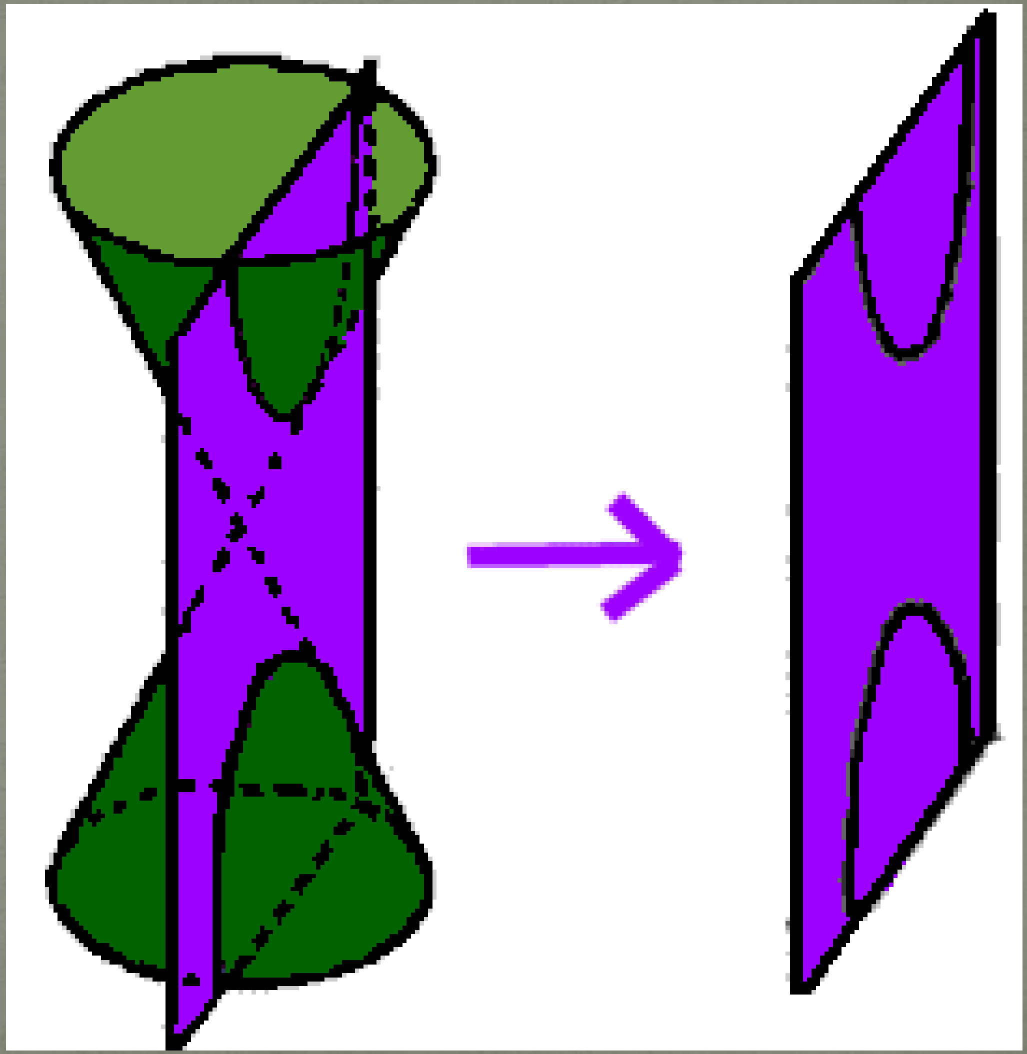
Now if we slice the cone in different ways…





Concentrate on just one slice…

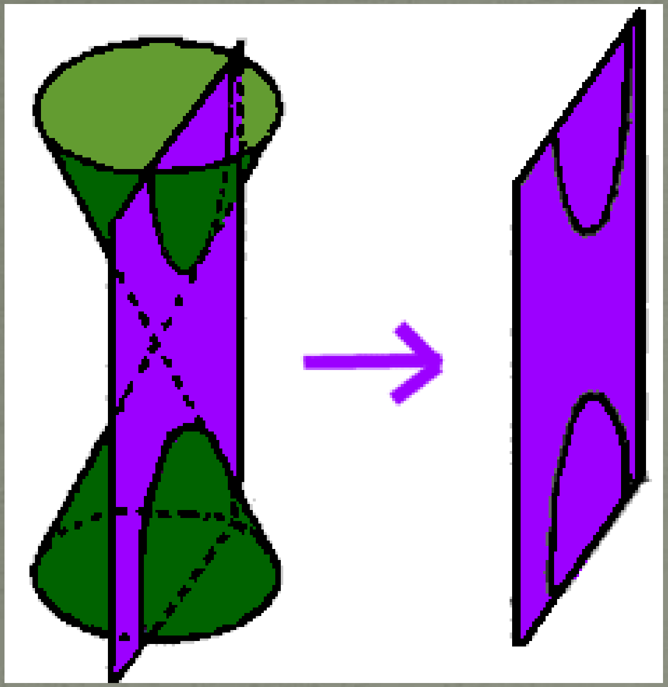
Let's look more carefully at just one particular slice…

