Math 305

Advanced Algebra and Trigonometry!

Complex Trigonometry!

from the world's most famous formula?!

Eleventh Class – Monday, July 14^{th}

- POTD
  - *But what does it mean?!*
- 2 x 2 =4, so what does *w* times *z* equal?!
  - *your conjectures…*
  - *(and explanations!)*
- Matrix transformations and i?
  - *a visual approach of the complex number line*
  - *…continued!*
- And now, time for more trig ids!
  - *some oldies…*
  - *and then create your own!*

POTD – symbol manipulation!

Remember those generating functions, such as

$$\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + \cdots?$$

Now find a generating function that equals its own derivative(?!)

$$1 + Ax + Bx^2 + Cx^3 + Dx^4 \ldots?$$

What function might this represent?

Some intriguing infinite polynomials!

$$e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 \ldots$$

Playing similar games one can find…

$$Cos\,(x) = 1 - \frac{1}{2}x^2 + \frac{1}{4!}x^4 - \cdots$$

$$Sin(x) = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \cdots$$

Can you get e^x by combining Cos and Sin?!

$$e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 \ldots$$

$$Cos(x) = 1 - \frac{1}{2}x^2 + \frac{1}{4!}x^4 - \cdots$$

$$Sin(x) = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \cdots$$

Okay, so now compare

e^{ix} *with* **cos(x) + i sin(x)***!*

POTD – round two!

So now if you think you've got it…
compare e^{ix} with $\cos(x) + i \sin(x)$

by thinking about their ratio…

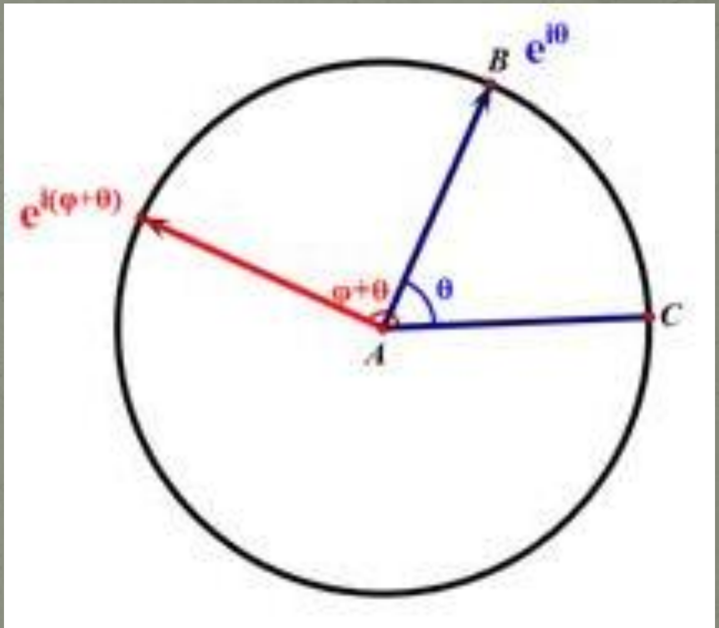
This is still a function of x, but a pretty simple one, right?

What is its derivative?(!)

POTD – now put it to work!

So if $e^{ix} = \cos(x) + i\sin(x)$

then where “does it live”?



now what is

$e^{i\varphi}$ *times* $e^{i\theta}$*?*

More to the point, what does this tell you about trigonometry?

Trig Identities!

