

Welcome to Math S305!

Math for Teaching Advanced
Algebra and Trigonometry

*if Math is the
language of Science...*

*then Algebra is the
language of Math!*

First Class – Thursday, June 26, 2014

- Introduction to course
 - *Wait, more algebra?!*
- Course nuts and bolts
 - *aka “the fine print”*
- Introducing each other
 - *what makes you unusual?!*
- First problem solving session
 - *how many sets of coins can you make?*

... Good Luck!

Find the value of $P(99)$ if

$$P(x) = x^3 + 3x^2 + 3x + 2014$$

Find two factors of 99,999,991

(both positive integers greater than 1)

Determine the value of

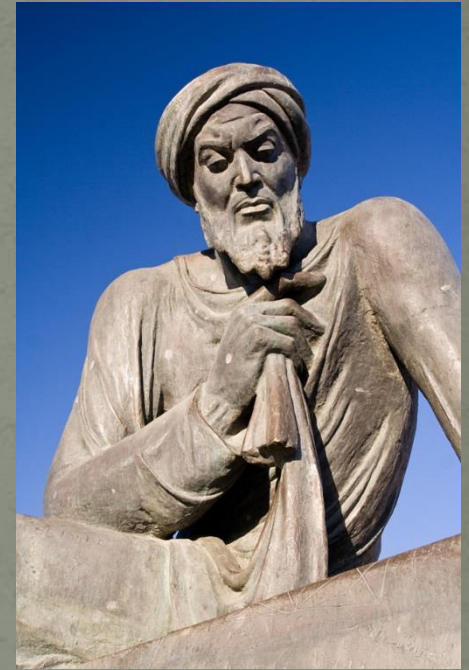
$$\cos \left[\frac{\pi}{2} + \frac{\pi}{4} + \frac{\pi}{8} + \frac{\pi}{16} + \dots \right]$$

...and the value of $\cos \left[\frac{\pi}{2} \right] - \cos \left[\frac{3\pi}{2} \right] + \cos \left[\frac{5\pi}{2} \right] - \dots$

Algebra... an interesting word!

From the mists of time...

“Algebra” itself comes from **al-ğabr** (Arabic), associated with a document from about 820 written by **Muhammad Al-Khwarizmi** (a Persian mathematician).



The title of his work was ***Kitāb al-muḥtaṣar fī ḥisāb al-ğabr wa-l-muqābala***

“The Compendious Book of Calculation by Completion and Cancellation”

... or “Restoration and Balancing”

Restoration...

Jabr is used when we go from something like $x - 4 = 8$ to $x = 12$. The left-side of the first equation, where x is reduced by 4, is "restored" or "completed" back to x in the second equation.

and Balancing...

Muqabalah occurs when we go from something like $x + y = y + 7$ to $x = 7$ by "cancelling" or "balancing" the two sides of the equation.

What did Algebra originally look like?

Thymaridas (around 400 BCE) wrote that:
(this is known as the “*bloom of Thymaridas*”!)

If the sum of n quantities be given, and also the sum of every pair containing a particular quantity, then this particular quantity is equal to $1/(n - 2)$ of the difference between the sums of these pairs and the first given sum...

...Huh?

In modern terms...

If $x + x_1 + x_2 + \dots + x_{n-1} = s$

and

$$x + x_1 = m_1$$

$$x + x_2 = m_2 \dots$$

$$x + x_{n-1} = m_{n-1}$$

then

$$x = \frac{(m_1 + m_2 + \dots + m_{n-1}) - s}{n - 2} = \frac{(\sum_{i=1}^{n-1} m_i) - s}{n - 2}$$

that's a bit better... isn't it?

Modern notation...(?!)

Diophantus (Greek, lived around 250 CE), was one of the first to use symbols for unknown numbers, as well as abbreviations for operations and relationships

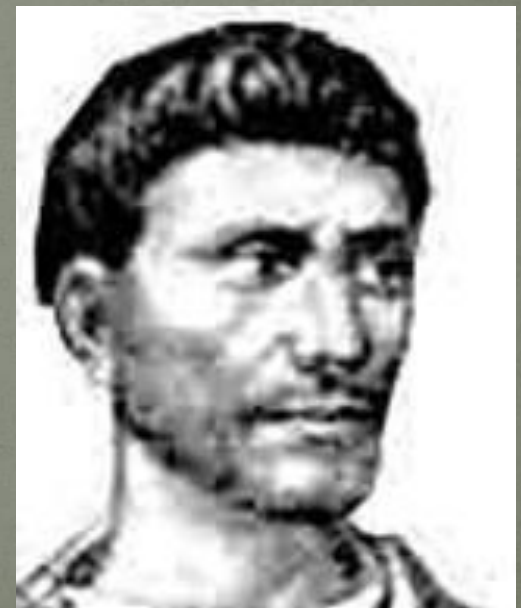
but even this turned out not quite the same thing as our modern use of symbols, however, as you'll see!