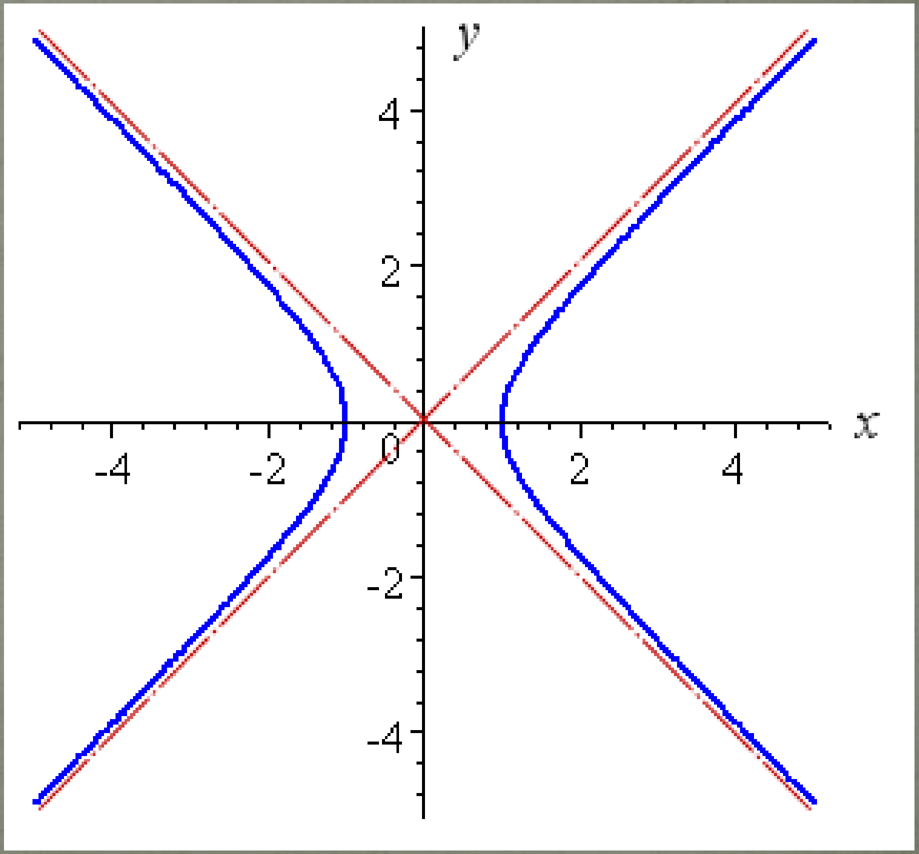


Hyperbolas!

Think about

$x^2 - y^2 = 1$





what functions could model this shape?

Back to those intriguing infinite polynomials…

$$e^{x} = 1 + x + \frac{1}{2}x^{2} + \frac{1}{3!}x^{3} + \frac{1}{4!}x^{4} \ldots$$

Also remember those other ones…

$$Cos\,(x) = 1 - \frac{1}{2}x^{2} + \frac{1}{4!}x^{4} - \cdots$$

$$Sin(x) = x - \frac{1}{3!}x^{3} + \frac{1}{5!}x^{5} - \cdots$$

Back to those intriguing infinite polynomials!

$$e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 \ldots$$

So yesterday we found a new function…

$$Cosh\,(x) = 1 + \frac{1}{2}x^2 + \frac{1}{4!}x^4 + \cdots$$

Neat… and so this turns out to be equal to $\frac{e^x + e^{-x}}{2}$ and it's also equal to Cos(ix)

Back to those intriguing infinite polynomials!

$$e^{x} = 1 + x + \frac{1}{2}x^{2} + \frac{1}{3!}x^{3} + \frac{1}{4!}x^{4} \ldots$$

So what would happen if we played with Sin(x)… what is Sin(ix)?

$$Sin(x) = x - \frac{1}{3!}x^{3} + \frac{1}{5!}x^{5} - \cdots$$

what about iSin(ix)… or… -iSin(ix)…!