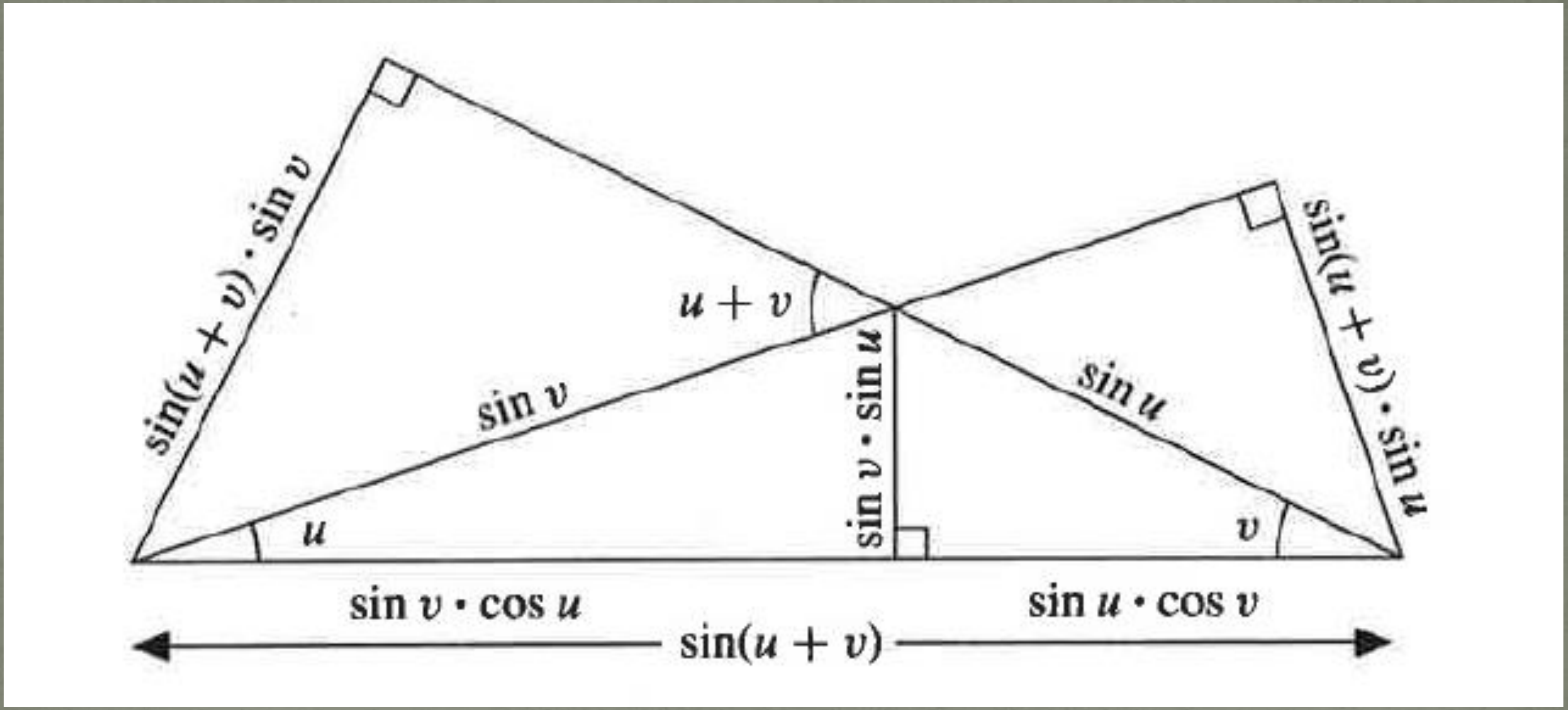
By breaking up $e^{i\varphi}$ times $e^{i\theta}$ into real and imaginary parts we get the famous *addition formulas* for Cosine and Sine!

Cos(A + B)
= Cos(A)Cos(B) – Sin(A)Sin(B)

Sin(A+ B)
= Sin(A)Cos(B) + Cos(A)Sin(B)

Or if you prefer pictures…

Contemplate the following!



not quite obvious what's going on here!

POTD – and now the famous…!