For example for the following equation,

$$x^3 - 2x^2 + 10x - 1 = 5$$

Diophantus would have written:

Yikes! $\kappa^{\gamma} \bar{\alpha}\zeta \bar{\Gamma}\eta \Delta^{\gamma} \bar{\beta} \mathbf{M} \bar{\alpha}'\iota\sigma \mathbf{M} \bar{\varepsilon}$

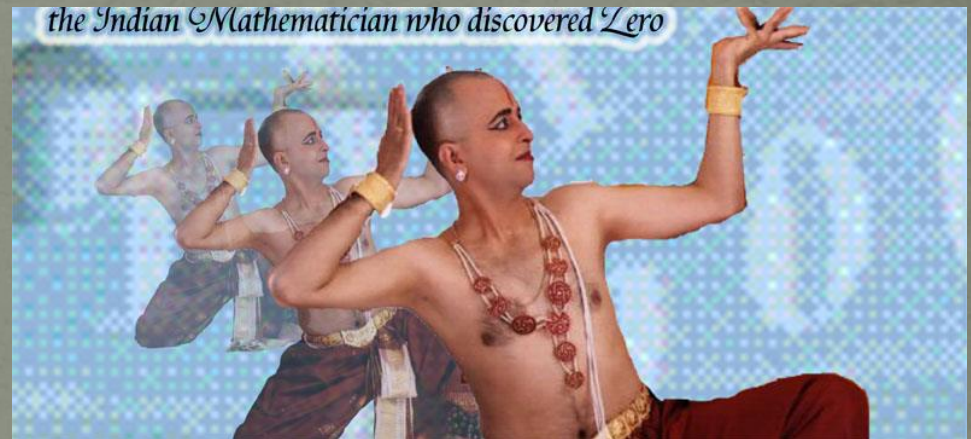


and now the magic decoder ring...

Symbol	Representation
$\bar{\alpha}$	represents 1
$\bar{\beta}$	represents 2
$\bar{\varepsilon}$	represents 5
$\bar{\iota}$	represents 10
ς	represents the unknown quantity (i.e. the variable)
ἴσ	(short for ἴσος) represents "equals"
Ϡ	represents the subtraction of everything that follows it up to ἴσ
M	represents the zeroth power of the variable (i.e. a constant term)
Δ^Y	represents the second power of the variable, from Greek δύναμις, meaning strength or power
K^Y	represents the third power of the variable, from Greek κύβος, meaning a cube
$\Delta^Y\Delta$	represents the fourth power of the variable
ΔK^Y	represents the fifth power of the variable
$K^Y K$	represents the sixth power of the variable

Other famous (?!) Algebraists

Aryabhata – an Indian mathematician (476–550 CE) – was one of the first to follow what can be considered “modern conventions” for solving equations (as opposed to using special approaches for individual cases)



He showed, for instance, that the sum of the first n squares...

$$1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

and that (curiously!) the sum of the first n cubes is equal to the square of the sum of the first n numbers...(?!)

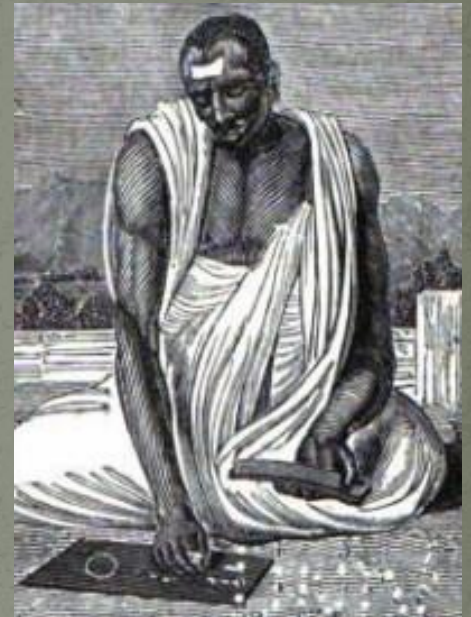
Other famous Algebraists

Brahmagupta (597 – 668 CE),
another Indian mathematician –
wrote the **Brahma Sphuta Siddhanta**
Correctly Established Doctrine of Brahma

Brahmagupta was the first to give a general solution to the linear Diophantine equation $ax + by = c$, where a , b , and c are integers.