and this equals $\frac{e^{x} - e^{-x}}{2}$ = Sinh(x)

New “Trig” functions!

$$Cosh\,(x) = 1 + \frac{1}{2}x^2 + \frac{1}{4!}x^4 + \cdots$$

$$Sinh(x) = x + \frac{1}{3!}x^3 + \frac{1}{5!}x^5 + \cdots$$

Neat, and so… **cosh(x) + sinh(x) = ?**

what about… **cosh(x) - sinh(x) = ?**

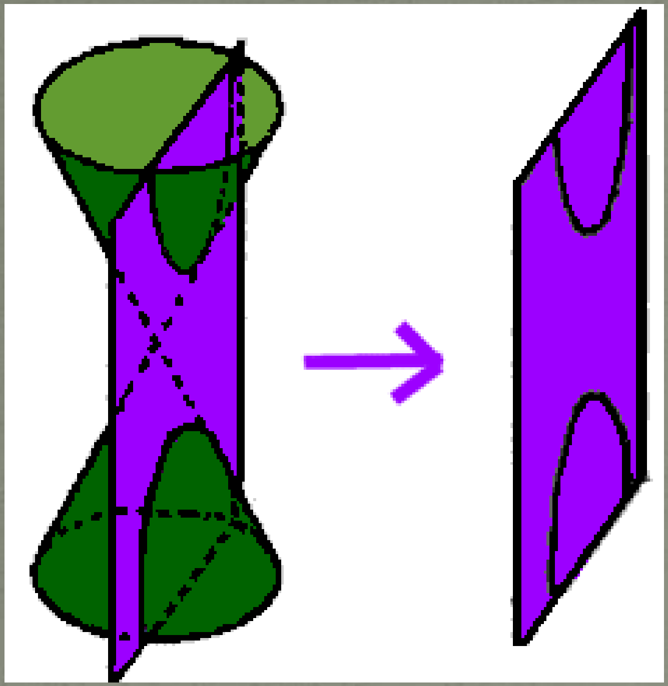
so what is … **$\cosh^2(x) - \sinh^2(x)$ = ?**

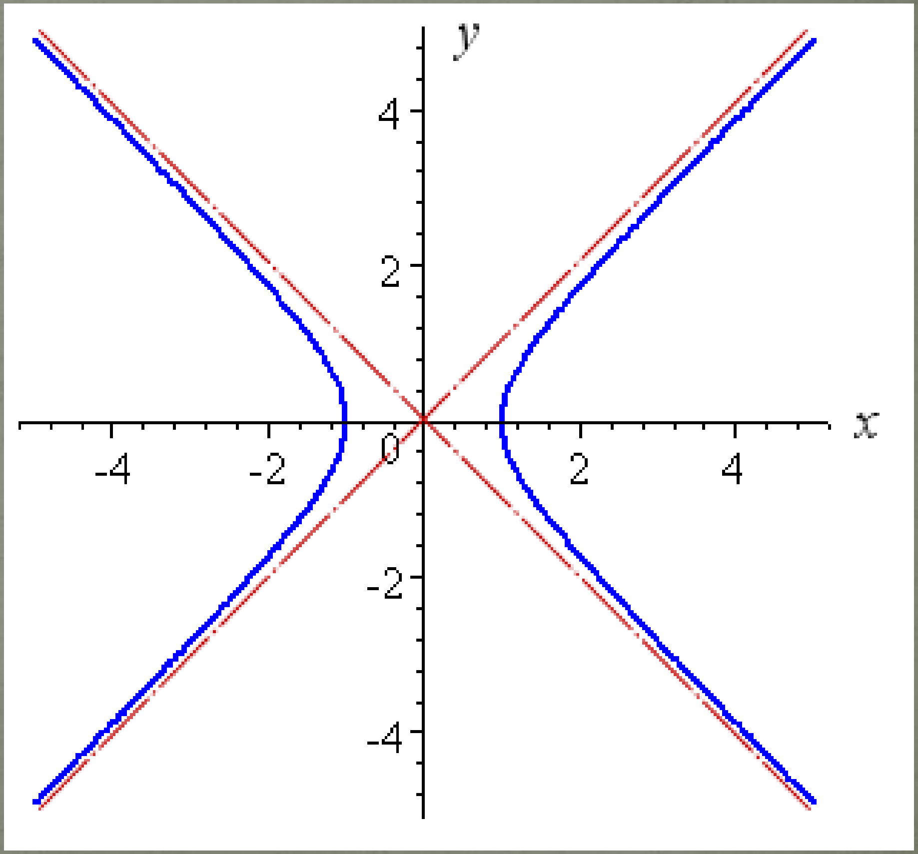
recall $e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 \ldots$

Aha - Hyperbolas!