Now if $e^{ix} = \cos(x) + i \sin(x)$

then what is $e^{i\pi}$*?*

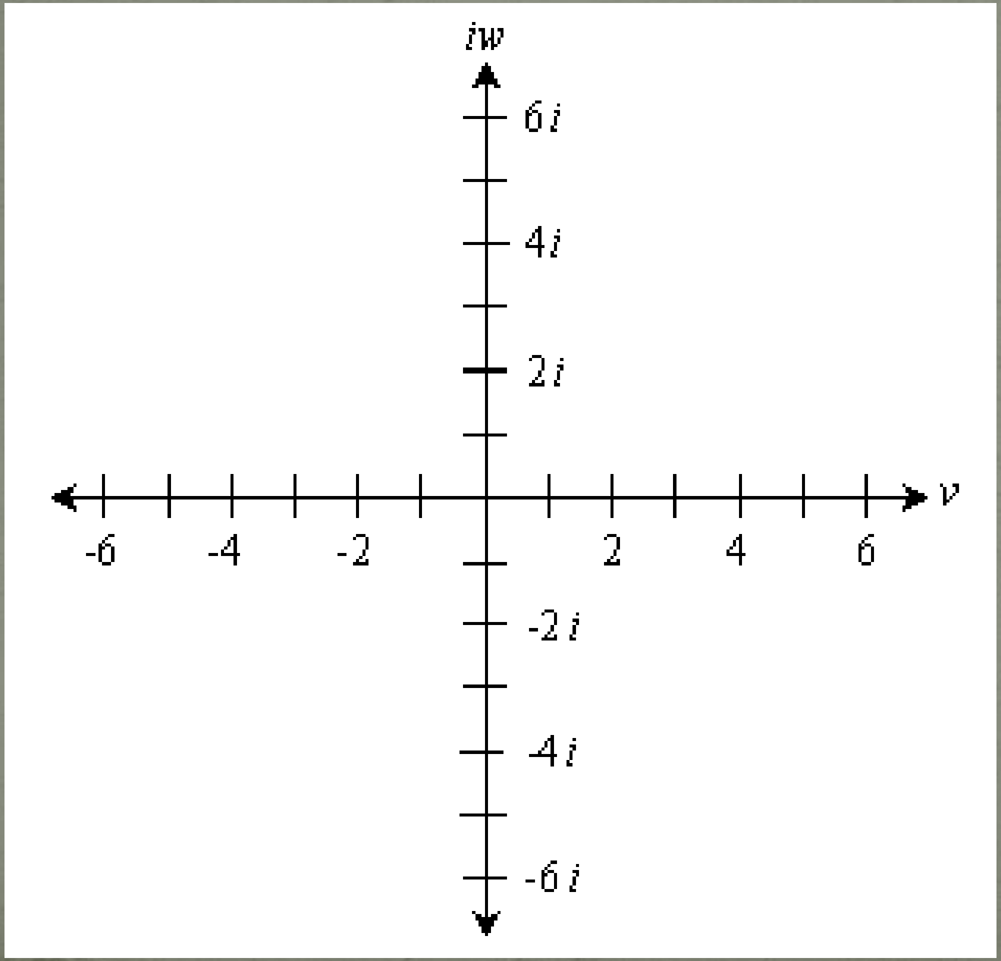
and while we're here…

okay, now write i *as* $e^{i\theta}$

…so what is i^i?

Back to our Complex Number Line!

The “complete” number line!



where all the action happens!

Back to -1 …

thinking back to multiplication by -1…

$$\begin{bmatrix} a \\ b \end{bmatrix} = a\begin{bmatrix} 1 \\ 0 \end{bmatrix} + b\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$\begin{bmatrix} -a \\ -b \end{bmatrix} = a\begin{bmatrix} -1 \\ 0 \end{bmatrix} + b\begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}$$

Last time … finding "i"…!

so now…

if multiplication by -1 is equivalent to multiplication by

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

then find a matrix A such that

$$A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

wow! a quadratic equation… in matrices!

Aha!

so multiplying by **1** is equivalent to

the matrix transformation $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

and **i** is equivalent to $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

so multiplying by a + bi is equivalent to the matrix transformation…?

$$a \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + b \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

Multiplying complex numbers ...as matrices!

Now try out $\begin{bmatrix} a & -b \\ b & a \end{bmatrix} \begin{bmatrix} c & -d \\ d & c \end{bmatrix}$

and we got $\begin{bmatrix} ac - bd & -ad - bc \\ ad + bc & ac - bd \end{bmatrix}$

which is equivalent to...