Unlike Diophantus who only gave one solution to an indeterminate equation, Brahmagupta gave *all* integer solutions

He was probably influenced by earlier Greek mathematicians, as well as familiar with works by the Babylonians.



Other famous Algebraists

And finally, **Zhu Shih-chieh** (a Chinese mathematician working in the late 13th century CE) wrote the **Ssy-yüan yü-chien** (in 1303) or *The Precious Mirror*, this work is considered the peak of development of Chinese algebra.



The *Precious Mirror* opens with a diagram of the Pascal's Triangle, using a round zero symbol, but Chu Shih-chieh didn't take credit for the triangle, given that the same triangle, without the zero symbol, appears in Yang Hui's work from 1261, but without the zero symbol

On to our Modern Algebra!

*Why have Algebra 2? Why more algebra?
- it's often just a grab-bag of topics
as a filler before precalc/calc*

Let's put Algebra back in its rightful place!

Three main goals for Math 305:

- 1) Recognize algebra's place in the building of calculus*
- 2) Expand our knowledge of new "number" systems*
- 3) Provide a stepping stone to "formal mathematics"*

And now for the fine print...

**Harvard University Extension School
Math S305 Mathematical Connections: Advanced Algebra and Trigonometry
Course Information Summer, 2014**

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Classes: daily, 9 to 11:30 in Sever Hall room 106
– the first class is on Thursday, June 26th, final is on Friday, July 18th

Course Description and Goals: Algebra is often considered the language of mathematics for good reason. In this class we continue the journey that began for many of you in Math E-303 (Math for Teaching Algebra), exploring this rich, fascinating subject, taking on topics involving the algebra of polynomials, such as the division algorithm for polynomials, and touching on the idea of representing functions using infinite polynomials.

Backdoor(?) to Calculus!

Calculus is often said to be
based on the study of limits...

...but is that the only way to get to calculus?

**Time to
introduce
Delta!**



Delta at work!

Suppose you have the following input/output table for a function...

input	output	Δ
0	1	3
1	4	3
2	7	3
3	10	3
4	13	3
5	16	3

*What's
a possible
polynomial
function
matching
this table?*

Finding a mystery function...

Suppose the following gives an input/output table for a polynomial function...

input	output	Δ	Δ^2
0	1	-1	4
1	0	3	4
2	3	7	4
3	10	11	4
4	21	15	
5	36		

Expanding Our World!

Algebra is just about solving for “x” ...

right?

Actually, algebra is a great place to study new number systems

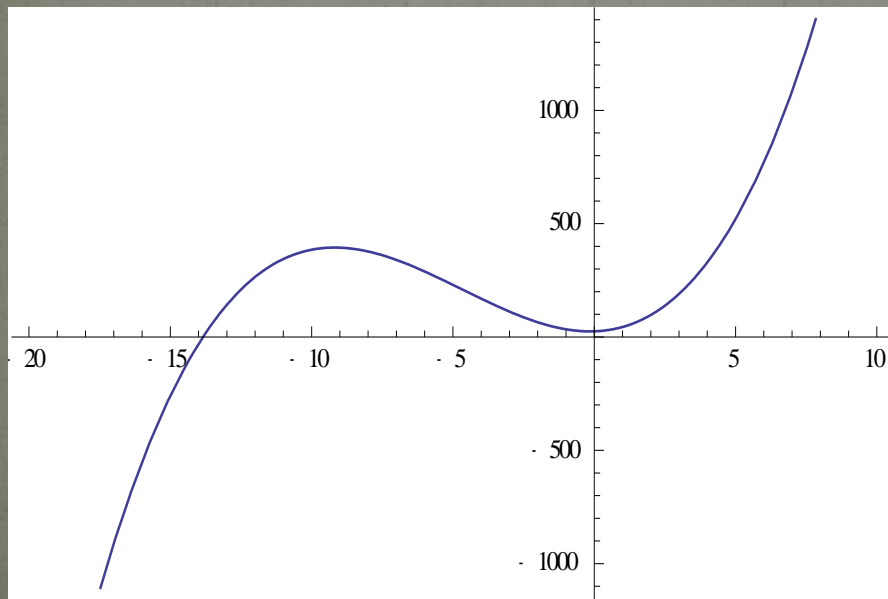
– *go beyond the “natural”!*



Time to get Formal!

Algebra is abstract math,
right?

Isn't it abstract enough?



What does
 $x^3 + 14x^2 + 4x + 25$
mean to you?