Think about

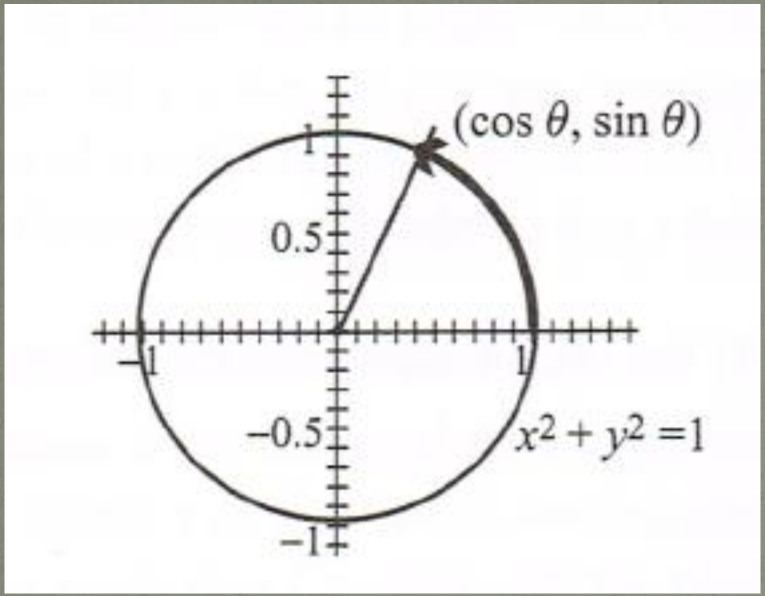
$x^2 - y^2 = 1$



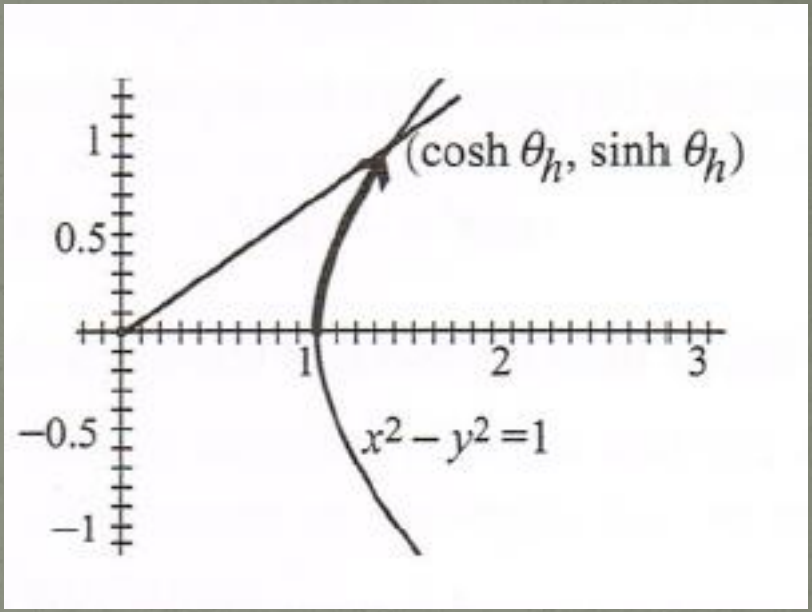


what functions could model this shape?

Hyperbolic trigonometry!