$(a + bi)(c + di)$

$= (ac - bd) + i(ad + bc)$

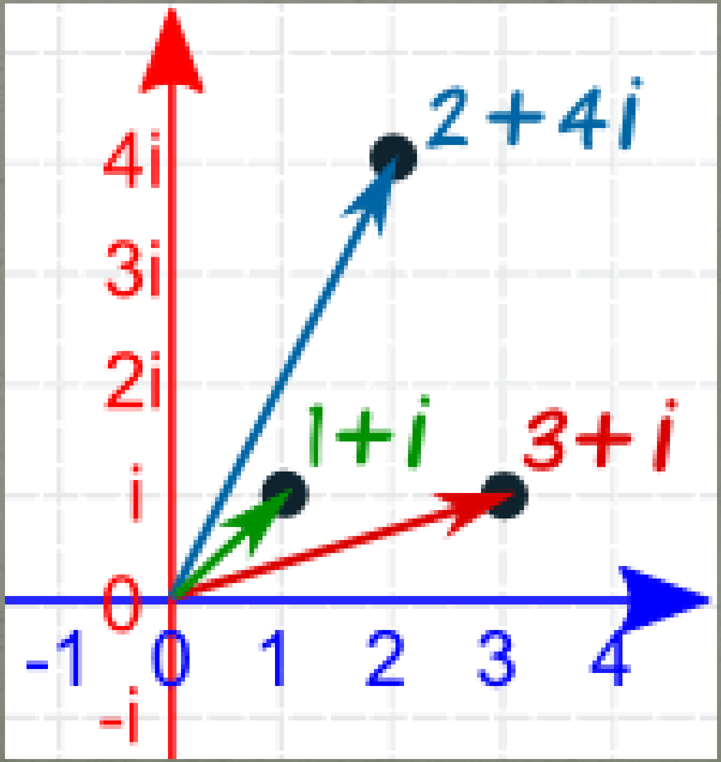
Let's get visual - geometrically…

Going back to our
complex number line… *Argand!*

what does the product

$$\begin{bmatrix} a & -b \\ b & a \end{bmatrix}\begin{bmatrix} c & -d \\ d & c \end{bmatrix}$$

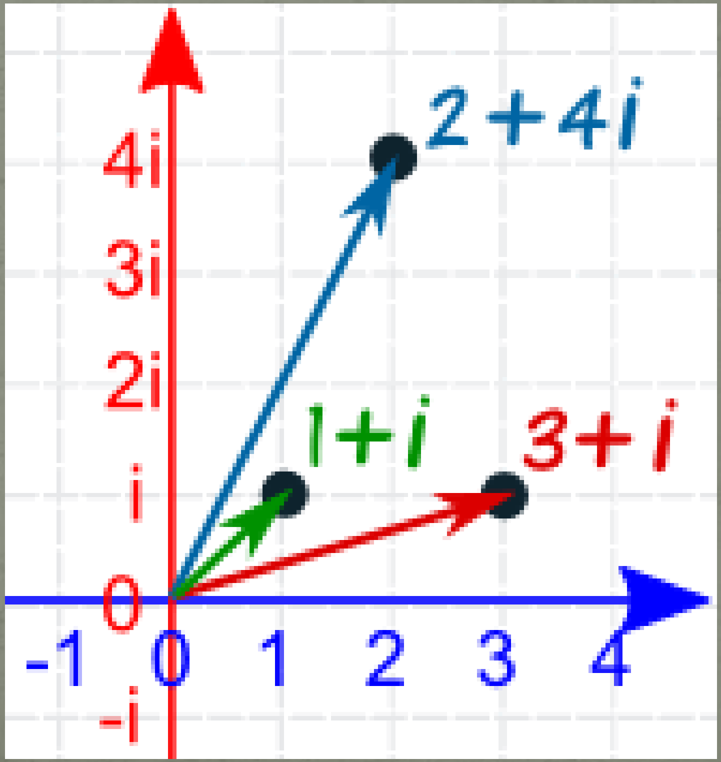
correspond to?



Let's get visual - geometrically…

Going back to our complex number line… *Argand!*

If $z = a+bi$
and $w = c + di$
then how does
wz *compare*
to w *and* z*?*



Midterm solution review…

MATH S305, Summer 2014
Advanced Algebra
Midterm Test

You have the rest of the class time to take this midterm (i.e. until 11:30am). Try to pace yourself by keeping track of how many problems you have left to go and how much time you have left. Don't feel obliged to answer the problems in any particular order - you should move on to another problem if you find you're getting stuck on one problem. There are 7 problems on the test, for a total of 50 points.

Please be sure to write down all your work for each of the problems so that even if you don't get the correct final answer you can still receive partial credit for the work that you have done. Remember that the explanations for your answers are as important as the answers themselves so be sure to write down your reasoning wherever possible – note that unjustified answers might not receive full credit.