Here’s a way that we worked on finding a closed formula for the Fibonacci sequence. First, just a quick reminder of the recursive definition for the sequenc – it can be given as $a_1 = 1$, $a_2 = 1$, and $a_n = a_{n-1} + a_{n-2}$
First we saw (using a plot in Mathematica) that the Fibonacci sequence is

1) not geometric (and not arithmetic either), but that
2) it looks geometric "in the long run"

Next we asked whether there are any geometric sequences that satisfy the recursion formula $a_n = a_{n-1} + a_{n-2}$ and this led us to solve the equation

$$X^2 = X + 1.$$

which led us to Phi (the golden ratio), and it's buddy which I called Psi - the other root of the same equation.

Then we checked that any sequence of the form (with constants A and B):

$$a_n = A \times (\text{Phi})^n + B \times (\text{Psi})^n$$

would also satisfy the same recursive formula. Then, finally, calculating for constants A and B so that $a_1 = 1$ and $a_2 = 1$ led us to finding a closed formula for a_n (and you can read more about this result if you check online for Binet's formula)