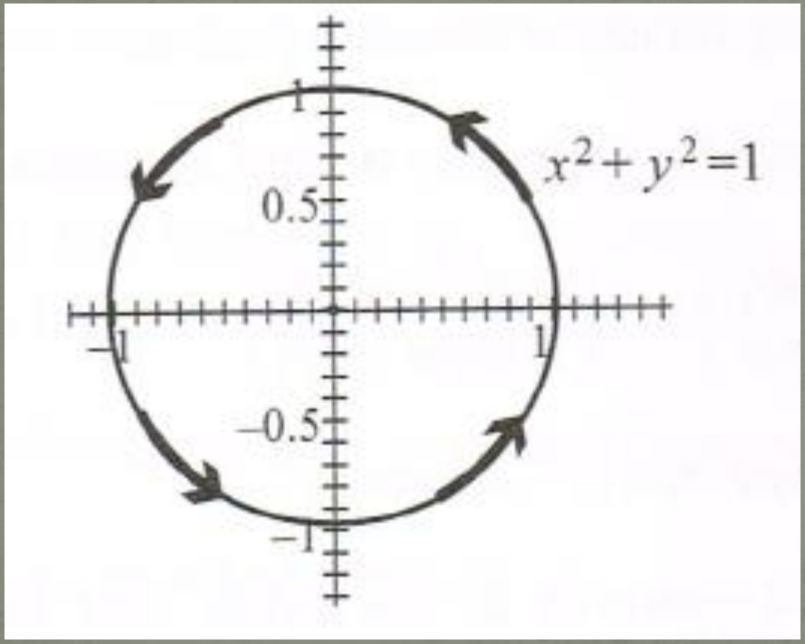
Old! Circular Trigonometry

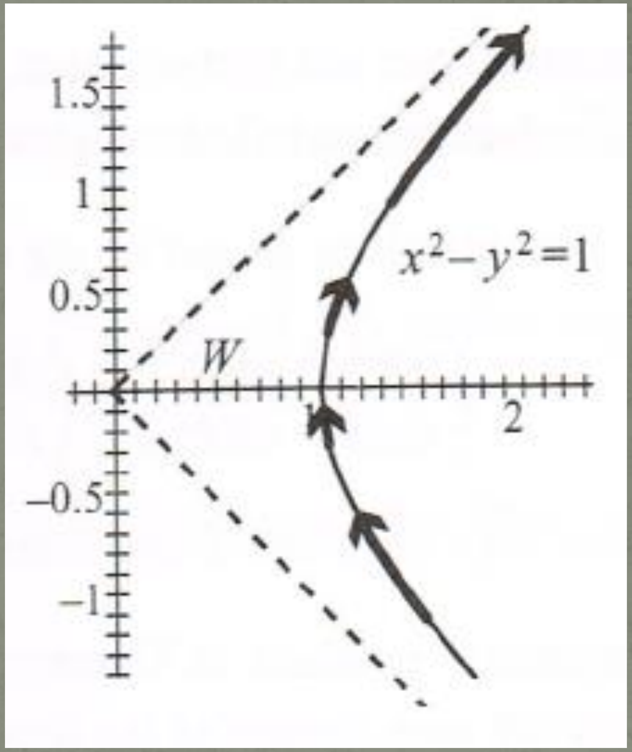


New! Hyperbolic Trigonometry

Hyperbolic angles

could go with making the angle equal to the arc length…



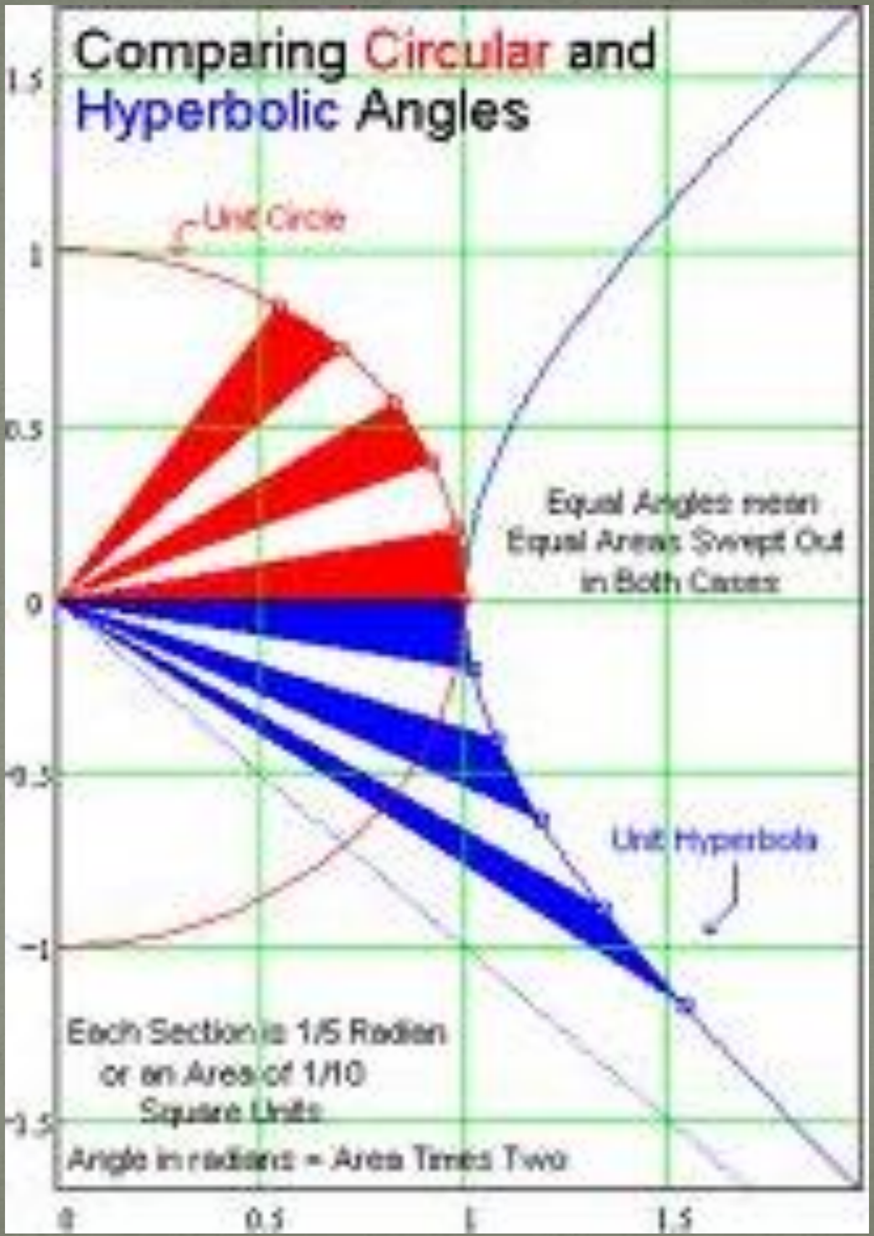


but that's problematic for a few reasons…

Hyperbolic angles

So instead we go with angle related to the ***area*** *of the sector that's swept out.*

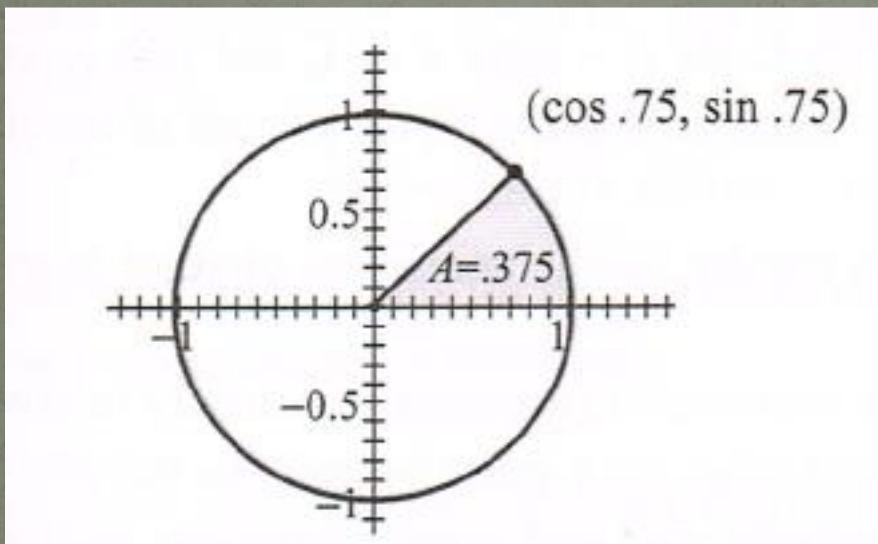
These give the same angle in circular trig, but not for hyperbolic trig

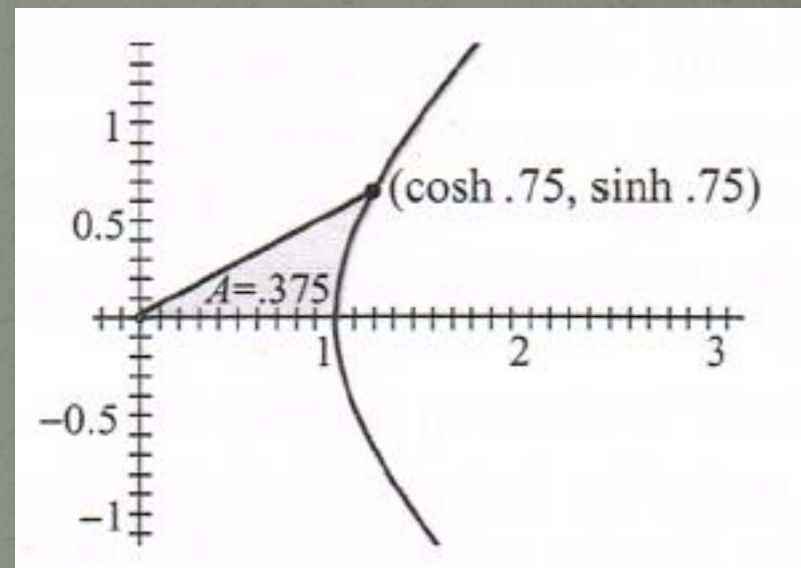


Finding Cosh(.75) and Sinh(.75)

Sweeps out half the sector area…

thus it sweeps out area .375





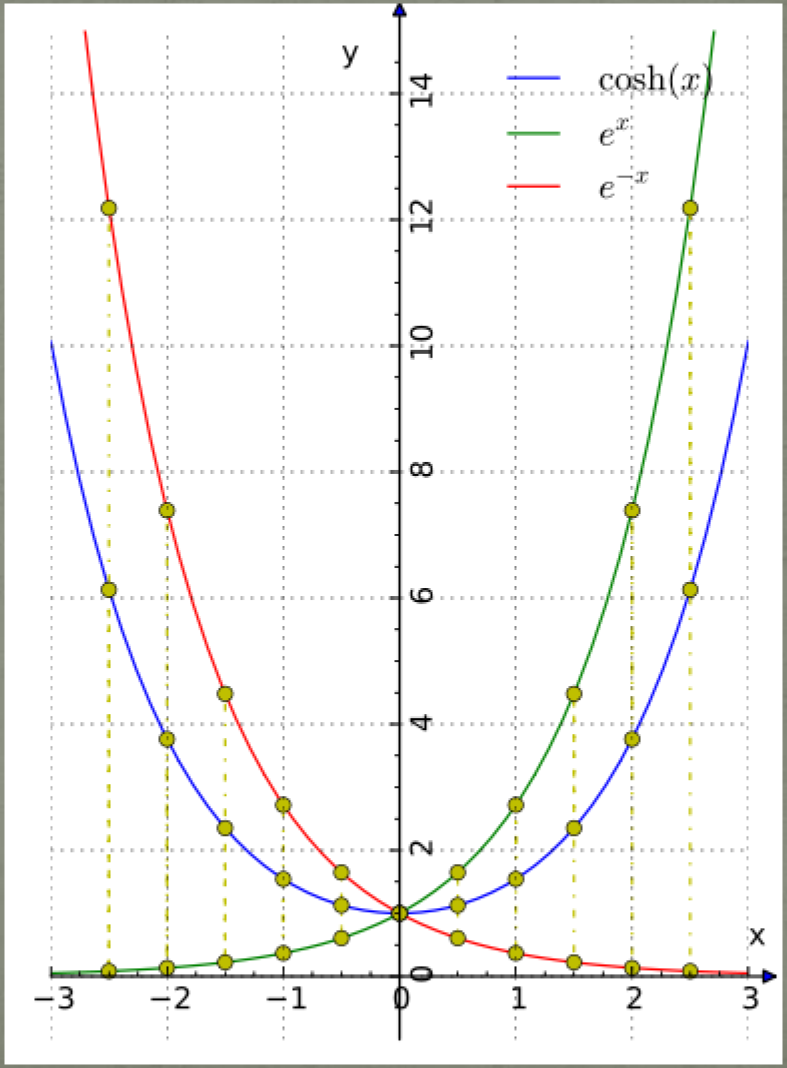
First, in circular trigonometry...

Note that one has to take half the area as the circumference of the unit circle is 2π whereas the total area of a unit circle is just π.

Plotting the Hyperbolic Cosine

Look at the definition of cosh in terms of the exponential function

So is cosh odd or even?



Plotting the Hyperbolic Cosine

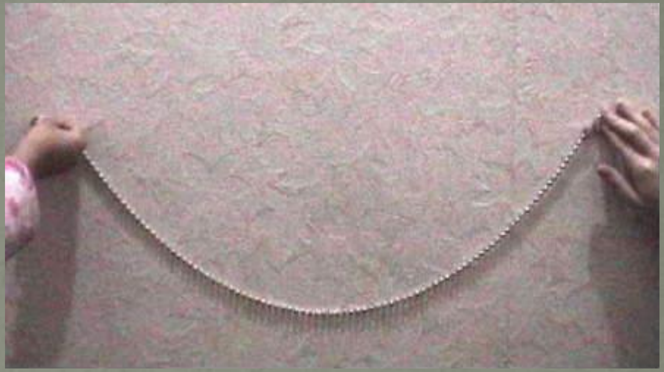
Curiously the cosh function is in the shape of a catenary

modeling with cosh curves is clearly useful in engineering!

Catenary versus Parabola…

a famous catenary!

Note that an unsupported hanging cable is in the form of a catenary

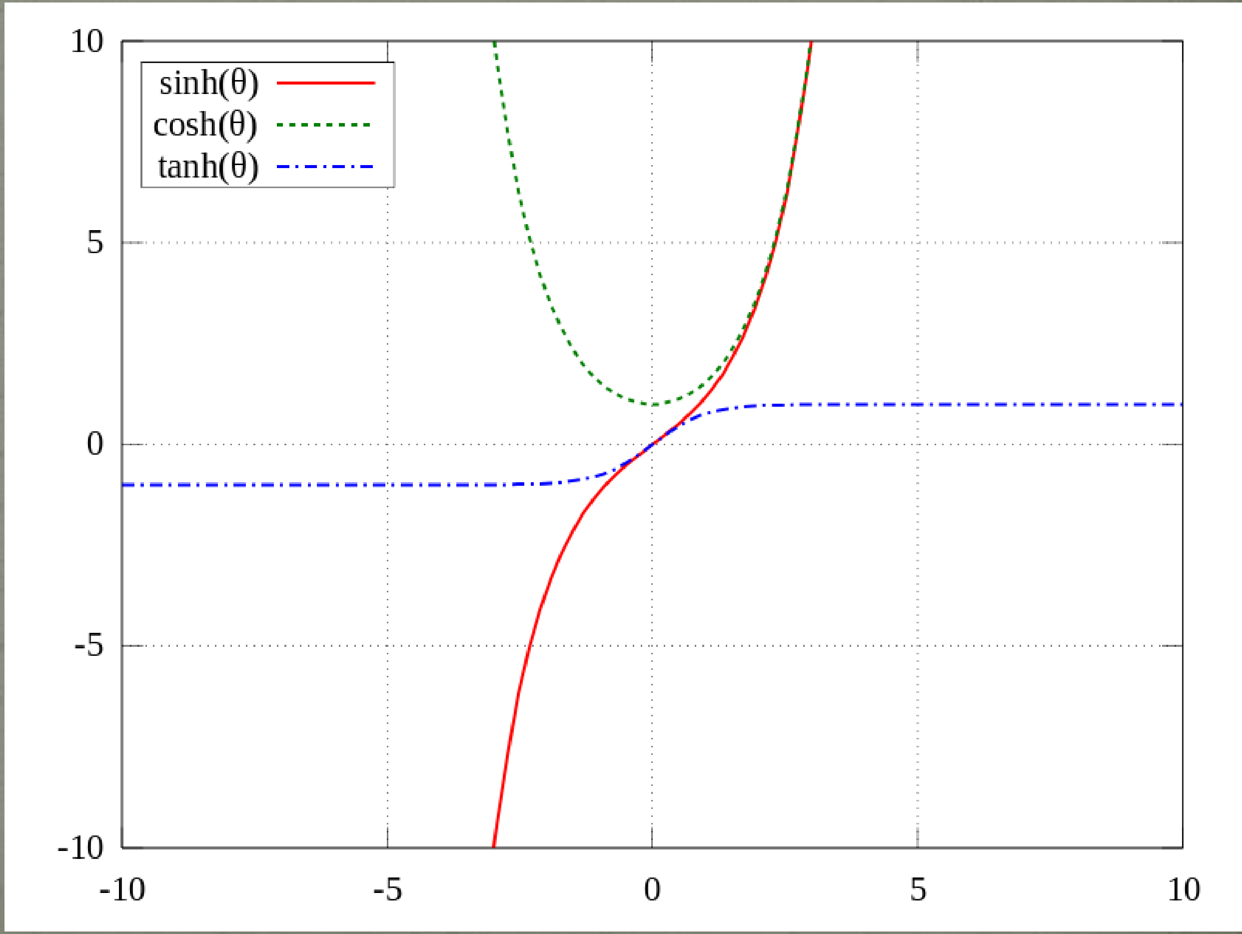


but a stressed (loaded) cable forms a parabolic shape instead

Plotting Other Hyperbolic Functions

What are the domain and range of cosh and sinh?

How about the domain and range of tanh?



Comparing Circular and Hyperbolic Trigonometry

First, the classic…

$$\cos^2(x) + \sin^2(x) = 1$$

Which equates to what in hyperbolic trigonometry?

$$\cosh^2(x) - \sinh^2(x) = 1$$

Comparing Circular and Hyperbolic Trigonometry

What about derivatives?

$$\frac{d}{dx}(\cos(x)) \qquad \frac{d}{dx}(\sin(x))$$

Which ends up as what in hyperbolic trigonometry?

$$\frac{d}{dx}(\cosh(x)) \qquad \frac{d}{dx}(\sinh(x))$$

Comparing Circular and Hyperbolic Trigonometry

And finally what about those angle sum formulas?

$\cos(A+B) =$

$\cos(A)\cos(B) - \sin(A)\sin(B)$

Now what happens to this in hyperbolic trigonometry?!

$\cosh(A+B) = ?$ $\sinh(A+B) = ?$